AE 422
Homework

3s.3 In a previous problem you were asked to determine the drag and base bending moment on a radio antenna to be mounted vertically on the top of a VW Beetle. Using the information provided you found the drag and bending moment produced at the antenna base assuming the antenna was exposed to $V_\infty = 100$ mph. This provided an estimate, but did not take into account the change in the flow filed caused by the car itself.

Find the drag of the antenna by modeling the car as a cylinder in cross-flow. You can find the velocity field using potential flow and then integrate the drag along the length of the antenna (located at $\theta = \pi/2$) using the local velocity.

The antenna is 2.5 ft long with a diameter of 1/8". Aerodynamic tests have shown that in the speed range of interest a two-dimensional cylinder has a drag coefficient of 1.3 based on frontal area. Assuming the car has a height of 5 feet and is traveling at 100 mph, model the car as a cylinder with a diameter of 10 feet with $U_\infty = 100$ mph.

1) Use this information to estimate the total drag of the antenna. How does this value compare to the drag found in problem 3s.2? Where would you expect the drag of the actual antenna to fall?

2) Plot the velocity distribution along the length of the antenna using potential theory.

3) The moment for the problem 3s.2 was found by the resultant drag was located at $1/2$ the antenna length. Describe approximately where the resultant force would be located for this problem and why.

For a cylinder in cross-flow

\[ V_0 = -U_\infty \sin(\theta) \left( 1 + \frac{R^2}{r^2} \right) \]

\[ V_r = U_\infty \cos(\theta) \left( 1 - \frac{R^2}{r^2} \right) \]

The antenna is located at $\theta = \pi/2$ so $\sin(\theta) = 1$ $\cos(\theta) = 0$

and

\[ V_0 = -U_\infty \left( 1 + \frac{R^2}{r^2} \right) \quad V_r = 0 \]
1) Find the Drag $D = \frac{1}{2} \rho V^2 S C_D$

\[
D = \frac{1}{2} \rho C_D \int_{R_0}^{R} V_0^2 r dr = \frac{1}{2} \rho C_D \int_{R_0}^{R} V_0^2 (1 + \frac{R^3}{r^3}) dr
\]

\[
D = \frac{1}{2} \rho C_D V_0^2 \left\{ \int_{R}^{R_0} r dr + \int_{R}^{R} \frac{R^2}{r^2} dr + \int_{R}^{R_0} \frac{R^4}{r^4} dr \right\}
\]

\[
D = \frac{1}{2} \rho C_D V_0^2 \left\{ [r]_{R}^{R_0} - 2R^2 \left[ \frac{1}{r^2} \right]_{R}^{R_0} - \frac{R^4}{3} \left[ \frac{1}{r^3} \right]_{R}^{R_0} \right\}
\]

\[
D = \frac{1}{2} \rho C_D V_0^2 \left\{ (R_0 - R) - 2R^2 \left( \frac{1}{R_0} - \frac{1}{R} \right) - \frac{R^4}{3} \left( \frac{1}{R_0^3} - \frac{1}{R^3} \right) \right\}
\]

Assume $\rho = 0.002377 \text{ slug/ft}^3$

\[
D = \frac{1}{2} (0.002377)(1.3)(0.01042)(146.7)^2 \left\{ (7.5 - 5) - 2(5) \left( \frac{1}{7.5} - \frac{1}{5} \right) \right\}
\]

\[
D = 0.3463 \left\{ 2.5 + 3.333 + 1.174 \right\}
\]

\[
D = 2.43 \text{ lb}
\]

This is 2.8 times the value found previously ($D = 0.86554$).

The actual drag will be between these values because the car is 3-D which will reduce the velocity flow field from the 2-D case.

b) see next sheet

c) The resultant was in the center of the antenna on the first problem because we had a uniform $V$ dist. It will be close to the base in reality because of higher $V$ at lower $r$. 