17.2 – Sample Aliasing

**Analog to Digital Conversion**
- Analog signals are continuous in time and output.
- Digital signals are broken into discrete “steps”

**Conversion Issues** – How many of these “steps” are required to accurately represent an analog signal? This depends on:
1) Frequency content of measure analog system
2) Increment size between each discrete sample
3) The total sample period
4) Behavior of signal to be measured (periodic?)

**Sampling Rate and Aliasing**

**Sample Rate**
\[ f_s = \frac{1}{d t} \]

**Aliasing**
If an insufficient sample rate is used the signal measured will differ from the actual signal.

**Nyquist Sampling Theorem**
A sample rate of at least two times the maximum signal frequency is required to prevent aliasing.
\[ f_{sc} > 2f_n \]

**Nyquist Frequency**
\[ f_N = \frac{f_s}{2} \]
7.3 Digital devices

Systems use:

- discrete steps in time and amplitude.
- binary system
  - Base 2 system. It is on or off.
  - A digit or bit is the smallest unit of measure (0, 1)
  - A word is a collection of bits used to express a number.
  - An 8 bit word is a byte.
- a physical location used to store a word is a register.

- We use different combinations of bits to form words representing decimal (base 10) numbers.

<table>
<thead>
<tr>
<th>Bit Width</th>
<th>Combinations</th>
<th>Decimal Integer</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 bit</td>
<td>$2^2 = 4$</td>
<td>0, 1, 2, 3</td>
</tr>
<tr>
<td>4 bit</td>
<td>$2^4 = 16$</td>
<td>0 to 15</td>
</tr>
<tr>
<td>8 bit</td>
<td>$2^8 = 256$</td>
<td>0 to 256</td>
</tr>
<tr>
<td>16 bit</td>
<td>$2^{16} = 65,536$</td>
<td>0 to 65,535</td>
</tr>
</tbody>
</table>

- Conversion from Binary to Decimal
  a) Straight Binary Code (5 bit example)

<table>
<thead>
<tr>
<th>Bit 4</th>
<th>Bit 3</th>
<th>Bit 2</th>
<th>Bit 1</th>
<th>Bit 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 x $2^4 = 16$</td>
<td>0 x $2^3 = 0$</td>
<td>1 x $2^2 = 4$</td>
<td>0 x $2^1 = 0$</td>
<td>1 x $2^0 = 1$</td>
</tr>
<tr>
<td>16 + 0 + 4 + 1 = 21</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b) Binary Coded Decimal

- In this system, each digit of a decimal number is individually coded into binary.
- EX. 532 is represented by

<table>
<thead>
<tr>
<th>0</th>
<th>0</th>
<th>1</th>
<th>1</th>
<th>0</th>
<th>1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>3</td>
<td>2</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
</tbody>
</table>

Using the Folding Diagram

- $f_s(f_s/2)$ is the maximum frequency that can be represented (measured).
- If the signal is a higher frequency, this will result in an aliased signal with an output < $f_s$.

Example

- Actual Signal to be Measured
  - $f_{signal} = 168$ Hz
- Sample Frequency
  - $f_s = 200$ Hz
- Nyquist Frequency
  - $f_N = f_s/2 = 100$ Hz
- Result
  - Divide $f_{signal}$ by $f_s$ to get $f_{signal}/f_s = 1.68$
  - *find this point on the folding diagram and drop straight down to get 0.32
  - *So, our output (what we read) will be 0.32$f_s$ = 32 Hz, so $f_{measured} = 32$ Hz!!!
  - Due to low sampling rate, we measure a wave with $f = 168$ Hz as $f = 32$ Hz!!!
7.4 Transmitting Digital Numbers

- Binary code is signaled using voltage "switches"
- High voltage represents 1 (on) and low 0 (off)

![Image](image)

7.5 Voltage Measurements

**Analog to Digital Converter**
- Analog Side: $E_{FSR}$ (Full Scale Range)
  - Ex. 10 V
- Digital Side: $2^M$ binary numbers
  - Ex. 8 bit (256 steps)
- Resolution

$$ Q = \frac{E_{FSR}}{2^M} = \frac{10V}{2^8} = 0.03906 \text{ V/count} $$

**Saturation**

If a signal that is above or below the $E_{FSR}$ is measured it will be converted at the limit value.
- Ex. 11 V will give the same reading as 10 V

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**A/D Errors**

1) **Quantization**
   - Error between the actual voltage and the indicated voltage.
   - Ex. 3 V at $2^2$ give outputs of 0, 1, 2, 3

Ex. 0-4 Volt board with a 0-10 "count" output.

2) **Saturation Error**
   - Error associated with exceeding the limits of the A/D converter.
   - Ex. Using the above, 5 V will be converted as 3 V for an error of $5 - 3 = 2$ V

3) **Conversion Error**
   - A/D Problems such as settling time, signal noise, temperature effects, power fluctuations, etc.
   - Yield errors like hysteresis, linearity, zero drift, repeatability

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**A/D Errors (cont.)**

Signal-to-Noise Ratio (SNR) due to Quantization

Consider the power of the signal given by Ohm’s law

$$\frac{E^2}{R}$$

and the power that can be resolved by the converter

$$\frac{E^2}{R2^M}$$

(where $M$ is the number of bits)

then the ratio of these in terms of decibels is

$$\text{SNR}[\text{dB}] = 20 \log(2^M)$$