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Social networks and non-market valuations

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ABSTRACT

This paper considers the role of social networks in the non-market valuation of public goods. In the model individuals derive utility both from their own direct enjoyment of the public good and from the enjoyment of those in their network. We find that network structure almost always matters, both for utility and for valuation. The network increases aggregate valuation when it assigns higher importance, that is, stronger connections, to individuals with higher private values for the public good. The model provides a theoretical foundation for the idea of opinion leaders who have disproportionate influence over their communities. Specifically, opinion leaders are individuals assigned high importance by the network, and projects favored by opinion leaders tend to be favored by the network as a whole. The model can also guide future empirical studies by enabling a more structural approach to non-market valuation in a socially connected group.

Introduction

For the most part, the theoretical public good valuation literature considers decision-makers in social isolation. There are two reasons why social structure might impact valuation. One is that individuals may be altruistic and care about public goods that benefit their friends even if they do not benefit themselves.[1] For example, the presence of a park might not generate any private utility for the individual, but if the park gives her friends utility and she values those friends′ utility, she might have positive willingness to pay for the park due to social utility.[2] A second reason is that people might use the public good in groups.[3] For example, someone might like going to a park, but not alone, so to get enjoyment from the park her friends must also like the park. She gets utility from going to the park with friends, but might also get utility from going with friends′ friends, and so on. Whichever the channel, altruism or joint use, the utility that one gets from the public good may be affected by friends′ utility. Furthermore, friends may behave in the same way and the utility of friends of friends may affect friends′ utility. This leads to network effects.

The purpose of this paper is to construct a model of public good valuation that can accommodate both these network effects. As argued by Jackson (2009, p. 491), “Many economic interactions are embedded in networks of relationships and the structure of the network plays an important role in governing the outcome.” As a result, network models have been

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[1] This is consistent with the finding of directed altruism by Leider et al. (2009). In their field experiment subjects allocate 52% more to close friends than to strangers in dictator games.

[2] As private utility we mean the direct (or own) utility that one receives from consuming a public good in social isolation, i.e. ignoring social effects. Social utility is the overall (or total) utility from the public good, which includes one′s private utility and (possibly) the social utility from friends.

[3] For instance, Morey and Kritzberg (2010) provide evidence that the presence of a companion changes the willingness to pay for biking trails.

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developed to explain a wide range of economic phenomena.[4] Our primary result links aggregate willingness-to-pay to network centrality. In particular, societies are willing to pay more in aggregate for a public good when that public good provides more benefit to people more central to the society. A straightforward implication is that public projects that pass the cost-benefit test and ultimately receive funding tend to favor more central agents.

To capture network interactions we use the sociometric approach in which the interaction patterns of agents are captured through the rows of a matrix (see DeGroot, 1974; DeMarzo et al., 2003). The matrix-based approach proves well-suited for the problem of computing individuals′ valuations for a public good when their valuations depend on those of others in their social network.[5] We assume that each individual has her own private value of the public good, and this private value is the one that would pertain if the public good were consumed in social isolation. Each individual′s social value of the public good may depend on how much others in her social network enjoy it, though, and so the individual′s social value of the public good may differ from her private value. We show that all network effects, including feedback effects, can be captured by a single weighting matrix so that each individual′s social value is a weighted average of the population private values. In particular, each individual′s centrality to the network is captured by the relevant column sum of the resulting weighting matrix. We refer to this column sum as an agent′s *importance*.

The thought exercise pursued in the paper involves a comparison between the valuations assigned to a public good when individuals are socially isolated and the valuations assigned when society has a network structure, holding the original vector of private valuations constant across the two settings. The paper concentrates on when, and how, the network structure impacts the social value of the public good. For individual valuation of a public good the requirement for a network effect is very weak: the individual′s social value of the public good differs from her private value if she cares about at least one agent with a different private value than her own. In other words, the structure of the network almost always impacts an individual′s valuation for a public good. The paper also identifies when the aggregate social value of the public good depends on the network, and this occurs if agents in the population are not uniformly important. If more important agents have higher private values of the public good, the population′s aggregate social valuation is higher.

The paper provides an economic foundation to a widely used idea in the other social sciences, that of an opinion leader whose position in a community makes him or her instrumental in affecting social change. This idea has been used, among other places, in such diverse areas as agricultural development (Monge et al., 2008), corporate training programs (Lam and Schaubroeck, 2000), and microfinance diffusion (Banerjee et al., 2011). Opinion leadership is clearly tied to the idea of network centrality (see Katz, 1953; Friedkin, 1991). However, the model in this paper ties opinion leadership directly to an influence on others′ willingness to pay for a public good. The results show that this leadership is easily identified with the agents whose columns have the largest sums in the social weighting matrix. Furthermore, the paper establishes situations in which projects valued more by opinion leaders are also valued more by the entire network.

The results have important implications for policy analysis. When the network matters, sampling values from the population provides the right information for performing a cost-benefit analysis for that population, but that same sample cannot be used as the basis for cost-benefit analysis for a similar public project benefiting a different population. In other words, even when two populations are very similar, e.g. they have similar distributions of relevant socio-economic characteristics, benefit transfer cannot be done without placing restrictions about the shape of the social networks. Because of the network, one population might find it worthwhile to provide the public good while the other does not.[6]

The paper adds to the economics literature linking social preferences and public good provision. A group of papers concentrates on whether social values should be considered in cost-benefit analyses of public projects.[7] Flores (2002) and Bergstrom (2006) demonstrate that there are cases where welfare-improving public good projects would be rejected if cost-benefit analysis was based only on private values as opposed to social values. Therefore, with social preferences, a public project may be Pareto-enhancing even if the cost of the project exceeds the sum of all agents′ private values.

Our contribution to this literature involves the use of a social network structure to explore the differences between the private and social welfare generated by public good provision. In doing so, our framework is similar to that of Bergstrom (1999) and Bramoullé (2001) in which a weighting matrix distinguishes private values from social values. The paper differs from the prior literature in the manner in which others′ utility impact own utility. Bergstrom (1999) looks at a system of benevolent utility functions in which social connections automatically add to an individual′s utility. Bramoulle′s (2001) treatment also involves adding friends′ social utility to an individual′s utility, however, he allows for individuals to be envious toward other agents and, in this case, other agents′ social utilities are subtracted from own utility. Our paper uses a different utility structure so that social connections neither automatically add nor automatically subtract welfare, thereby disentangling the effects of social preferences and network structure.

[4] Network models have been used to explain labor market outcomes (see Calvó-Armengol and Jackson, 2004, 2007), risk sharing (see Fafchamps and Lund, 2003; Bramoullé and Kranton, 2007b), and opinion formation (see DeGroot, 1974; Friedkin and Johnsen, 1990; DeMarzo et al., 2003; Neilson and Winter, 2008).

[5] As discussed by Wasserman and Faust (1994, Chapter 3), the graph-theoretic approach, common in the work of Jackson and others (e.g. Jackson and Watts, 2002; Jackson, 2005; Jackson and Rogers, 2007), proves to be beneficial for modeling networks with multiple relations. The sociometric notation is, however, a simple way to model directed networks in which links between agents have different strength.

[6] This result is in line with experimental evidence that social preferences are stronger towards socially connected agents. For instance, Leider et al. (2009) distinguish baseline altruism towards strangers from directed altruism that favors friends.

[7] See Bergstrom (2006) for a review of theoretical work.

The paper contrasts with the literature on local public goods in networks. In these papers the public good has the same value to everyone, but individuals only obtain access to the public good when they are connected directly to someone who provides it. Bramoullé and Kranton (2007a) present the first model of such public goods. They show that there always exists an equilibrium in which some agents free ride, and that in some cases the most efficient equilibrium entails provision by the central agent in the network. Their model concentrates on provision, which is made interesting by the localness of the public good, while ours concentrates on valuation for global public goods in the presence of networks.[8]

The joint-use interpretation of our model provides a theoretical foundation for the empirical recreation-site choice literature. Using a choice experiment, Morey and Kritzberg (2010) demonstrate that the presence of a companion can significantly change the value of mountain bike trails. They take their large estimates of the effect of a companion on the value of trails as evidence that real world site-choice data may be influenced by social interactions. Commensurate with these findings, other empirical papers find significant effects of party size on recreational values (see Kaoru et al., 1995; Massey et al., 2006). Along the same lines, Timmins and Murdock (2007) find evidence that some congestion can be desirable. Specifically, they estimate the value of a large recreational fishing site in Wisconsin (Lake Winnebago) accounting for congestion effects, and conclude that ignoring congestion leads to an understatement of the lake′s value by more than 50%. Although these papers do not account for social networks explicitly, they provide some empirical support for our results by showing that social interactions affect valuation. Our results also inform this literature by suggesting that the strength of social ties to the companions, and not just the number of companions, affects valuation.

The remainder of the paper is organized as follows. Section 2 describes the preferences underlying the analysis, and Section 3 presents the model. Section 4 analyzes social network effects on an individual′s utility and willingness to pay for public projects. Section 5 investigates social network effects on welfare and aggregate non-market valuation. Section 6 explores the role of opinion leaders in the choice among public projects. Section 7 concludes.

Preferences

A population consists of $n \geq 2$ agents indexed by $i = 1, \ldots, n$. Agents obtain utility from the consumption of a private good x and a public good g. Utility is assumed to be quasilinear. Agent i′s overall utility is

$$V_i(x_i, g) = x_i + v_i(g), \tag{1}$$

where $v_i(g)$ is agent i′s social utility from the public good.

The public good g is exogenously provided to the entire population, without congestion, such that every agent can benefit from its consumption. There are two channels through which the provision of g can affect i′s social utility $v_i(g)$. First, agent i obtains *private* utility $u_i(g)$ from the consumption of g. This is the component of social utility that is obtained from own consumption of g and is independent of social effects. Second, agent i may care about the enjoyment of her friends and, as a result, may obtain utility from the *social* utility of friends.[9]

Friendships are represented by a social network. Formally, let agent j be a *friend* of agent i if j is directly connected to i.[10] The social network is represented by the (possibly asymmetric) row stochastic matrix $\mathbf{A}$, with dimensions $n \times n$. An element a_{ij} is positive if j is a friend of i, and zero otherwise.[11] The diagonal of $\mathbf{A}$ is equal to zero reflecting the fact that an agent is not a friend of herself.

Utility received from friends is assumed to be a weighted average of friends′ social utility of the public good, with weights determined by the rows of $\mathbf{A}$.[12] Formally, agent i′s social utility from the public good is defined as

$$v_i(g) = (1-\lambda_i)u_i(g) + \lambda_i \sum_j a_{ij} v_j(g), \tag{2}$$

where $\lambda_i \in [0, 1)$ is a parameter that reflects the extent to which social utility of friends is relevant to agent i.

The term $(1-\lambda_i)$ is the weight that agent i places on her own private enjoyment $u_i(g)$. Hence, the parameter λ_i is intuitively denoted as i′s degree of social interaction in the consumption of g. Agent i is said to be *socially isolated* if i′s social utility from the public good is not influenced by the social utility of friends. Thus, when $\lambda_i = 0$, agent i′s social utility $v_i(g)$ is equal to i′s private utility $u_i(g)$. Social isolation shuts down the social channel through which the provision of g affects i′s utility and the model simplifies to a standard utility model without network effects.

The preferences described by expression (2) can be thought of in at least four different ways. First, the good g can reflect a paternalistic public good so that individuals care about how much their network connections consume but not how much

[8] More recent research stemming from Bramoullé and Kranton (2007a) develops different network models of public goods. Newton (2010) analyzes the effect of coalitional behavior on local public goods provision. O′Dea (2010) examines the relationship between local public good provision and social network formation. Cho (2010) studies endogenous formation of networks for local public goods in sequential bargaining games. Chih (2010) incorporates interactive costs and social perception of free-rider behavior in a model of local public goods and network formation.

[9] As discussed in Section 1, altruism and group consumption are two possible reasons for the influence of friends on an individual′s social utility.

[10] In the network literature, connected agents are often referred to as neighbors.

[11] A row stochastic matrix is a square matrix of nonnegative real numbers, with each row summing to 1. Therefore, we implicitly assume that every agent has at least one friend.

[12] From i′s perspective, the intensity of the friendship between i and j is captured by a_{ij}. The element a_{ij} captures j′s influence on i′s social utility. The same friendship may have different intensities from j′s perspective such that a_{ij} may be different from a_{ji}.

their connections pay for it. This sort of specification allows for, say, parents wanting their children to consume education regardless of their children's wishes. Alternatively, the good g can represent a combination of a public good and a vector of individual tax payments to pay for it. In this case the utility functions u_i and v_i can be thought of as the utility of a *net public good*, that is, the utility of the public good less the disutility of the assigned tax payment. Such a specification would allow individuals to care about not only how much their connections enjoy the public good but also how much they pay for it. It requires a slight reinterpretation of any ensuing willingness-to-pay measure, though, because the WTP measure would compute the additional amount (above the required tax allocation) that the individual would be willing to pay for the policy. This interpretation of (2) is altruistic but not paternalistic because individuals care about payments and not just use.

Two other interpretations arise from considering the good g as one that is used jointly. Individual i gains utility from her own use but also from the enjoyment of those she uses the public good with. For example, the individual values a public park, but might value it even more if her friends and their friends also use it and enjoy it with her. This interpretation of (2) requires no altruism, and it also requires no consideration of payment by others. Instead, individual i gains utility if her connections consume at the same time as she does, whether or not they pay.

Rather than being a jointly used public good, g could be a jointly used private good. For example, the individual could consider building a swimming pool in her yard, and she receives some private utility from having the pool. She also receives utility from having friends over to enjoy the pool with her, and the social utility component of (2) allows for this. The representation does not require altruism, but instead allows for the owner to enjoy the enjoyment of others. The primary difference between the two joint-use interpretations is that aggregate willingness to pay is relevant for the public good but only individual WTP is relevant for the private good. After all, no one would ask her neighbors to help pay for the construction of a private swimming pool.

Solving the model

Let $\mathbf{v} = (v_1(g), \ldots, v_n(g))'$ denote the social utility profile of all agents. Using matrix notation, $\mathbf{v}$ can be written as

$$\mathbf{v} = (\mathbf{I} - \Lambda)\mathbf{u} + \Lambda\mathbf{A}\mathbf{v}, \tag{3}$$

where $\mathbf{I}$ is the identity matrix, Λ is a diagonal matrix with λ_i in the ith row, and $\mathbf{u} = (u_1(g), \ldots, u_n(g))'$ is the population's private utility profile. Bergstrom (1999) and Bramoullé (2001) study systems of utility functions using a similar framework: $\mathbf{v} = \mathbf{u} + \mathbf{A}\mathbf{v}$. In Bergstrom's (1999) treatment agents are benevolent, thus, $\mathbf{A}$ is a nonnegative matrix. In Bramoulle's (2001) formulation entries of $\mathbf{A}$ are either positive (if there is an altruistic social connection) or negative (if the social connection is envious). Either way, adding a friend (or enemy) to an agent's network automatically increases (decreases) that agent's utility. While this might be realistic, it prohibits disentangling the impact of a change in size of a network from a change in its shape. In Eq. (2) adding a friend for agent i requires reconfiguring the i-th row of $\mathbf{A}$, retaining the requirement that the row sum to one. Consequently adding a friend does not automatically add to utility.[13]

The network component of (3) captures the social utility obtained by straight links to friends' social utility. The influence of friends' social utility on own social utility is determined by the matrix $\Lambda\mathbf{A}$. Borrowing Bramoullé's terminology we refer to $\Lambda\mathbf{A}$ as the *primary network*. Rearranging (3) yields

$$\mathbf{v} = (\mathbf{I} - \Lambda\mathbf{A})^{-1}(\mathbf{I} - \Lambda)\mathbf{u}. \tag{4}$$

To simplify notation make $\mathbf{W} = (\mathbf{I} - \Lambda\mathbf{A})^{-1}(\mathbf{I} - \Lambda)$. Again, borrowing Bramoullé's terminology, we refer to $\mathbf{W}$ as the *induced network*. Elements of $\mathbf{W}$ correspond to circuitous links between agents emerging from links in the primary network. Links in the induced network account for the impact of friends of i's friends on i's social utility, plus the impact of friends of friends of i's friends on i's social utility, and so on. Mathematically, this arises from the Neumann series approximation $(\mathbf{I} - \Lambda\mathbf{A})^{-1} = (\mathbf{I} + (\Lambda\mathbf{A}) + (\Lambda\mathbf{A})^2 + (\Lambda\mathbf{A})^3 + \cdots)$.[14] More intuitively, consider the three-person population with

$$\Lambda = \begin{pmatrix} 0.8 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.2 \end{pmatrix} \quad \text{and} \quad \mathbf{A} = \begin{pmatrix} 0 & 0.5 & 0.5 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}.$$

Then

$$\mathbf{W} = \begin{pmatrix} 0.28 & 0.28 & 0.44 \\ 0.14 & 0.64 & 0.22 \\ 0.06 & 0.06 & 0.88 \end{pmatrix}.$$

From the matrix $\mathbf{A}$ we see that agent 1 is friends with agents 2 and 3 (because a_{12} and a_{13} are both positive) but agents 2 and 3 are not friends with each other (because $a_{23} = a_{32} = 0$). Nevertheless, because agent 2 cares about agent 1's utility which in turn depends on agent 3's utility, in the end agent 2 places weight on agent 3's utility and $w_{23} = 0.22 > 0$. The same

[13] A second difference in our model is that network effects influence only one type of good (public good) but not the other type of good (private good).
[14] See Meyer (2000).

reasoning explains why $w_{32}>0$ even though $a_{32}=0$. The rationale for $w_{23}>w_{32}$ is that $\lambda_2>\lambda_3$, so that agent 2 places more weight on others' well-being than agent 3 does.

It follows from (4) that agent i's social utility can be expressed as a function of the elements of the private utility profile $\mathbf{u}$. The following expression represents the social utility function of agent i and corresponds to the ith row of system (4):

$$v_i(g)=\sum_j w_{ij}u_j(g), \tag{5}$$

where w_{ij} is an element of the square matrix $\mathbf{W}$. Lemma 1 formally describes i's social utility function.

Lemma 1. *Agent i's social utility is a convex combination of the private utilities of all agents, i.e. for all and j,* $w_{ij}\in[0,1]$ *and* $\sum_j w_{ij}=1$.

Proof. All proofs are found in the Appendix.

Lemma 1 establishes that agent i's social utility of the public good really is a weighted average of the private utilities of the agents in the economy, that is, that the weights in (5) are all nonnegative and sum to one. In addition, it implies that the primary network $\mathbf{\Lambda A}$ contains all of the information needed to determine how much weight agent i places on j's private utility of g, accounting for all possible induced interactions among all agents.

Agent i's overall utility function is obtained by plugging (5) into (1):

$$V_i(x_i,g)=x_i+\sum_j w_{ij}u_j(g). \tag{6}$$

It is worth highlighting some conceptual differences between the social utility function v_i and the overall utility function V_i. The collection of equations (2) forms a system of interdependent utilities because the social utility of an agent is a function of the social utility of other agents, i.e. $v_i(g,v_{j\neq i})$. In contrast, the overall utility function (6) does not induce interdependence of utilities because overall utility is a function of private utilities, i.e. $V_i(x_i,u_i,u_{j\neq i})$. This type of utility structure is known as Bergson utility.[15]

We use Eq. (6) to define agent i's willingness to pay for an increase in the provision of the public good accommodating possible network effects.[16] Normalizing the price of the private good, the compensating welfare measure associated with a discrete public project that yields an increase in g from g^0 to g^1 is defined by C_i that solves

$$V_i(m_i,g^0)=V_i(m_i-C_i,g^1) \tag{7}$$

where m_i represents agent i's income. Two compensating measures are defined. The first is agent i's willingness to pay under network interaction. It is defined by $C_i^{network}$ that solves

$$m_i+\textstyle\sum_j w_{ij}u_j(g^0)=m_i-C_i^{network}+\displaystyle\sum_j w_{ij}u_j(g^1)$$

or just,

$$C_i^{network}=v_i(g^1)-v_i(g^0). \tag{8}$$

The second is the traditional compensating welfare measure that only accounts for private willingness to pay, that is, the measure that pertains if agent i is socially isolated ($\lambda_i=0$). With social isolation, the social utility v_i simplifies to u_i (see Eq. (2)), and therefore private willingness to pay is defined as follows:

$$C_i^{private}=u_i(g^1)-u_i(g^0). \tag{9}$$

Combining Eqs. (5), (8) and (9) yields a relationship between the vectors $\mathbf{C}^{network}$ (that collects $C_i^{network}$ for $i=1,\dots,n$) and $\mathbf{C}^{private}$ (that collects $C_i^{private}$ for i=1,…,n):

$$\mathbf{C}^{network}=\mathbf{W}\mathbf{C}^{private}. \tag{10}$$

The same induced network $\mathbf{W}$ determines both social utility and willingness to pay under network interaction.

We can also use Eq. (6) to identify a single agent's impact on society. The amount $w_{1j}u_j$ measures j's contribution to agent 1's social utility, $w_{2j}u_j$ measures j's contribution to agent 2's social utility, and so on. Agent j's total contribution to the society is then $\sum_i w_{ij}u_j(g)$. This motivates the following definition.

[15] Consumption externalities lead to another type of interdependent preferences. With consumption externalities, the utility of an agent depends on her own consumption bundle c_i and also on the consumption bundle of others ($c_{j\neq i}$), i.e. $V_i(c_i,c_{j\neq i})$. Refer to Bramoullé (2001) for a review.

[16] When g is a net public good, so that it combines the public good with a vector of tax charges, this willingness to pay measures the additional amount the individual is willing to pay above what she owes on her tax bill. A negative willingness-to-pay measure is possible and means that the individual values g less than what she is expected to pay for it.

Definition 1. Agent j's importance is defined as $\delta_j = \sum_i w_{ij}$.

Since **W** is a row normalized matrix, agent importance is the sum of the elements of the jth column of the induced network and can be intuitively thought as a measure of the "popularity" of agent j. This measure of importance is closely related to a number of measures of network centrality.[17] The next lemma further characterizes agent importance.

Lemma 2. *Every agent in the network has positive importance, i.e. $\delta_i > 0$ for all i.*

The maximum value of δ_j approaches n and the minimum approaches 0. Consequently, every agent has at least a little importance to society and no single agent is a dictator. Average agent importance is 1.

One might be interested in how changes in the primary network affect the weights and the agents' importance in the induced network. We discuss these effects in Propositions 1 and 2. None of the results are meant to be surprising. Rather, they are meant to confirm one's intuition about how the component matrices $\mathbf{\Lambda}$ and **A** impact **W**.

Proposition 1. *If agent i reallocates ε weight from a_{ik} to a_{ij} in the social network **A**, i.e. the weight that i places on the friendship with j increases while the weight placed on k decreases by the same amount, then the following results hold*:

(i) *The weight placed by i on j in the induced network **W** increases, i.e. $\partial w_{ij}/\partial\varepsilon \geq 0$.*
(ii) *The importance of j increases, i.e. $\partial\delta_j/\partial\varepsilon \geq 0$.*

Intuitively, the proposition suggests that larger weights in **A** are translated into larger weights in **W**, and that smaller weights in **A** translate into smaller weights in **W**. Likewise, it suggests that an important agent in the social network is also an important agent in the induced network. The second proposition describes the relationship between $\mathbf{\Lambda}$ and **W**.

Proposition 2. *If the degree of social interaction λ_i of agent i increases, then the following results hold*:

(i) *The weight placed by i on herself in the induced network **W** decreases, i.e. $\partial w_{ii}/\partial\lambda_i \leq 0$.*
(ii) *The importance of i decreases, i.e. $\partial\delta_i/\partial\lambda_i \leq 0$.*
(iii) *The weight placed by i on a friend j in the induced network **W** increases, i.e. $\partial w_{ij}/\partial\lambda_i \geq 0$.*
(iv) *The importance of j increases, i.e. $\partial\delta_j/\partial\lambda_i \geq 0$.*

An increase in λ_i intensifies the effect of friends' social utilities on i's utility, making i more sensitive to feedback loops in the network. The weight and importance of a friend j in the induced network increase as i becomes more socially interactive and cares more about friends. According to Lemma 1, the weight that i places on herself must decrease when the weight of friends increases. In other words, when i cares less about herself, the weight and importance of friends increase and, as a result, i's own weight and importance to the network decrease.

Networks and individual valuation

This section analyzes the relationship between agent i's social and private utility. We explore differences between the traditional utility model in social isolation and our network model by studying how the shape of the social network affects non-market values. We begin by defining network neutrality.

Definition 2. A network is neutral if, for the entire population, social utility is equal to private utility, i.e. for every private utility profile $(u_1, \ldots, u_n)$ we have $v_i = u_i \ \forall i$.

Under network neutrality, the social structure imposed by the system of interdependent utilities (4) is irrelevant. Stated differently, there are no network externalities as agents' overall utilities are not affected by network interactions. Identification of situations that lead to network neutrality becomes important because doing so also identifies situations where the network does matter, and Proposition 3 presents conditions that lead to network neutrality.[18]

Proposition 3 (Network neutrality). *Network neutrality holds if and only if all agents are socially isolated (i.e. $\lambda_i = 0 \ \forall i$).*

[17] In an unweighted network **X**, where an element x_{ij} equals 1 if i is connected to j, 0 otherwise, degree centrality of node j is defined as $\sum_i x_{ij}/(n-1)$ (see Jackson, 2010). In a weighted network **Y**, where an element y_{ij} is positive if i is connected to j, 0 otherwise, the equivalent centrality measure $\sum_i y_{ij}/(n-1)$ is defined by Friedkin (1991) as the *total effects centrality* of node j. Both centrality measures relate to average column sum while δ captures the column sum. Refer to Opsahl et al. (2010) for a discussion of centrality measures in weighted networks.

[18] The network could also be irrelevant if all individuals have identical private tastes, that is, if $u_i(g) = u_j(g) \forall i, j$. The irrelevance of the network then follows because every agent's social utility is a weighted average of the private utilities, which in turn are all equal. It is also possible, but extremely unlikely, that for some particular value of g the vectors of social and private values end up being identical. In real world applications, with large social networks, a combination of values in ΛA such that $v_i = u_i \ \forall i$ is essentially impossible.

Mathematically, network neutrality holds if and only if the primary network ΛA is a matrix of zeros. If this is the case, the induced network does not contain any (direct or indirect) connections between agents. In fact, **W** is the identity matrix.[19] Network effects are expected to be small if there are weak primary networks with little social interaction in the consumption of the public good. For example, one would be hard pressed to argue that λs are high when the public good in question is a sewer system. Of course altruism is always a possible reason for the existence of social preferences. However, it is probably safe to assume that a population's average λ for a park (possibly a jointly consumed public good) is higher than the average λ for a sewer system (a public good that is consumed individually). It may be the case that social networks are neutral if the public good is a sewer system. Importantly, though, Proposition 3 implies that when some agents care about friends' utility (so that $\lambda_i > 0$ for some agents) the shape of the network matters for social utility.

Proposition 3 implies that the network matters more when agents are more socially connected (so that λs are high), and it also follows from the structure of the model that, relative to a world of social isolation, network effects can significantly change individuals' well-being in environments in which agents have large disparities in private utilities. On the flip side, network effects are expected to be small if the population is homogeneous. For instance, consider a group of peasants of a small village in a developing country. Assume that they are a very homogeneous group that obtains natural resources from a watershed. Despite the fact that there may be strong social utility associated with the consumption of the watershed (i.e. λs are not zero), one could imagine the private utilities from the watershed as being the same for every peasant. In this case, any random peasant is a perfect representative agent and the welfare generated by the watershed can be perfectly assessed by the welfare of a single peasant. This is a case in which the social network is neutral for a specific public good, but not in general.

The following proposition formalizes the obvious implication that when a network has no impact on individuals' social utility levels, that is, when network neutrality holds, it also has no impact on individual willingness to pay. It does so by comparing the network compensating measure $C_i^{network}$ to the private measure $C_i^{private}$.

Proposition 4 (Individual valuation neutrality). *If network neutrality holds, the willingness to pay measure $C_i^{network}$ is equal to the private measure $C_i^{private}$.*

If the social network is not neutral, it may have a significant effect on non-market values. A natural next step is to examine how the social network affects social utility. We now study the setting in which agents have heterogeneous private utilities and are not socially isolated. When network neutrality fails, the utility of g is determined by the social utility v, and it is different from the private utility u. The next proposition characterizes how the shape of the social network affects social utility and establishes the conditions in which the network generates a positive externality such that the social utility v_i is greater than the private utility u_i.

Proposition 5 (Network effects on utility). *In non-neutral networks (i.e. $w_{ii} \neq 1$), the network benefits agent i, i.e. $v_i(g) > u_i(g)$, if and only if*

$$u_i(g) < \frac{\sum_{j \neq i} w_{ij} u_j(g)}{\sum_{j \neq i} w_{ij}}. \tag{11}$$

The left-hand side of expression (11) is individual i's own private utility, and the right-hand side is the weighted average of her network's private utilities. If her induced social network values the public good more than she does, on average, her social utility from the public good exceeds her private utility. Conversely, if she values it more than her network does, on average, the impact of the network is to reduce her social utility. So, for example, if i likes the beach more than any of her friends do, i receives lower social utility from going to the beach than she would if she were socially isolated.

As a consequence of Proposition 5, willingness to pay under network interaction ($C_i^{network}$) is expected to be different from private willingness to pay in social isolation ($C_i^{private}$). Proposition 6 describes the circumstances in which the network generates higher valuations than those generated under social isolation.

Proposition 6 (Network effects on individual valuation). *In non-neutral networks (i.e. $w_{ii} \neq 1$), $C_i^{network} \geq C_i^{private}$ if and only if $(u_i(g^1) - u_i(g^0)) \leq \sum_{j \neq i} w_{ij}[u_j(g^1) - u_j(g^0)] / \sum_{j \neq i} w_{ij}$.*

Proposition 6 demonstrates that for agents with small private willingness to pay the network generates higher valuations than the ones in social isolation. In a social network environment, low private valuation agents are willing to pay more for an increment in g because they benefit from the gains of higher private valuation friends. To see this, consider the

[19] Proposition 3 indicates that if Λ is a matrix of zeros, then the induced network is equal to the identity matrix ($\mathbf{W} = \mathbf{I}$). There is no mathematical condition that imposed on **A** would lead to network neutrality. In fact, mathematically, if $\mathbf{A} = \mathbf{I}$ then $\mathbf{W} = \mathbf{I}$, regardless of Λ. However, this is ruled out by the model construction as the diagonal of **A** has zeros reflecting the fact that agent i is not a friend of herself.

following example. The induced network is given by

$$\mathbf{W} = \begin{pmatrix} 0.5 & 0.1 & 0.4 \\ 0.2 & 0.6 & 0.2 \\ 0.3 & 0.3 & 0.4 \end{pmatrix},$$

and the three individuals in the society differ in how much they value the change in the public good from g^0 to g^1. Let

$$\mathbf{u}(g^1) - \mathbf{u}(g^0) = \begin{pmatrix} 10 \\ 5 \\ 2 \end{pmatrix},$$

so that agent 1 has the highest private utility gain from the policy change and agent 3 has the smallest. Restricting attention to agent 3, note that

$$\frac{\sum_{j \neq i} w_{ij}(u_j(g^1) - u_j(g^0))}{\sum_{j \neq i} w_{ij}} = \frac{0.3(u_1(g^1) - u_1(g^0)) + 0.3(u_2(g^1) - u_2(g^0))}{0.6} = 7.5$$

which is larger than agent 3′s private value of the change, $(u_3(g^1) - u_3(g^0)) = 2$. According to the proposition, agent 3′s social value of the change should exceed her private value, and this is indeed the case as can be observed when one computes the social values

$$\mathbf{W}(\mathbf{u}(g^1) - \mathbf{u}(g^0)) = \begin{pmatrix} 6.3 \\ 5.4 \\ 5.3 \end{pmatrix}.$$

The example highlights the importance of recognizing networks to study non-market values that are influenced by social interactions between agents. When eliciting valuations from a population, subjects naturally report their true values, which are their social values. Part of the variation in these values arises from heterogeneous private values, which may be correlated with individual characteristics. The variation in elicited values is also affected by the shape of the network, though, and so studies that ignore the nature of the network may be misspecified.

Networks and aggregate valuation

This section investigates economic welfare generated by the provision of a public good. It considers non-neutral networks in which at least one agent is not socially isolated ($\exists \lambda_i$ s.t. $\lambda_i > 0$) and at least two agents have different private utility functions ($\exists \{u_i, u_j\}$ s.t. $u_i \neq u_j$ for $i \neq j$). The following definitions of welfare are discussed.

Definition 3.

(**A**) Social network welfare is defined as $\sum_i v_i$
(**B**) Social isolation welfare is defined as $\sum_i u_i$
(**C**) Welfare neutrality is defined by $\sum_i v_i = \sum_i u_i$

In non-neutral networks, v_i is typically different from u_i.[20] However, this may or may not have welfare implications. In some cases, network neutrality fails but welfare is unchanged such that the social network welfare is equal to the social isolation welfare. Hence, the existence of social network effects on the provision of public goods does not necessarily affect the population′s welfare but may nevertheless reorganize the distribution of social utility. Agent importance (δ, defined in Section 3) is a fundamental concept for our network welfare analysis. The following proposition characterizes welfare neutrality.

Proposition 7 (Welfare neutrality). *If every agent in the network has the same importance, then the social network welfare is equal to the social isolation welfare.*

It is important to acknowledge that the social network may have relevant individual welfare implications even in the environments covered by Proposition 7. The proposition states that there are populations in which the aggregate welfare generated by the provision of g is unaffected by the social structure. Under welfare neutrality, the social network acts as a smoothing operator, re-distributing utility among agents and decreasing well-being concentration. The following example provides an illustration.

[20] Recall that according to Definition 2, network neutrality implies that $v_i = u_i$ $\forall i$. Throughout this work, we refer to non-neutral networks as the counterpart of the neutral networks presented in Definition 2. Thus, we use the term "neutral networks" to refer to Definition 2, and not to welfare neutrality as in Definition 3C.

Consider two separate populations of size $n=3$ with identical private utility profiles for the public good but different networks. In both cases the private utility profile is $\mathbf{u}=(5,10,15)'$, for social isolation welfare, or aggregate private utility, of 30. The two different social structures are given by the induced network matrices

$$\mathbf{W_1}=\begin{pmatrix}0.5 & 0.1 & 0.4\\ 0.2 & 0.6 & 0.2\\ 0.3 & 0.3 & 0.4\end{pmatrix} \quad \text{and} \quad \mathbf{W_2}=\begin{pmatrix}0.5 & 0.3 & 0.2\\ 0 & 0.6 & 0.4\\ 0.5 & 0.1 & 0.4\end{pmatrix}.$$

Both populations are welfare neutral because in each of them every column sums to one. For the private utility profile $\mathbf{u}$ given above, the resulting social utility profiles are $\mathbf{v_1}=(9.5,10.0,10.5)'$ and $\mathbf{v_2}=(8.5,12.0,9.5)'$. Both of these have the same social network welfare, or aggregate social utility, of 30. This demonstrates welfare neutrality. Network neutrality does not hold, however, as can be seen by the fact that in both populations agent 1′s social utility exceeds her private utility, and in both cases agent 3′s private utility exceeds her social utility. Furthermore, even though the presence of welfare neutral network effects does not change the average utility of the group, it changes the distribution of utilities in a variety of ways. Network $\mathbf{W_1}$ preserves the median utility level at 10, but network $\mathbf{W_2}$ reduces the median to 9.5. This second network also changes the ordering of who gains the most utility, with agent 3 having the highest private utility level but agent 2 having the highest social utility level. Finally, the second network obviously generates a larger standard deviation of social utility than the first network, and both these standard deviations are smaller than in the private utility profile.

The following definitions are used to discuss the aggregate value of public projects.

Definition 4.

A Aggregate network value is defined as $C^{network}=\sum_i C_i^{network}$
B. Aggregate private value is defined as $C^{private}=\sum_i C_i^{private}$
C. Aggregate valuation neutrality is defined by $C^{network}=C^{private}$

As a consequence of Proposition 7, valuation of public projects is independent of social structure when the population has a social network that is welfare neutral. The next proposition formalizes this result.

Proposition 8 (Aggregate valuation neutrality). *If welfare neutrality holds, then* $C^{network}=C^{private}$.

Proposition 8 provides sufficient conditions for aggregate valuation neutrality. It implies that in networks in which agents have the same importance, i.e. $\delta_1=\cdots\delta_n=1$, the aggregate value of a public good can be measured by either $C^{network}$ or $C^{private}$. Aggregate valuation neutrality does not require individual valuation neutrality. In fact, in non-neutral networks, $C_i^{network}$ is typically different from $C_i^{private}$ even when $C^{network}$ is equivalent to $C^{private}$. Therefore, standard non-market valuation measures can be used if: (i) welfare neutrality holds, *and* (ii) the objective is to obtain a measure of aggregate willingness to pay *or* mean willingness to pay *and not* median willingness to pay. This can be highlighted with an example similar to the one above. Let

$$\mathbf{W}=\begin{pmatrix}0.5 & 0.3 & 0.2\\ 0 & 0.6 & 0.4\\ 0.5 & 0.1 & 0.4\end{pmatrix}.$$

This is the same as induced network $\mathbf{W_2}$ in the previous example. Let $\mathbf{u}(g^1)-\mathbf{u}(g^0)=(4,12,16)'$. Then $\mathbf{v}(g^1)-\mathbf{v}(g^0)=(8.8,13.6,9.6)'$. The aggregate network value and the aggregate private value are equal at 32, but every agent′s valuation changes. Importantly, the median social valuation of 9.6 is lower than the median private valuation of 12, demonstrating that welfare neutral networks can change the quantiles of the valuation distribution even though they do not change the mean valuation.

The preceding results highlight when a network does or does not impact welfare, but they do not address how the network impacts aggregate welfare and aggregate valuation. To this end, we define welfare-increasing social networks as follows.

Definition 5. A social network is welfare-increasing if $\sum_i v_i > \sum_i u_i$.

To facilitate welfare comparisons, it is useful to write the social network welfare as a weighted sum of the private utilities of all agents with weights determined by the importance of agents as defined in Section 2:

$$\sum_j \delta_j u_j,$$

where δ_j is agent j′s importance as presented in Definition 1, i.e. $\delta_j=\sum_i w_{ij}$. Index agents by increasing values of private utility such that agent 1 is the lowest private utility agent and agent n is the highest private utility agent. Hence, $\mathbf{u}=(u_1,\ldots,u_n)'$ is a sorted private utility profile such that $u_1 \le u_2 \le \cdots \le u_n$. It is now easy to see that social network welfare increases as the importance of high private utility agents increases and, as a consequence, the importance of low private utility agents decreases. To formalize this intuition, define the *distribution of importance* as the vector $(\delta_1/n, \ldots, \delta_n/n)$. This is a distribution because, recalling that $0 < \delta_i < n$ for each i, every element in the distribution of importance lies between zero and one. With this in mind, the next proposition formalizes the idea that the network increases social welfare by shifting importance to agents with higher private values of the public good.[21]

Proposition 9 (Welfare-increasing networks). *For all sorted private utility profiles* $\mathbf{u}$, *if the distribution of importance of a network* $\mathbf{W}$ *first order stochastically dominates* (*FOSD*) *that of the social isolation case*, *then* $\mathbf{W}$ *is a welfare-increasing network.*

Under social isolation, the induced network $\mathbf{W}$ is equal to the identity matrix. As a result, every agent in the network has the same importance $\delta = 1$. Thus, if the network's distribution of importance FOSD the (social isolation) uniform distribution of importance, then the provision of a public good in the network will generate welfare greater than the sum of the private values. Proposition 9 has important implications as the welfare generated by the provision of a public good can be enhanced or diminished by social networks with the outcome depending on the distribution of importance.

One implication is that public goods policy should target high importance individuals. This view provides new insights to questions like "Should the government fund fine arts?". A traditional approach to this problem would consider the potentially high costs associated with benefits to a select group of individuals with significant high utility from fine arts. However, if these individuals are important (or popular) individuals in the social network, the positive externalities generated from these policies may justify such public investments.

Accordingly, Proposition 9 provides structure to the idea of opinion leadership, that is, the existence of agents who can facilitate change. For a given population governed by a given network, projects valued more highly by agents with higher importance tend to be the projects valued by the entire population. In extreme cases, efforts to undertake projects valued highly by the single individual with the greatest importance tend to be more successful than those valued negatively by that same individual. Thus, opinion leadership and importance are linked in our framework. This is consonant with the conclusions from Flores (2002) and Bergstrom (2006) highlighting the fact that social utility can play an important role in cost-benefit analysis. A public project may be Pareto improving even though the sum of private values is not large enough to justify the public investments. That Pareto improvement comes from the high private values of highly important agents, or opinion leaders.[22]

If the social network is capable of generating striking changes in social welfare, it is important to understand which types of network are more desirable. This is formalized in the next proposition that facilitates comparisons of networks focusing on social welfare.

Proposition 10 (Network welfare comparisons). *For all sorted private utility profiles* $\mathbf{u}$, *if the distribution of importance of a network* $\mathbf{W}$ *FOSD that of another network* $\mathbf{W}'$, *then* $\mathbf{W}$ *generates greater social network welfare than* $\mathbf{W}'$.

According to Proposition 10, a network that favors high private utility agents generates greater social network welfare than one that favors low private utility agents. The following example stresses the relevance of this result. Consider two geographically separated populations of same size, with separated social networks but with identical private utility profile $\mathbf{u}$. Assume that a central planner has the resources to implement a public project in one of the two populations. In which population should the project be implemented? If the cost of implementation is the same in both populations, private benefit-cost analysis would indicate that the central planner should be indifferent between the two options. Social benefit-cost analysis leads to a different conclusion. Considering Definition 3A of social network welfare, if the two populations have different social networks, the project should be implemented in the network that places more weight on agents with higher private utility. Proposition 11 considers network effects on aggregate valuation.

Proposition 11 (Network effects on aggregate valuation). $C^{network} \geq C^{private}$ *if and only if* $\sum_i (\delta_i - 1)[(u_i(g^1) - u_i(g^0)] \geq 0$.

The proposition indicates that a social network has positive effects on aggregate valuation when the weighted sum of private willingness to pay is positive, with the weights determined by deviations from the mean importance. Intuitively, the more the distribution of importance favors agents with high private valuation, the greater is the aggregate network valuation. This result has important implications.

For example, suppose the public project is one that targets the improvement of attributes of a beach frequented by n residents of a certain neighborhood. Suppose *few* residents are surfers. As committed surfers, they love to be at the beach and have high private willingness to pay for an increase in beach quality. Now suppose that these few surfers have several friends and, as a result, are very popular residents of this neighborhood. Moreover, assume that this is a high enough combination of popularity and private valuation such that the conditions for Proposition 11 hold. The consequence is that these few surfer residents may be responsible for a significant boost in the value of the public project making $C^{network} > C^{private}$. Now imagine that the surfers leave the neighborhood. Clearly, if high valuation agents are not considered, the aggregate value of the public project decreases. However, because of the network structure, the aggregate value may drop further. Shocks in the network can make second-order valuation effects (network effects) larger than first-order effects.

[21] Proposition 9 is a special case of Proposition 10.

[22] These ideas in turn provide an intuitive rationale why, for example, the United Nations might name a Hollywood actress such as Angelina Jolie as a Goodwill Ambassador. Refer to http://www.unhcr.org/pages/49c3646c56.html. Accessed on October 26, 2011.

As a consequence, the condition in Proposition 11 may be no longer satisfied in a neighborhood without surfers. The example emphasizes how sensitive aggregate valuation can be to changes in social structure.

Project choice and opinion leadership

Sections 3 and 4 concerned how the characteristics of the network affect individual and aggregate valuation, respectively, for a given public project. In this section we turn to the issue of project choice. In some instances policy-makers consider a number of public projects simultaneously, but can only provide one. This section illustrates how the characteristics of the network impact the decision of which project to undertake.

To isolate the effects of network structure, consider the case of a policy-maker deciding between two public projects, g and g^*, that generate private utility profiles $\mathbf{u}$ and $\mathbf{u}^*$, respectively. Assume that both projects generate the same aggregate private utility: $\sum_i u_i = \sum_i u_i^*$. Are there characteristics of the network that would lead the policy-maker to prefer one of the projects to the other, that is, that would make one project have higher aggregate social utility than the other?

The answer relates to how the distribution of private utilities relates to the distribution of importance. In particular, as the next proposition shows, if the project g^* shifts private utility from a less-important to a more-important individual, then g^* generates higher aggregate social network welfare than g does.

Proposition 12 (Agent-to-agent benefit transfers). *A public project that transfers $\varepsilon > 0$ private utility from an agent to another constitutes a Kaldor–Hicks improvement if and only if the project transfers private utility from a less important agent to a more important agent.*

In our model, a Kaldor–Hicks improvement is simply an increase in social network welfare, so that $\sum_i v_i^* \geq \sum_i v_i$. The if-and-only-if nature of the proposition says that, when filtered through the network, a transfer of private utility from a less-important to a more-important agent increases social network welfare, while a transfer from a more-important to a less-important agent reduces social network welfare. Thus, the proposition corroborates the overall theme of the paper that social welfare is determined by the correlation between private utility and network importance.

The proposition also suggests a special role for the most important agent. Let $\Delta_i = \{\delta | \delta_i > \delta_j$ for all $j \neq i\}$, where δ is an importance vector. Consequently, Δ_i is the set of all importance vectors that assign the highest importance to agent i. The following proposition links social network welfare to the private benefits accruing to the network′s most important agent.

Proposition 13 (Opinion leaders and public projects). *A public project that transfers $\varepsilon > 0$ private utility from each of $n-1$ agents to the remaining agent constitutes a strict Kaldor–Hicks improvement for every $\delta \in \Delta_i$ if and only if the recipient is agent i.*

This proposition further highlights the special role that an opinion leader, or the most important agent, plays in the provision of a public good. Proposition 10 explored the impact of changing the network structure while holding the underlying private utility vector constant, and stated that network changes that shift importance toward those who are already important increases the value of the public project. Proposition 13, in contrast, holds the network fixed and looks at changes in the distribution of private utilities. Holding aggregate private utility fixed, a project that shifts private utility to the most important person necessarily increases aggregate welfare. Tellingly, a project that shifts private utility to anyone else might reduce aggregate welfare, so that the opinion leader is the only member of the network who has the following distinction: projects that are valued more by the opinion leader are valued more by the entire network.

When one interprets the public good as a net public good, that is, as a combination of a public good and a tax assignment vector, another interpretation of Proposition 13 arises. It is possible to conduct transfers of the type described in the proposition by reducing the opinion leader′s tax burden and spreading it over the remaining $n-1$ agents, and the proposition shows that such a reallocation improves welfare. Taking this to the extreme, in the net public good setting the opinion leader′s tax payment should be minimal.

A similar consideration arises from thinking about g as a joint-use private good. In this case only one individual pays, and an implication of the proposition is that the payer will favor private goods most liked by the opinion leader. Without the network one might surmise that the payer would choose goods to favor the friend she likes most, but the proposition says that she would choose goods to favor the connection valued by the most people and not the connection valued the most by herself. Thus, opinion leaders influence individual choices even in a private good setting.

Proposition 13 also allows for the possibility of a lower information requirement for a policy maker who wants to take advantage of the network. Rather than identifying the entire importance vector, the policy maker could instead identify the most important member of the network, the opinion leader. The proposition shows that aggregate-private-utility redistributions favoring the opinion leader are welfare improving, so the policy maker can simply target this one individual when selecting the public project. Of course, this under-identification of the network might lead to mistakes if the projects yield different levels of aggregate private utility, but targeting the opinion leader remains a viable strategy for some project choices.

The role of opinion leaders was early-on recognized in other disciplines such as marketing and sociology. In marketing, the literature focuses on the effect of leadership on opinion formation and the diffusion of innovations.[23] Recent research investigate these topics in the context of social networks. van Eck et al. (2011) use simulations to show that in networks with

[23] Refer to Watts and Dodds (2007) for a brief review.

active opinion leaders, information spreads faster, the product diffuses faster, and the adoption percentage is significantly higher than in a network without opinion leaders. Other papers empirically investigate leadership effects using real-world social network data. Nair et al. (2010) find an opinion leader effect on physician prescription behavior. Iyengar et al. (2011) find that both sociometric and self-reported opinion leadership affect the adoption of a new drug by physicians.[24] Aral and Walker (2012) use a randomized field experiment in a large sample of Facebook users to study the adoption of a product. They detect highly influential individuals and conclude that these individuals may be instrumental in the spread of a product in a network. In sociology, the literature has studied how opinion leaders influence mass communications (see Katz and Lazarsfeld, 1955), voting behavior (see Berelson et al., 1954), and discrimination (see Dean and Rosen, 1955), to name a few examples.

Our paper contributes to the non-market valuation literature by introducing the notion of opinion leadership to a valuation model of public goods in social networks. Opinion leaders are important in choosing among projects because they disproportionately influence the aggregate value of public goods.[25] In our model, leadership is easily identified through δ, i.e. the importance of agents. Our results have important consequences for the design of public projects. In practice, policy makers use behavioral data (revealed preference) or survey data (stated preference) to learn how different project characteristics and socio-economic variables affect the distribution of non-market values. Information about social networks and opinion leaders are not traditionally used in these studies. Our paper indicates, however, that this type of information may enable policy makers to design better public projects, and may be a valuable resource in settings in which tight budget constraints make an increase in aggregate private benefits unfeasible.

Conclusion

Directed altruism towards friends or joint consumption of public goods with friends are possibly two important reasons to consider social structure in non-market valuation approaches. The paper builds a network model for analyzing provisions of public goods accounting for the presence of social utility operating through social connections. The model assumes that individuals′ private values are the ones that pertain in the absence of social network effects while social values weight own private utility and social utilities of friends. This framework allows us to study the effects of the shape of the connections on non-market values, holding constant the effect of network size.

Current research on public goods in networks study environments in which links are used to share non-excludable goods, i.e. local public goods. Differently, the focus of our research is not to study incentives problems related to the production of local public goods. Instead, we present a valuation model in social networks. The model delivers two measures of willingness to pay for an increase in the provision of public goods: willingness to pay under network interaction, a measure that accounts for the influence of connected friends and feedback effects; and standard willingness to pay in social isolation, a special case of the model that arises when the network structure is neutral.

By comparing these two measures, the paper demonstrates that non-market values can significantly be affected by social networks. For example, if the network is such that connections with high private utility agents are more intense, private willingness to pay understates the true value of non-market goods. However, if agents are equally "popular" in the social network, i.e. all agents receive the same amount of attention from their friends, the social structure may affect individual values but the overall welfare generated by the provision of the public good is the same of that generated in an environment of complete social isolation. We demonstrate that social welfare changes as a function of the distribution of popularity of agents in the network. When popular agents have high private valuation, the second-order (networks) effects have high impact on aggregate valuation.

The network model presented in this research can potentially guide empirical work. If the underlying consumption decisions involve considerations about the well-being of socially connected agents, conventional non-market valuation approaches may mislead econometric identification by not taking into account an important source of variation in the willingness to pay of agents: the social network. With network interaction, the value an individual attributes to a public good is a function of the values that friends attribute to the public good, and the value that friends attribute to the public good is a function of the individual′s valuation. Manski (1993, 2000) refers to this as the *reflection problem*. If this is the case, the estimation of non-market values becomes even more challenging.[26]

Future empirical research should focus on the development of econometric models and survey techniques to facilitate estimation of non-market values accounting for the possible social network effects demonstrated in this paper. Future theoretical work should focus on generalizations of the analyses developed in this research. These may include, for instance, the study of environments with multiple (substitute or complementary) public goods or the investigation of congestion effects.

[24] As discussed by Iyengar et al. (2011), sociometric opinion leadership measures are obtained through sociometric techniques, i.e. network centrality scores. Self-reported opinion leadership measures are directly obtained through surveys.

[25] This is not to say that opinion leaders exert excessive influence on the network. In our setting, leadership comes from the network structure while opinion arises from the leader′s private value of the public good, $u_i(g)$. Nothing in this setting is normative, so other network members have no need to "correct" for the opinion leader′s influence. Instead, the other network members value the opinion leader′s consumption of the pubic good, which is why the opinion leader′s preferences have a large weight in the aggregate valuation.

[26] Readers interested in econometric identification of peer effects through social networks should refer to Bramoullé et al. (2009).

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Appendix A

Lemma 1. *Agent i's social utility is a convex combination of the private utilities of all agents, i.e. for all i and j,* $w_{ij} \in [0,1]$ *and* $\sum_j w_{ij} = 1$.

Proof. First notice that $(\mathbf{I}-\mathbf{\Lambda}\mathbf{A})$ is a strictly diagonally dominant matrix and, by the Levy–Desplanques theorem, cannot be singular (see Taussky, 1949, Theorem I). Hence $\mathbf{W}$ always exists. $(\mathbf{I}-\mathbf{\Lambda}\mathbf{A})^{-\mathbf{1}}$ is a nonnegative matrix. To see this, note that the matrix $(\mathbf{I}-\mathbf{\Lambda}\mathbf{A})^{-\mathbf{1}}$ can be written as the Neumann series $(\mathbf{I}+(\mathbf{\Lambda}\mathbf{A})+(\mathbf{\Lambda}\mathbf{A})^2+(\mathbf{\Lambda}\mathbf{A})^3+\cdots)$, i.e. a sum of nonnegative matrices. Since $(\mathbf{I}-\mathbf{\Lambda})$ is a nonnegative matrix, $\mathbf{W}=(\mathbf{I}-\mathbf{\Lambda}\mathbf{A})^{-\mathbf{1}}(\mathbf{I}-\mathbf{\Lambda})$ is also a nonnegative matrix. To prove Lemma 1 it must be demonstrated that $\mathbf{t}$, the row sum vector of the matrix $\mathbf{W}$, is a vector whose entries are all 1. The row sum vector of a matrix can be obtain by post-multiplying the matrix by a column vector $\mathbf{i}$ whose entries are all 1. Thus, $\mathbf{t}$ can be written as

$$\mathbf{t}=(\mathbf{I}-\mathbf{\Lambda}\mathbf{A})^{-\mathbf{1}}(\mathbf{I}-\mathbf{\Lambda})\mathbf{i} \tag{A.1}$$

By construction, $(\mathbf{I}-\mathbf{\Lambda}\mathbf{A})$ and $(\mathbf{I}-\mathbf{\Lambda})$ have the same row sum column vector $\mathbf{r}$, with the i-th entry equal to $1-\lambda_i$. As a consequence,

$$(\mathbf{I}-\mathbf{\Lambda}\mathbf{A})\mathbf{i}=\mathbf{r} \tag{A.2}$$

$$(\mathbf{I}-\mathbf{\Lambda})\mathbf{i}=\mathbf{r} \tag{A.3}$$

Plugging (A.3) into (A.1) yields to

$$\mathbf{t}=(\mathbf{I}-\mathbf{\Lambda}\mathbf{A})^{-\mathbf{1}}\mathbf{r}$$

According to (A.2), $\mathbf{i}=(\mathbf{I}-\mathbf{\Lambda}\mathbf{A})^{-\mathbf{1}}\mathbf{r}$. Thus, $\mathbf{t}=\mathbf{i}$. □

Lemma 2. *Every agent in the network has positive importance, i.e.* $\delta_i > 0$ *for all i.*

Proof. Rewrite $\mathbf{W}=(\mathbf{I}-\mathbf{\Lambda}\mathbf{A})^{-\mathbf{1}}(\mathbf{I}-\mathbf{\Lambda})$ as $\mathbf{W}=\mathbf{X}\mathbf{Y}$. Notice that an element of $\mathbf{W}$ can be written as $w_{ii}=x_{i1}y_{1i}+x_{i2}y_{2i}+\cdots+x_{ii}y_{ii}+\cdot+x_{in}y_{ni}$, where x_{ij} and y_{ij} are elements of $\mathbf{X}$ and $\mathbf{Y}$, respectively. Also, notice that $x_{ii}\geq 1$. To see this, recall that $\mathbf{X}$ can be written as the Neumann series $(\mathbf{I}+(\mathbf{\Lambda}\mathbf{A})+(\mathbf{\Lambda}\mathbf{A})^2+(\mathbf{\Lambda}\mathbf{A})^3+\cdots)$, which is a sum of the identity matrix with nonnegative matrices. Moreover, $0\leq\lambda_i<1 \Rightarrow 0<y_{ii}\leq 1$. Therefore, since $\mathbf{X}$ and $\mathbf{Y}$ are nonnegative matrices, and $x_{ii}y_{ii}>0$, it follows that $w_{ii}>0$ for all i, thus $\delta_j=\sum_i w_{ij}>0$. □

Proposition 1. *If agent i reallocates* ε *weight from* a_{ik} *to* a_{ij} *in the social network* $\mathbf{A}$*, i.e. the weight that i places on the friendship with j increases while the weight placed on k decreases by the same amount, then the following results hold*

(i) *The weight placed by i on j in the induced network* $\mathbf{W}$ *increases, i.e.* $\partial w_{ij}/\partial\varepsilon \geq 0$.
(ii) *The importance of j increases, i.e.* $\partial\delta_j/\partial\varepsilon \geq 0$.

Proof. Without loss of generality, assume that $i=1$, $k=3$, and $j=2$, such that agent 1 transfers weight ε from agent 3 to agent 2 in $\mathbf{A}$. Let the resulting network be denoted as $\tilde{\mathbf{A}}=\mathbf{A}+\epsilon\mathbf{M}$, with elements of $\mathbf{M}$ equal to $m_{12}=1$, $m_{13}=-1$, and $m_{ij}=0$ else. Define $\tilde{\mathbf{W}}=(\mathbf{I}-\mathbf{\Lambda}\tilde{\mathbf{A}})^{-\mathbf{1}}(\mathbf{I}-\mathbf{\Lambda})$. To prove statement (i) it must be shown that the element of the first row and second column of the matrix $\partial\tilde{\mathbf{W}}/\partial\varepsilon$ is nonnegative, i.e. $\partial\tilde{w}_{12}/\partial\varepsilon \geq 0$. Using the rule of derivatives of inverse matrices yields

$$\frac{\partial\tilde{W}}{\partial\varepsilon}=\mathbf{V}=(\mathbf{I}-\mathbf{\Lambda}\tilde{\mathbf{A}})^{-\mathbf{1}}\mathbf{\Lambda}\mathbf{M}(\mathbf{I}-\mathbf{\Lambda}\tilde{\mathbf{A}})^{-\mathbf{1}}(\mathbf{I}-\mathbf{\Lambda}).$$

We must demonstrate that the element $v_{12}\geq 0$. Let $\mathbf{X}=(\mathbf{I}-\mathbf{\Lambda}\tilde{\mathbf{A}})^{-1}$ (see proof of Lemma 1 to see that $\mathbf{X}$ is non-singular). The matrix $(\mathbf{I}-\mathbf{\Lambda})$ is a diagonal matrix with nonnegative diagonal elements. Therefore, to show that $v_{12}\geq 0$, we must show that the element z_{12} of the matrix $\mathbf{Z}=\mathbf{X}\mathbf{\Lambda}\mathbf{M}\mathbf{X}$ is nonnegative. $\mathbf{\Lambda}\mathbf{M}$ is a matrix with λ_1 in the first row, second column; $-\lambda_1$ in the first row, third column; 0 elsewhere. $\mathbf{X}\mathbf{\Lambda}\mathbf{M}$ is a matrix of zeros, except for columns 2 and 3. The second of column is equal to $\lambda_1\mathbf{X}_{i1}$, and the third column is equal to $-\lambda_1\mathbf{X}_{i1}$, where $\mathbf{X}_{i1}$ represents the first column of the matrix $\mathbf{X}$. We are now able to compute element $z_{12}=\lambda_1 x_{11}x_{22}-\lambda_1 x_{11}x_{32}$. Therefore, $v_{12}\geq 0$ if and only if the condition below is satisfied:

$$x_{22}\geq x_{32} \tag{A.4}$$

Notice that $(\mathbf{I}-\mathbf{\Lambda}\tilde{\mathbf{A}})$ is an M-matrix, i.e. it can be written in the form $\alpha\mathbf{I}-\mathbf{P}$, where (i) $\mathbf{P}$ is nonnegative; (ii) α is greater than the spectral radius (or the absolute value of the largest eigenvalue) of $\mathbf{P}$. In our case, $\alpha=1$, which is greater than the absolute value of the largest eigenvalue of $\mathbf{\Lambda\tilde{A}}$. To see this, note that the largest eigenvalue of a row stochastic matrix is 1, and $\mathbf{\Lambda\tilde{A}}$ has entries smaller than the row-stochastic matrix $\mathbf{A}$.

Let $\mathbf{C}$ be the inverse of a M-matrix. $\mathbf{C}$ has the following property (see Johnson, 1982, p. 200):

$$|c_{ii}| > |c_{ij}|, j \neq i \tag{A.5}$$

Since the inverse of any non-singular M-matrix is a non-negative matrix, condition (A.5) proves that condition (A.4) holds for $\mathbf{X}$. Therefore, $\partial \tilde{w}_{12}/\partial \varepsilon \geq 0$ and statement (i) is true.

To prove statement (ii) we need to show that $\partial \tilde{w}_{i2}/\partial \varepsilon \geq 0$ for all i. Following the same logic of part (i), $\partial \tilde{w}_{i2}/\partial \varepsilon \geq 0 \Leftrightarrow v_{i2} \geq 0 \Leftrightarrow z_{i2} \geq 0$. It follows that $z_{i2} = \lambda_1 x_{i1} x_{22} - \lambda_1 x_{i1} x_{32}$. For any i, this is true if and only if the condition (A.4) holds. The first part of the proof demonstrates that the condition holds, hence statement ii) is true. □

Proposition 2. *If the degree of social interaction of agent i increases, then the following results hold*:

(i) *The weight placed by i on herself in the induced network* $\mathbf{W}$ *decreases, i.e.* $\partial w_{ii}/\partial \lambda_i \leq 0$.
(ii) *The importance of i decreases, i.e.* $\partial \delta_i/\partial \lambda_i \leq 0$.
(iii) *The weight placed by i on a friend j in the induced network* $\mathbf{W}$ *increases, i.e.* $\partial w_{ij}/\partial \lambda_i \geq 0$.
(iv) *The importance of j increases, i.e.* $\partial \delta_j/\partial \lambda_i \geq 0$.

Proof. Without loss of generality, assume that $i=1$. Let $\tilde{\mathbf{\Lambda}} = \mathbf{\Lambda} + \epsilon \mathbf{M}$, with elements of $\mathbf{M}$ equal to $m_{11} = 1$, 0 elsewhere. Let $\tilde{\mathbf{W}} = (\mathbf{I} - \tilde{\mathbf{\Lambda}}\mathbf{A})^{-1}(\mathbf{I} - \tilde{\mathbf{\Lambda}})$. To prove statements (i) and (ii) we must demonstrate that $\partial \tilde{w}_{j1}/\partial \varepsilon \leq 0$ for all j. To prove statements (iii) and (iv) we must demonstrate that $\partial \tilde{w}_{1j}/\partial \varepsilon \geq 0$ for $j > 1$. Using two rules; the derivative of inverse matrices and the derivative of the product of matrices, and evaluating the derivative at $\varepsilon = 0$, yields

$$\frac{\partial \tilde{W}}{\partial \varepsilon} = \mathbf{V} = (\mathbf{I} - \mathbf{\Lambda A})^{-1}\mathbf{M A}(\mathbf{I} - \mathbf{\Lambda A})^{-1}(\mathbf{I} - \mathbf{\Lambda}) - (\mathbf{I} - \mathbf{\Lambda A})^{-1}\mathbf{M} = (\mathbf{I} - \mathbf{\Lambda A})^{-1}\mathbf{M}[\mathbf{AW} - \mathbf{I}]$$

The matrix $[\mathbf{AW} - \mathbf{I}]$ has non-positive diagonal elements and nonnegative off-diagonal elements. To see why, note that the product of two row stochastic matrices is also a row stochastic matrix. The matrix $\mathbf{Z} = \mathbf{M}[\mathbf{AW} - \mathbf{I}]$ has $z_{11} \leq 0$, $z_{1j} \geq 0$ for $j > 1$, and 0 elsewhere. From the proof of proposition 1, recall that $\mathbf{X} = (\mathbf{I} - \mathbf{\Lambda\tilde{A}})^{-1}$ is an inverse M-matrix, hence is a nonnegative matrix. This structure implies that $\mathbf{XZ}$ will have non-positive elements in the first column, nonnegative elements elsewhere. Thus, we have demonstrated that $v_{i1} \leq 0$ for all i and $v_{1j} \geq 0$ for $j > 1$, hence, all statements of the proposition are true. □

Proposition 3 (Network neutrality). *Network neutrality holds if and only if all agents are socially isolated* (*i.e.* $\lambda_i = 0 \forall i$).

Proof. According to Definition 2, a network is neutral if, for the entire population, social utility is equal to private utility, i.e. for every private utility profile $(u_1, \ldots, u_n)$ we have $v_i = u_i \ \forall i$. For the first direction, notice that if agent i is socially isolated $(\lambda_i = 0)$, then $v_i = (1-\lambda_i)u_i + \lambda_i \sum_j a_{ij} v_j = u_i$. Suppose to the contrary that $\lambda_i > 0$ for some i but that $v_j = u_j$ for all j. Because $v_i = (1-\lambda_i)u_i + \lambda_i \sum_j a_{ij} v_j$, the fact that $v_i = u_i$ implies that $\lambda_i u_i = \lambda_i \sum_j a_{ij} v_j$, which in turn reduces to $\mathbf{u} = \mathbf{Av}$ in matrix notation. Plugging this into Eq. (4) yields

$$\mathbf{v} = (\mathbf{I} - \mathbf{\Lambda A})^{-1}(\mathbf{I} - \mathbf{\Lambda})\mathbf{u}$$

$$\mathbf{Au} = (\mathbf{I} - \mathbf{\Lambda A})^{-1}(\mathbf{I} - \mathbf{\Lambda})\mathbf{u}$$

and this holds for all vectors $\mathbf{u}$, which means that $\mathbf{A} = (\mathbf{I} - \mathbf{\Lambda A})^{-1}(\mathbf{I} - \mathbf{\Lambda})$. Left-multiplying both sides by $(\mathbf{I} - \mathbf{\Lambda A})$ yields

$$(\mathbf{I} - \mathbf{\Lambda A})\mathbf{A} = \mathbf{I} - \mathbf{\Lambda}$$

which is only true if $\mathbf{A} = \mathbf{I}$. But this contradicts the assumption that $a_{ii} = 0$ for all i. □

Proposition 4 (Individual valuation neutrality). *If network neutrality holds, the willingness to pay measure* $C_i^{network}$ *is equal to the private measure* $C_i^{private}$.

Proof. $C_i^{network} = \sum_j w_{ij} u_j(g^1) - \sum_j w_{ij} u_j(g^0)$ (see Eq. (8)), or just $C_i^{network} = v_i(g^1) - v_i(g^0)$. If the network is neutral, $v_i(g) = u_i(g)$. It follows that $C_i^{network} = u_i(g^1) - u_i(g^0) = C_i^{private}$ (see Eq. (9)). □

Proposition 5 (Network benefits on utility). *For* $w_{ii} \neq 1$, *the network benefits agent i, i.e.* $v_i(g) > u_i(g)$, *if and only if* $u_i(g) < \sum_{j \neq i} w_{ij} u_j(g) / \sum_{j \neq i} w_{ij}$.

Proof. $v_i(g) > u_i(g) \Leftrightarrow \sum_j w_{ij} u_j(g) > u_i(g) \Leftrightarrow w_{ii} u_i(g) + \sum_{j \neq i} w_{ij} u_j(g) > u_i(g) \Leftrightarrow \sum_{j \neq i} w_{ij} u_j(g) > (1 - w_{ii}) u_i(g) \Leftrightarrow \sum_{j \neq i} w_{ij} u_j(g) / \sum_{j \neq i} w_{ij} > u_i(g)$. □

According to Lemma 1, $\sum_{j \neq i} w_{ij} = (1 - w_{ii})$. Thus, for $w_{ii} \neq 1$, $\sum_{j \neq i} w_{ij} > 0$.

Proposition 6 (Network effects on individual valuation). *In non-neutral networks with* $w_{ii} \neq 1$, $C_i^{network} \geq C_i^{private}$ *if and only if* $(u_i(g^1) - u_i(g^0)) \leq \sum_{j \neq i} w_{ij}(u_j(g^1) - u_j(g^0)) / \sum_{j \neq i} w_{ij}$.

Proof. $C_i^{network} > C_i^{private} \Leftrightarrow v_i(g^1) - v_i(g^0) > u_i(g^1) - u_i(g^0) \Leftrightarrow \sum_j w_{ij} u_j(g^1) - \sum_j w_{ij} u_j(g^0) > u_i(g^1) - u_i(g^0) \Leftrightarrow (1 - w_{ii}) u_i(g^1) - (1 - w_{ii}) u_i(g^0) < \sum_{j \neq i} w_{ij} u_j(g^1) - \sum_{j \neq i} w_{ij} u_j(g^0) \Leftrightarrow (u_i(g^1) - u_i(g^0)) < \sum_{j \neq i} w_{ij} (u_j(g^1) - u_j(g^0)) / \sum_{j \neq i} w_{ij}$.

According to Lemma 1, $\sum_{j \neq i} w_{ij} = (1 - w_{ii})$. Thus, for $w_{ii} \neq 1$, $\sum_{j \neq i} w_{ij} > 0$. □

Proposition 7 (Welfare neutrality). *If every agent in the network has the same importance, then the social network welfare is equal to the social isolation welfare.*

Proof. $\sum_i v_i = u_1 \sum_j w_{j1} + u_2 \sum_j w_{j2} + \cdots + u_n \sum_j w_{jn} = \sum_i \sum_j w_{ji} u_i$. When agents have the same importance, the columns of the induced network sum to one, i.e., $\sum_j w_{ji} = 1$. To see this, note that since the rows of $\mathbf{W}$ sum to 1, i.e. $\sum_j w_{ij} = 1$ (see Lemma 1), the sum of all entries in $\mathbf{W}$ is equal to $\sum_i \sum_j w_{ij} = n$. If all agents have the same importance, the importance of a single agent is obtained by dividing n evenly among the n columns of $\mathbf{W}$. If this is the case, the column sum vector of $\mathbf{W}$ is a vector of ones. Hence, equality of agents' importance implies $\sum_j w_{ji} = 1$. Therefore, $\sum_i v_i = \sum_i \sum_j w_{ji} u_i = \sum_i u_i$. □

Proposition 8 (Aggregate valuation neutrality). *If welfare neutrality holds, then* $C^{network} = C^{private}$.

Proof. $C^{network} = \sum_i C_i^{network} = \sum_i v_i(g^1) - \sum_i v_i(g^0)$. If welfare neutrality holds, $\sum_i v_i(g) = \sum_i u_i(g)$. Then, $C^{network} = \sum_i u_i(g^1) - \sum_i u_i(g^0) = C^{private}$. □

Proposition 9 (Welfare increasing networks). *For all sorted private utility profiles* $\mathbf{u}$, *if the distribution of importance of a network* $\mathbf{W}$ *first order stochastically dominates (FOSD) that of the social isolation case, then* $\mathbf{W}$ *is a welfare increasing network.*

Proof. See Proposition 10 with $\mathbf{W}'$ equal to the identity matrix. □

Proposition 10 (Network welfare comparisons). *For all sorted private utility profiles* $\mathbf{u}$, *if the distribution of importance of a network* $\mathbf{W}$ *FOSD that of another network* $\mathbf{W}'$, *then* $\mathbf{W}$ *generates greater social network welfare than* $\mathbf{W}'$.

Proof. It will be shown that, for all sorted private utility profile, if distribution of importance of a network $\mathbf{W}$ FOSD that of another network $\mathbf{W}'$, then $\mathbf{W}$ generates greater social network welfare than $\mathbf{W}'$. Hence, it must be demonstrated that, $\sum_{i=1}^{k} \delta_i \leq \sum_{i=1}^{k} \delta_i'$ implies $\sum_{i=1}^{n} \delta_i u_i \geq \sum_{i=1}^{n} \delta_i' u_i$, for all sorted private utility profile $\mathbf{u}$.

$\mathbf{W}$ generates greater social network welfare than $\mathbf{W}'$ when

$$\sum_i \delta_i u_i > \sum_i \delta_i' u_i \tag{A.6}$$

(see proof of Proposition 7). Construct $p_i = \delta_i / n$ and re-write (A.6) as

$$\sum_i p_i u_i > \sum_i p_i' u_i. \tag{A.7}$$

Since $\mathbf{p} = (p_1, \ldots, p_n)$ represents a probability vector, $U_i = \sum_i p_i u_i$ is an expected utility function. Let $P(k) = \sum_{i=1}^{k} p_i$ be the cdf that governs the probability vector $\mathbf{p}$. If $P(k)$ *FOSD* $P'(k)$, i.e. $\sum_{i=1}^{k} p_i \leq \sum_{i=1}^{k} p_i'$ for all k, then the expected utility under P is greater than the expected utility under P', $U_i > U_i'$, as in (A.7). □

Proposition 11 (Network effects on aggregate valuation). $C^{network} \geq C^{private}$ *if and only if* $\sum_i (\delta_i - 1)((u_i(g^1) - u_i(g^0)) \geq 0$.

Proof. $C^{network} \geq C^{private} \Leftrightarrow \sum_i C_i^{network} \geq \sum_i C_i^{private} \Leftrightarrow \sum_i v_i(g^1) - \sum_i v_i(g^0) \geq \sum_i u_i(g^1) - \sum_i u_i(g^0) \Leftrightarrow \sum_i \delta_i u_i(g^1) - \sum_i u_i(g^1) - \sum_i \delta_i u_i(g^0) + \sum_i u_i(g^0) \geq 0 \Leftrightarrow \sum_i (\delta_i - 1)((u_i(g^1) - u_i(g^0)) \geq 0$. □

Proposition 12 (Agent-to-agent benefit transfers). *A public project that transfers* $\varepsilon > 0$ *private utility from an agent to another constitutes a Kaldor–Hicks improvement if and only if the project transfers private utility from a less important agent to a more important agent.*

Proof. Let k index be the agent that receives $\varepsilon > 0$ private utility from agent j. The welfare impact of the policy change is

$$\varepsilon \delta_k - \varepsilon \delta_j = \varepsilon (\delta_k - \delta_j).$$

Clearly, the policy change strictly increases aggregate welfare if and only if $\delta_k > \delta_j$. □

Proposition 13 (Opinion leaders and public projects). *A public project that transfers* $\varepsilon > 0$ *private utility from each of* $n-1$ *agents to the remaining agent constitutes a strict Kaldor–Hicks improvement for every* $\delta \in \Delta_i$ *if and only if the recipient is agent i.*

Proof. Consider a policy change that increases private utility by $(n-1)\varepsilon$ for agent $k=i$ and reduces private utility by ε for each of the other agents. The welfare impact of the policy change is

$$(n-1)\varepsilon \delta_i - \varepsilon \sum_{j \neq i} \delta_j = \left[(n-1)\delta_i - \sum_{j \neq i} \delta_j \right] \varepsilon.$$

By construction, $\delta_i + \sum_{j \neq i} \delta_j = n$. Hence, the welfare impact of the policy can be re-written as

$$[(n-1)\delta_i - (n - \delta_i)]\varepsilon = n(\delta_i - 1)\varepsilon > 0,$$

which is strictly positive because $\delta_i > 1$. To see this, notice that average importance is 1 and $\delta \in \Delta_i$ assigns higher importance to agent i. Hence, δ_i is above average. Therefore, the policy change increases aggregate welfare.

For the other direction, choose any $\delta \in \Delta_i$ such that $\delta_i > n-1$. Since $\delta_i + \sum_{j \neq i} \delta_j = n$, it follows that $\delta_j < 1$ for all $j \neq i$. Consider a policy change that increases private utility by $(n-1)\varepsilon$ for agent $k \neq i$ and reduces private utility by ε for each of the other agents. The welfare impact of the policy change is

$$(n-1)\varepsilon\delta_k - \varepsilon \sum_{j \neq k} \delta_j = \left[(n-1)\delta_k - \sum_{j \neq k,i} \delta_j - \delta_i \right] \varepsilon.$$

Note that $\delta_j > 0$ for all j, and that $\delta_k < 1$ and $\delta_i > n-1$ by construction. Consequently,

$$(n-1)\delta_k - \sum_{j \neq k,i} \delta_j - \delta_i < (n-1) \cdot 1 - \sum_{j \neq k,i} 0 - (n-1) = 0$$

and the policy change reduces aggregate welfare. □

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