Chapter 9
Supersonic and Transonic Flows

Effect of Compressibility on Streamlines

3-D, Steady, Compressible, Inviscid

• Continuity
  \[ \frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} + \frac{\partial (\rho w)}{\partial z} = 0 \]

• Momentum (neglecting body forces)
  \[ x: \quad \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial x} \]
  \[ y: \quad \frac{\partial v}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial v}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial y} \]
  \[ z: \quad \frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial z} \]

Perturbation Velocity Components

\[ u = U_\infty + u' \]
\[ v = v' \]

Coordinate Transformation

Linearized Transformation for a Wing

Note: corresponding points in each flow have the same \( \phi \) values.
Effect of Compressibility ($C_{l0}$)

\[ \frac{dC_l}{dM} = \frac{2a}{\pi(1 - M^2)} \]

Mach effect on drag for a P-38.

Transonic Flow Characteristics

Mach Distribution on an Airfoil

Mach number $M$ for different velocities:

- $M = 0.75$
- $M = 0.81$
- $M = 0.89$
- $M = 0.98$
- $M > 1.0$

C$_p$ for a NACA 0012 at $M_{\infty} = 0.8$

Lift Coefficient for NACA 0012
Condensation Shocks

Super Critical Airfoils

Wing Sweep

Variable Geometry Wings

Variable Geometry Wings

Pivot Wing
Whitcomb’s Area Rule

Convair F-102 and F-102A

Convair F-102A

T-38

Second Order Effects

Boeing 747

Ideally, we should also account for the stream tube expansion caused by compressible flow.
Forward Swept Wings

Sukhoi S37