4.3. Consider two-dimensional, incompressible flow over a cylinder. For ease of use with the nomenclature of the current chapter, we will assume that the windward plane of symmetry (i.e., the stagnation point) is $\theta = 0$ and that $\theta$ increases in the streamwise direction. Thus,

$$u_r = 2U_\infty \sin \theta \quad \text{and} \quad x = R\theta$$

Determine the values of $\beta$, at $\theta = 30^\circ$, at $\theta = 45^\circ$, and at $\theta = 90^\circ$.

$$\beta = \frac{2S}{U_\infty} \frac{du_r}{ds}$$

Find $S$

$$S(\theta) = \int_0^\theta U_\infty \, dx = \int_0^\theta 2U_\infty \sin \theta \, d\theta = 2U_\infty R \int_0^\theta \sin \theta \, d\theta$$

$$S = 2U_\infty R [-\cos \theta]_0^\theta = 2U_\infty R [1 - \cos \theta]$$

So, $\frac{du_r}{ds} = \frac{du_r}{d\theta} \frac{d\theta}{ds}$

$$\frac{du_r}{ds} = \frac{\cos \theta}{R \sin \theta}$$

Now

$$\beta = 2 \frac{2U_\infty R [1 - \cos \theta]}{2U_\infty \sin \theta} \left( \frac{\cos \theta}{R \sin \theta} \right)$$

$$\beta = \frac{2 \cos \theta [1 - \cos \theta]}{\sin^2 \theta}$$

$\theta = 30^\circ$

$$\beta = \frac{2 \cos(30) [1 - \cos(30)]}{\sin^2(30)}$$

$$\beta(30^\circ) = 0.9287$$

$\theta = 45^\circ$

$$\beta = \frac{2 \cos(45) [1 - \cos(45)]}{\sin^2(45)}$$

$$\beta(45^\circ) = 0.8284$$

$\theta = 90^\circ$

$$\beta = \frac{2 \cos(90) [1 - \cos(90)]}{\sin^2(90)}$$

$$\beta(90^\circ) = 0$$