Tournaments, Risk Taking, and the Role of Carrots and Sticks

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Abstract

We study a Lazear-Rosen tournament in which players choose both work effort, which determines the mean of their output distribution, and the variance of output. The variance can be increased above its natural level but it is costly to do so. We show that tournaments involving more than two players generate incentives for risk-neutral players to pursue high-risk projects. However, the incorporation of a penalty for the player who finishes last in addition to a prize to the player who finishes first can eliminate this risk-seeking and retrieve efficiency in the tournament. More generally, the model illustrates that the balance of carrots and sticks in rank-dependent compensation will influence workers attitude toward risk taking.

Keywords: tournaments, risk-seeking (JEL J33, C72)
1 Introduction

When workers compete for promotion, bonuses, or other rewards tied to their relative performance ranking vis-à-vis colleagues, competition is likely to drive not just work effort but, where imperfect monitoring permits, other choices at the workers’ discretion. In this paper we ask how rank-order tournament compensation influences workers’ attitudes toward risk. Unlike previous literature, we assume that it is costly for workers’ to indulge any discretion they have to alter the variance of their output distribution. We find that, despite the cost, workers may indeed engage in wasteful risk-seeking, but only when they face multiple competitors and tournament payoffs are asymmetric with the reward for ranking first not balanced by a penalty for ranking last.

Since the pioneering work of Lazear & Rosen (1981), Nalebuff & Stiglitz (1983) and others, rank-order tournaments have been used to model worker incentives in a variety of labor market settings. In particular executive compensation has frequently been modeled as a tournament in which low-level executives are motivated by the prospect of climbing the corporate ladder and obtaining the large salaries attendant to top-ranking executives (Rosen, 1986; Bognanno, 2001). Similarly, the behavior of investment fund managers who compete for top fund-rankings has been modeled as a tournament (Brown, Harlow and Starks, 1996). The distinguishing feature of a tournament is that workers are paid based on the rank of their output relative to others rather than the level output. Prize levels are set in advance and competition to win generates the incentive to exert effort to increase output.

Lazear & Rosen (1981) showed that rank-order tournaments can provide an efficient incentive framework when workers are risk neutral. A substantial literature has compared the efficiency of tournaments with alternative incentive schemes, such as piece rates, with regard to
such factors as the risk-aversion of workers and the flexibility of the incentive framework to environmental uncertainty (e.g. Nalebuff & Stiglitz, 1983). However, little work has explored incentives in a tournament setting when workers choose not solely how much work effort to exert, but some other aspect of the work that is undertaken. In particular, executives may have substantial scope for influencing the output variance, or risk, of projects they undertake. Furthermore, they may be able to influence the mean output through choices other than work effort, such as by choice of production process or regulatory compliance.\(^1\) When monitoring is imperfect it is likely that workers’ incentives will not be perfectly aligned with those of their employer because they will have opportunities to increase their probability of winning the tournament by engaging in activities which do not serve the interest of the firm.

The most significant line of research in this area has been exploring “influence activities”: behavior that arises when workers can influence the choice of superiors regarding who is promoted or otherwise rewarded in an organization through actions which are non-productive, ranging from ingratiatingation to bribery and sabotage of competitors (Milgrom and Roberts, 1988; Prendergast and Topel, 1996; Kim et al., 2002; and Chen, 2003). Bronars (1987) was the first to address risk preference in tournament settings, and showed that leaders in sequential tournaments have an incentive to be risk-averse while followers are risk-seeking. Hvide (2002) first modeled a tournament setting where workers simultaneously choose both effort and risk, as we do here. Unlike our model, Hvide assumed that workers could costlessly alter the variance of their output, which yields significantly different implications as we describe below.

\(^1\) Nalebuff & Stiglitz (1983) term this issue the influence of prize on choice of technique, but do not model the problem. Stiglitz & Weiss (1981) address a related problem of the influence of the interest rate in bank lending on the risk involved in projects undertaken by borrowers.
Similarly to Hvide (2002) we develop a Lazear-Rosen tournament model in which players can influence both the mean and variance of their output distribution. In our model players incur a cost of increasing the variance of the output distribution, which reflects costs of searching for high-risk projects or a lower mean output (holding effort constant) associated with very risky projects.

The primary result from this model is that *multiplayer* \(n \geq 2\) tournaments which award a single prize to the winner (or single penalty to the loser) generate an incentive for inefficient risk seeking (risk aversion). This arises because, when workers are identical and effort levels are symmetric in equilibrium, one worker’s increasing the variance of his output distribution increases the probabilities of his ranking both first and last when facing multiple opponents. Hvide examined 2-player tournaments where players never incur a cost of increasing output variance and found that, if variance is unbounded the unique equilibrium involves both players choosing infinite variance and zero effort. When variance is bounded players simply choose the maximum variance and a sufficiently high prize spread will generate efficient incentives; inefficiency arises in this case only if players are risk averse. Conversely, our results illustrate the importance of the number of players competing in the tournament on the risk-incentives, and show that inefficiency arises from pursuit of high (or low) risk projects without relying on risk-averse preferences among players or the notion of infinite variance, which, as Hvide acknowledged, is “somewhat unclear”.

An important secondary result is that the prize structure of the tournament can be manipulated to eliminate the risk incentives generated by multiplayer tournaments. This arises from the fact that increasing output variance symmetrically increases the probabilities of ranking first and last. Therefore if a “penalty” is imposed on the lowest ranking player exactly equal to
the “prize” awarded to the winner the incentives precisely balance and players will not have an incentive to pursue riskier projects. More generally, the model illustrates the importance of the use of penalties for poor performance (“sticks”) as well as rewards for high performance (“carrots”) in rank-dependent compensation, and how their relative magnitudes influence the decision-making of workers.

This paper is organized as follows: section 2 models the choice of variance in a tournament with a single prize to the winner. Sections 3 shows that balancing the prize to the winner of the tournament with an equal penalty to the loser eliminates the workers’ incentive to pursue risky projects. Section 4 addresses the analogous issue of risk-aversion arising in tournaments with a penalty to the lowest-ranked player and no reward for finishing first. Section 5 concludes with a discussion of general implications regarding rank-dependant payoffs and risk preferences.

2 The Model

We consider a tournament where each of \( n \) identical risk-neutral players determines the mean and variance of the distribution of output he will produce, \( q_i \sim N(\mu, \sigma^2) \), which we assume to be normally distributed for simplicity and concreteness. Work effort directly determines the mean with cost of effort defined as \( c(\mu) \) and \( c'(\mu), c''(\mu) > 0 \). We assume there is exogenous variation in individual output, which determines the natural variance in output \( \sigma \). The individual can then choose \( \sigma \in [\sigma, \infty] \) with \( z(\sigma) = 0; z', z'' > 0 \). The notion here is that there is a natural level of uncertainty regarding the output of the type of projects the individual is employed to undertake, but he can achieve a mean-preserving increase in the spread of the distribution by seeking out unusually risky projects. Doing so, however, requires time and effort.
and is costly. Alternatively, one could assume that increasing the variance of the output
distribution does not entail a direct cost, but that beyond some point increasing the variance is
possible only by choosing a project with a reduced mean outcome—an assumption with similar
implications but which increases the complexity of the model.\(^2\)

Our assumption that it is costly for individuals to increase the variance of their output is
central to the results we derive and merits some discussion. In particular, it may seem strangely
at odds with the common notion that investment decisions typically involve a trade-off of greater
expected returns in exchange for incurring increased risk. Our view is that workers have some
discretion in the projects they pursue which includes a limited ability to vary the risk incurred,
but that the nature of the firms’ activities establishes a natural baseline variance in output,
represented by \(\sigma\) in our model. Workers can use their discretion to pursue projects that entail
greater risk, but doing so requires effort on their part to push the usual bounds of the firms’
activities or their role in the organization and may also involve choosing projects with a lower
expected return. We essentially assume that workers cannot transfer risk costlessly: if they wish
to increase output variance from its natural level this must involve some cost. Similarly, if an
individual wished to reduce output variance below the natural level they could do so but only at a
cost. An incentive to engage in such risk-averse behavior would arise in a tournament where no
prize is awarded, only a penalty is meted out to the player who finishes last. We address this
scenario in greater depth in section 4 below. We emphasize that the natural variance (\(\sigma\)) in
workers’ output is exogenously determined and the total variance can be shifted only at some
cost.

\(^2\) Greater complexity arises because in such a model the cost of increasing output variance is indirect (through the
effect on the probability of winning) rather than direct, but the results parallel those presented here.
We follow the Lazear and Rosen (1981) tournament model in which individuals sell their labor in a competitive market where the value of their output is $V$ per unit. Initially we assume that there are only two prize levels, a payoff of $W_1$ to the highest ranked player and $W_2$ to all others. Applying the usual Cournot-type assumption that players optimize over the choice variables given their beliefs regarding the choices made by other players, we first solve for the workers’ best response functions and then impose symmetry to find their choices of effort and variance in Nash equilibrium as a function of the prizes. We then find the prize pair that maximizes workers’ expected utility subject to the constraint that firms earn zero profits (which arises from the assumption of a competitive labor market).

Let $P$ represent the probability of winning the larger payoff $W_1$, then workers maximize

$$\max_{\mu_i,\sigma_i} \pi_i = P_1 W_1 + (1 - P_1) W_2 - c(\mu_i) - z(\sigma_i)$$

$$= P_1 (W_1 - W_2) + W_2 - c(\mu_i) - z(\sigma_i)$$

(1)

The probability of winning is a function of the probability density function chosen by agent $i$, which we will denote $f(x)$, and the cumulative distribution function $G(x)$ representing player $i$’s expected probability of winning for any realized output, which is a function of his beliefs regarding the choices of the other players:

$$P_i = \int f(x) (G(x))^{n-1} dx.$$  

(2)

Expression (2) indicates that the probability of player $i$ winning the tournament is found by integrating over the player’s own density function (which is determined by his choice of effort

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3 Lazear and Rosen found that, where players choose only effort, it is sufficient to have only two payoffs to achieve efficiency from the tournament and that results from 2-player tournaments generalize to n-players. They also note that in their 2-player tournament model when individuals are risk-neutral the solution is not affected by the ability of individual’s to choose the variance. However, although these statements are separately correct, they do not hold in combination. In other words, we show that when there are more than two players the solution is affected by the ability of individuals to choose risk, and tournament incentives differ when more than two possible payoffs are considered.
and variance) multiplied by the probability of winning associated with each possible output that he may realize, represented by the expression \((G(x))^{n-1}\). Notice that as the number of opponents increases, the shape of this function changes: with two players (a single opponent) the function is symmetric around the opponent’s mean output. But as the number of players increases the function becomes increasingly skewed to the right of the opponents’ mean output, reflecting the fact that the probability of winning when facing multiple opponents remains very small unless one achieves an output substantially greater than the opponents’ mean.

Utilizing the assumption of normality on \(f\), we can write

\[
P_i = \int \frac{1}{\sigma_i \sqrt{2\pi}} e^{\frac{(x-\mu_i)^2}{2\sigma_i^2}} (G(x))^{n-1} \, dx.
\] (3)

Of course the probability of winning is strictly increasing in effort, as shown by the following expression:

\[
\frac{\partial P_i}{\partial \mu_i} = \int \frac{x - \mu_i}{\sigma_i^2} f(x) (G(x))^{n-1} \, dx > 0.
\] (4)

The effect of \(\sigma\) on the probability of winning is given by:

\[
\frac{\partial P_i}{\partial \sigma_i} = \int \frac{1}{\sigma} f(x) \left( \frac{(x - \mu_i)^2}{\sigma_i^2} - 1 \right) (G(x))^{n-1} \, dx.
\] (5)

PROPOSITION 1: For \(n>2\), when players’ effort levels are symmetric (\(\mu_i = \mu_{-i}\) for all \(i\)), each player’s probability of winning is increasing in his choice of output variance: \(\frac{\partial P_i}{\partial \sigma_i} > 0\). (Proof in the appendix.)
To see this, first note that the expression in (5) equals zero for \( n=2 \): for any two symmetric distributions with the same mean the probability of receiving a higher draw on one distribution is not affected by increasing the variance. However, for \( n>2 \) the \( G(x))^{n-1} \) term is no longer symmetric around \( \mu_i \), its mass is shifted to the right as discussed earlier. Of course increasing the variance of \( f \) reduces mass in the center of the distribution and places more of the mass in the tails. Increasing the mass in the left tail has only a small negative impact on the probability of winning since outcomes below the symmetric mean are extremely unlikely to win in any event, but increasing the mass in the right tail substantially increases the probability of winning because in a multiplayer competition a right-tail outcome is usually necessary to win. Moving probability mass from the center of the distribution to the tails increases the probability of finishing both first and last, but it is only the former that is relevant in this setting.

We can now solve for the symmetric Nash equilibrium of the game as a function of the payoffs. Nash best response functions are defined by the first-order conditions:

\[
\mu_i : (W_i - W_z) \frac{\partial P_i}{\partial \mu_i} - c'(\mu_i) = 0
\]

(6)

\[
\sigma_i : (W_i - W_z) \frac{\partial P_i}{\partial \sigma_i} - z'(\sigma_i) = 0.
\]

(7)

Imposing symmetry we find the Nash equilibrium is described by

\[
c'(\mu_i) = (W_i - W_z) \int \frac{x - \mu_i}{\sigma_i^2} f(x)(F(x))^{n-1} dx
\]

(8)

\[
z'(\sigma_i) = (W_i - W_z) \int \frac{1}{\sigma} f(x) \left( \frac{(x - \mu_i)^2}{\sigma_i^2} - 1 \right) (F(x))^{n-1} dx.
\]

(9)
By proposition 1 above the right hand side of (9) must be positive for \( n > 2 \), which allows us to state the following:

PROPOSITION 2: For \( n > 2 \) in the symmetric Nash equilibrium individuals choose \( \sigma > \sigma \), which is inefficient since it entails costly effort without raising expected output. Furthermore, players’ equilibrium choice of variance (and the magnitude of inefficiency) is increasing in the prize spread \( (W_1 - W_2) \).

We now solve for equilibrium prizes and choices of effort and risk. Let \( \mu \) and \( \sigma \) represent the symmetric choices of effort and risk of all players in Nash equilibrium. Then a firm’s expected revenue is the product of the number of workers it employs, their mean output, and the value of the output: \( n\mu V \). Competition in the labor market ensures expected revenues equal total prizes (or equivalently, the average prize equals average value of output \( V\mu \))

\[
nV\mu = W_1 + W_2(n-1) \quad \text{or} \quad V\mu = \frac{W_1 + W_2(n-1)}{n} = \frac{1}{n}(W_1 - W_2) + W_2. \tag{10}
\]

Noting that at symmetric equilibrium \( P_i = 1/n \) for all \( i \), then workers’ expected payoffs are

\[
\pi_i = \frac{1}{n}(W_1 - W_2) + W_2 - c(\mu) - z(\sigma), \quad \text{substituting from (11) above} \tag{12}
\]

\[
\pi_i = V\mu - c(\mu) - z(\sigma). \tag{13}
\]

Firms competing for workers choose the prize spread to maximize this expected payoff, and this maximum is found where

\[
[V - c'(\mu)](\partial \mu / \partial W_j) = z'(\sigma) \frac{\partial \sigma}{\partial W_j}, \quad j = 1, 2. \tag{14}
\]
PROPOSITION 3: For \( n > 2 \), since workers chose \( \sigma > \overline{\sigma} \) and \( z'(\sigma) > 0 \), expression (14) implies that \( V > c'(\mu) \) and therefore indicates that workers exert suboptimal effort.

In multiplayer tournaments increasing the prize spread increases the incentive to wastefully pursue risk while also encouraging greater work effort. Therefore, in competitive equilibrium the prize spread will not be sufficient to induce efficient effort.

3 Sticks and Carrots

The fact that the problem of wasteful risk-seeking arises in winner-take-all multiplayer tournaments suggests that the risk-seeking incentive may be eliminated by structuring the tournament with more than 2 prize levels. In particular, an obvious candidate is to penalize the player who ranks last as well as rewarding the player who ranks first and thereby establish symmetry in the game’s payoffs. We now consider a multiplayer tournament with three payoffs: \( W_1 > W_2 > W_3 \). Let \( P_i \) represent an individual’s probability of ranking first and earning \( W_1 \) as before, and \( R_i \) be the probability of ranking last and receiving \( W_3 \). An individual’s expected utility is now

\[
\pi_i = P(W_1 - W_2) - R(W_2 - W_3) - c(\mu) - z(\sigma)
\]  

(15)

with the probability of receiving \( W_i \) just as before

\[
P_i = \int \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} (G(x))^{\epsilon-1} dx
\]  

(16)

and the probability of receiving \( W_3 \)
\[ R_i = \int \frac{1}{\sigma_i \sqrt{2\pi}} e^{-\frac{(x - \mu_i)^2}{2\sigma_i^2}} (1 - G(x))^{\mu_i - 1} dx. \]  

(17)

The probability of finishing last is of course decreasing in effort

\[ \frac{\partial R_i}{\partial \mu_i} = \int \frac{x - \mu_i}{\sigma_i^2} f(x) (1 - G(x))^{\mu_i - 1} dx < 0. \]  

(18)

Just as the probability of finishing first is increasing in \( \sigma \) at symmetric effort levels, the probability of finishing last is also increasing in \( \sigma \)

\[ \frac{\partial R_i}{\partial \sigma_i} = \int \frac{1}{\sigma_i} f(x) \left( \frac{(x - \mu_i)^2}{\sigma_i^2} - 1 \right) (1 - G(x))^{\mu_i - 1} dx > 0 \text{ for } \mu_i = \mu_i. \]  

(19)

Furthermore, by symmetry of the \( G \) distribution around the common mean output \( \mu \),

\[ (1 - G(x))^{\mu - 1} = (G(x))^{\mu - 1} \text{ at } \mu. \]  

Therefore \( \frac{\partial R_i}{\partial \sigma_i} = \frac{\partial P_i}{\partial \sigma_i} \) at the symmetric Nash equilibrium.

The first order conditions are

\[ \mu_i : (W_1 - W_2) \frac{\partial P_i}{\partial \mu_i} - (W_2 - W_3) \frac{\partial R_i}{\partial \mu_i} - C'(\sigma) = 0 \]  

(20)

\[ \sigma_i : (W_1 - W_2) \frac{\partial P_i}{\partial \sigma_i} - (W_2 - W_3) \frac{\partial R_i}{\partial \sigma_i} - z'(\sigma) = 0. \]  

(21)

PROPOSITION 4: In a multiplayer tournament when winning and losing prize spreads are equal, \( (W_1 - W_2) = (W_2 - W_3) \), expression (21) indicates that workers choose not to expend effort pursuing high risk projects \( z'(\sigma) = 0 \). The tournament will therefore generate efficient incentives with \( c'(\mu) = V \) at the competitive equilibrium.
This result suggests that in multiplayer tournament settings where workers have scope for risk-seeking which cannot be perfectly monitored by the employer it is important that the tournament design incorporate both a carrot and a stick. The inclusion of a penalty for ranking last exposes the worker to both right and left tail effects from increasing output dispersion, thereby eliminating the incentive to pursue risky projects.

4 Risk Aversion in Tournaments with No Prize, Only a Penalty

Although it is conventional to model tournaments as settings in which workers compete for a high payoff or “prize”, it is easy to conceive of settings in which little reward is paid to workers with top-ranking output, but significant penalties are applied to those who finish last. For example, this may apply to organizations which offer little opportunity for promotion or advancement, but where relatively poorly performing workers are likely to be dismissed. Of course in two-player tournaments it is meaningless whether the higher payoff is designated a prize to the winner or the lower payoff a penalty to the loser. But multiplayer tournaments with a single penalty to the player with the lowest output generate very different risk-taking incentives than those in which a single prize is awarded to the player who finishes first.

The model discussed above can easily be altered to show that, when the tournament imposes only a penalty on the lowest-ranking player and workers can reduce the variance of their output but incur a cost of doing so, workers will engage in inefficient risk-aversion. All the results are parallel: for any symmetric effort level workers reduce their probability of finishing last by reducing the variance of output. This also reduces the probability of finish first, but since there is no prize for doing so workers ignore this effect. Workers therefore wastefully seek to reduce the variance of output, and the incentive to do so increases with the magnitude of the
penalty associated with finishing last. Just as in the winner-take-all multiplayer tournament described earlier, worker effort will be below the first-best in the symmetric equilibrium.

5 Conclusion

In many labor market settings performance incentives are likely to be a complex mix of monetary and non-monetary payoffs, some of which have the rank-dependant character of a tournament, while others are closer to piece-rates or playing against a standard (such as meeting a quota) in character. We believe the model presented here provides important insight into the manner by which the balance of payoffs—carrots versus sticks—influences the decision-making of workers. When rank-dependent payoffs involve primarily positive rewards such as bonuses and promotion for the few most productive workers, with comparatively little likelihood of dismissal or other penalties for those who rank at the bottom, workers will be encouraged to pursue high-risk strategies. Conversely, if rewards are few but unusually poor performers consistently face dismissal or other rebukes, workers will be discouraged from taking risks in their activities.

These results suggest that firms are likely to use the balance of payoffs to influence the risk preferences of their workers. In particular, large rewards for top-ranked performance may be a means for firms in industries where a willingness to take large risks is critical to success to balance the inherent risk-aversion of most employees. Firms in other industries where risk-taking by workers is undesirable may use primarily negative incentives to motivate employees without encouraging the risk-taking that bonuses and other positive incentives generate. In general, the nature of compensation and differences among firms and industries will reflect firms’ efforts not
just to motivate employee’s work effort, but to influence the decisions they make and the risks they incur on behalf of the firm.
Appendix A: Proofs

PROOF OF PROPOSITION 1: We first show that for \( n=2 \) \( P_i=1/2 \) for any \( \sigma_i \). This follows from the symmetry of both the density \( f \) and distribution \( G \) around a common mean \( \mu \):

\[
P_i = \int f(x)(G(x))^{n-1} \, dx = \int \left[ f(x)G(x) + f(x)(1-G(x)) \right] \, dx = \int_{-\infty}^{\infty} f(x) \, dx = \frac{1}{2}.
\]  

(22)

Since for \( n=2 \) \( P_i=1/2 \) at any symmetric effort equilibrium with common \( \mu \) then \( \frac{\partial P_i}{\partial \sigma_i} = 0 \) for \( n=2 \).

From equation (5) we know that generally

\[
\frac{\partial P_i}{\partial \sigma_i} = \int \frac{1}{\sigma} f(x) \left( \frac{x - \mu_i}{\sigma_i^2} - 1 \right) (G(x))^{n-1} \, dx.
\]

Notice that the integrand of this expression contains three terms, each of which is symmetric around \( \mu \) for \( n=2 \), and the integrand takes on negative values for \( \mu_i - \sigma_i > x > \mu_i + \sigma_i \) and positive values outside this range. The negative and positive values precisely balance, rendering the expression equal to zero. For \( n>2 \) the last term becomes asymmetrically skewed to the right, reducing the value of the integrand compared to the \( n=2 \) case for all \( x \), but reducing it progressively less for higher values of \( x \). This necessarily renders the expression positive by placing greater weight on the positive values of the integrand found where \( x > \mu_i + \sigma_i \) relative to the negative values that occur around \( \mu \).
References


