EXERCISES IN ENGINEERING EXPERIMENTATION:

A LABORATORY MANUAL FOR MECHANICAL, AEROSPACE, & BIOMEDICAL ENGINEERING (MABE) 345

2nd Edition
STUDENT VERSION

Joseph C. McBride
Dr. William S. Johnson

University of Tennessee, Knoxville
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FOREWORD

This lab manual is the second edition of *Exercises in Engineering Experimentation* originally written by Dr. William S. Johnson of the University of Tennessee, Knoxville to serve as an instruction manual for lab experiments conducted in conjunction with the course MABE 345: Instrumentation and Measurement offered through the Mechanical, Aerospace, and Biomedical Engineering Department of the College of Engineering at the University of Tennessee, Knoxville.

This latest edition incorporates additions to the original work which are designed to improve student understanding of the theory involved in each of the experiments presented. Theory sections have been added for each experiment to present essential information on the background scientific, mathematical, and/or statistical concepts which the experiments employ and are designed to reinforce. Necessary equations and concepts needed to complete the experiments are clearly presented in a coherent and logic manner in order to reduce the need for reference materials beyond this manual.

Pre- and post-lab questions have been added to test students’ grasp of essential concepts of background theory and observed results of experiments. The procedures for each lab have been rewritten to improve clarity and to update technical concepts.

Additionally, the INTRODUCTION section of this manual has been revamped to emphasize the importance of technical writing skills in engineering practice. The introduction includes a brief discussion of the essential components of and concepts needed for the writing of technical reports. A list of useful online reference materials for technical writing is also provided. To further emphasize the principles presented in the INTRODUCTION section, a template for technical lab reports on the experiments presented in this manual is provided in the section entitled TECHNICAL LAB REPORT TEMPLATE.

This edition of *Exercises in Engineering Experimentation* is built on the foundation of the original work of Dr. William S. Johnson of the University of Tennessee, Knoxville, whose contributions over the years to the instruction of MABE 345: Instrumentation and Measurement, the course for which this lab manual is intended, is acknowledged and greatly appreciated.

Sincerely,

Joseph C. McBride  
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University of Tennessee, Knoxville
INTRODUCTION

Overview

This lab manual has been composed to serve as an instruction manual for performing lab experiments designed to reinforce concepts presented in the course MABE 345: Instrumentation and Measurement offered through the Mechanical, aerospace, and Biomedical Engineering Department of the College of Engineering at the University of Tennessee, Knoxville. This manual contains background theory, detailed instructions, and data sheets for performing various experiments designed to illustrate basic concepts in analysis of instrumentation and measurement systems. Details on the function and operation of equipment used in the experiments are also provided.

The experiments presented in this lab manual offer students the opportunity for practicing technical writing by reporting on the results of experiments. An introduction to important concepts in technical writing and an example template for technical lab reports are included in this section and the next section entitled TECHNICAL LAB REPORT TEMPLATE. These sections have been added in this edition to emphasize the importance of technical writing in engineering and to provide students with a starting point for further developing their technical writing skills.

Introduction to Technical Writing

As an engineer, the importance of being able to write well cannot be overstated. According to a study published in the *The Technical Writing Teacher* in 1982, on average, engineers spend 25% of their job-related work time writing, 23% reading technical and other business related materials, 11% supervising the technical writing of others, and 7% giving oral presentations—thus, on average, 65% of an engineer’s time is occupied by the communication of information through writing or speaking. Moreover, this percentage was reported to increase as the position of an engineer in the company hierarchy increased, especially in the form of critiquing the writing of others [1]. Since the time of this study, the common use of email and other forms of electronic documentation have led to a greater increase in the time spent by engineers in preparing technical documents and communicating information. Communication has an important role in the success of a company and the success of oneself as an engineer. Poor technical writing skills will stall one’s career early.

Technical reports present facts and conclusions about projects, experiments, designs, etc. in a clear and concise manner. Generally, technical reports include technical concepts and graphical representations of data or designs and can include a wide range of types of technical information. There are many different formats for technical reports. For example, one may need to write a report describing the reasons behind the failure of a specific piece of equipment, in which case one would write a forensic report. If one were writing to describe the design of a new piece of equipment, one would use a design report format. If one wanted to also include the new design’s subsequent failure, one would write a combined forensic and design report.
To report the results of an experiment, one would generally use a lab report format. Laboratory reports are written for several reasons, such as to communicate laboratory results to management level employees or supervisors. Management decisions are often based on the results presented in technical lab reports; therefore, the ability to write clearly and concisely, and thereby improve the odds of management correctly comprehending the results of lab experiments, is crucial.

It is important to keep one’s audience in mind when writing a technical report. Depending on the intended audience, it may be necessary to include descriptions or definitions of concepts or equipment which may be common knowledge to the author, but which may not be known by members of the audience. As a student, one may assume that one’s technical reports’ audience is the laboratory instructor; however, this may not always be the case. The technical report template provided in the next section suggests that students assume that their audience has only a basic knowledge of engineering and is unfamiliar with the equipment and experiments presented in this manual. Lack of firsthand knowledge by the audience is often the case in practice. An audience with basic engineering knowledge is likely to understand the terminology used; however, one should always avoid technical jargon where possible. Technical reports should be written in such a manner that when other engineers read what has been written, they will be able to quickly locate and understand the information that interests them the most. In practice, engineers often must also be able to convey results or data to individuals without engineering knowledge. For this reason, the ability to describe data in “layman’s terms” is essential. Visual aids, including figures and tables, are also useful for conveying important information quickly and accurately.

The next section includes a template for writing technical lab reports on the results of the experiments presented in this manual. The text of the template provides additional information on the formatting, content, and other requirements expected in a technical lab report.

Below is a list of recommended resources for further information on technical writing:

Print Resources


Online Resources

- Pennsylvania State University’s website on technical writing. Available at www.writing engr.psu.edu.
- Colorado State University’s website on technical writing. Available at www.writing engr.colostate.edu.
- Swarthmore University’s website on technical writing. Available at www.swarthmore.edu/academics/writing-program/student-resources/engineering-writing-guide.xml.
EXAMPLE LAB REPORT TEMPLATE

A technical lab report should include a title or cover page. The cover page should clearly indicate the following: (1) the company name; (2) department name; (3) sub-department title; (4) identifying designation of the experiment; (5) the date of submission; (6) the name(s) of the author(s) with the corresponding/principal author clearly indicated; (7) the name of the intended recipient; and (8) contact information for corresponding author. Note: that headers and footers should not be used for a cover page. See the next page for a template of an appropriate cover page for one of the experiments in this manual.

Additional formatting and content information for technical lab reports to be prepared on the results of experiments in this manual are presented in the text of the lab report template beginning on the next page. Note: brackets ([ ]) indicate missing information which should be filled in appropriately; do not keep brackets after filling in.

In preparing technical lab reports for the experiments presented in this manual, the following standard information for the template provided should be used:

Sub-department: MABE 345: Instrumentation and Measurement, section [section number]
Note: May be abbreviated as MABE 345 ([section number])

Department: Mechanical, Aerospace, & Biomedical Engineering Department
Note: May be abbreviated as MABE Dept.

Super-department: College of Engineering

Company name: University of Tennessee, Knoxville

Date: Use “day-abbreviated month-year” format (e.g., 12 Apr. 2012)

Project designation: Experiment No. [#]: [title of experiment]
[date]

[project designation]

Technical brief prepared for:

[name of recipient]
[recipient’s title (e.g., Lab Instructor)]
[recipient’s sub-department]  
[recipient’s department]  
[recipient’s super-department]  
[recipient’s company name]

Prepared by:

[student’s name], [position (e.g., project leader)], corresponding author
[name of other relevant project member (e.g., lab partner’s name)], [position (e.g., lab partner)]
[name of other relevant project member (e.g., lab partner’s name)], [position (e.g., lab partner)]
[name of other relevant project member (e.g., lab partner’s name)], [position (e.g., lab partner)]

Please direct questions, comments, or concerns to [name of corresponding author] by [mode of communication (e.g., email)] at [corresponding author’s contact information (e.g., bobsmith@utk.edu)].
TECHNICAL BRIEF:

[PROJECT DESIGNATION]

Abstract

The abstract is a crucial part of any technical brief. In many cases, it is the only portion of your brief which will likely be read in full by the recipient. As such, it should provide a brief synopsis of the experiment, relevant results, and conclusions. The synopsis of the experiment should be concisely written and contain only as much information as is essential for the recipient to understand the premise and general procedure of the experiment. Avoid using technical jargon and symbols if possible. The abstract should also include a brief mention of relevant results and a few sentences describing the significance of the results/experiment itself and how the results/procedure can be applied to other problems (e.g., suggest what actions may be appropriate based on the results). For the experiments in this manual, it would be sufficient to discuss what concepts the experiment illustrates and a few possible applications of the concepts or techniques used in the experiment to real-life engineering problems. It is not uncommon to spend a significant time composing an abstract. It is often the last portion of the report to be written. Regarding formatting, the abstract should be inset relative to the rest of the text on the first page of the report. Alignment should be justified, as with the entire report. The word count should be kept to less than 250 words. A reduced font may be used but is not necessary.

I. Introduction

I.1. Content of Introduction

The Introduction section of a laboratory report should provide a brief description of the experiment conducted. It should include a list of objectives, explain the significance of the experiment, and provide a general theoretical background. The phrasing of objectives is important because these objectives are analyzed and assessed in the conclusion of the lab report to determine whether or not an experiment was successful. The background theory involved should be presented in a clear and cogent manner to allow the recipient to easily follow and understand the progression of concepts and how they are applicable/relevant to the current experiment. At the same time, one should avoid lengthy or overly detailed derivations and explanations which may confuse the recipient. It is often best to keep the language as simple as possible and only present that information which is necessary for the recipient to read and understand what was done in the experiment and how to interpret the results. Figures and tables may also be helpful as visual tools for conveying the theory, design, or significance of the experiment. In describing the design of the experiment, be sure to provide a brief description of the function of any equipment used with which the recipient of the lab report may not be familiar.
I.2. Audience Characteristics

It is important to keep in mind the audience when writing a technical report. For the technical reports written by students detailing the results of experiments in this manual, the lab instructor will be the audience. However, to better prepare students for real-life technical writing, it is suggested that the intended audience be assumed to be individuals who (1) have only a basic knowledge of engineering, (2) have not read this manual, (3) have not performed the experiment, and (4) may wish to replicate your experiment based on the report. Students should keep these characteristics of their audience in mind when writing their reports.

I.3. General Formatting Guidelines

I.3.1. Body of Text

A few general formatting guidelines for the body of the text should be addressed. Firstly, the overall length of the technical report should be as short as possible while satisfying the requirements indicated for each section of the report. For the experiments in this manual, four to six pages (no more than six) should be adequate (excluding cover page and appendices). If the student finds this short length difficult to cope with, he/she should review the content of his/her report and try to remove unnecessary details while preserving the significance of the content. The goals of any technical brief are to be both concise and to contain only necessary information.

Page numbers should appear at the bottom of the all pages except the cover page, with the numbering beginning on the first page (cover page is not included). Page numbers should be in Arabic numerals. The Roman numerals used in this template are used to be consistent with the rest of the manual. Note: that the page numbers in this template are enclosed in brackets ([ ]) indicating that they should be changed by students when writing their own lab reports.

The entire body of the text should have a justified alignment. The font should be a simple and standard font, such as Times New Roman or Calibri. The font size should be large enough to be easily read, such as 10 or 12 pt. The entire document should use single space line spacing. Paragraphs may be indented and stacked immediately after each other to save space, or may be separated by a clear space, as is the case in this manual. The title of the report should be in a larger font than the body of the text, say 14 pt, and may be presented in all caps; see top of previous page. Section titles should begin with a capital letter and be presented in bold font. A clear space above and below a section title is generally sufficient to clearly separate sections. When a reference is made to a section in the text, the title of the section should still begin with a capital letter and be presented in bold font, as in the following sentence. See the Abstract section of this report for details on special conditions for the abstract. A section title should not appear at the bottom of a page without at least the first sentence of the section also appearing on the same page. Page breaks may be necessary to ensure that this does not occur. Subsection titles should also be bolded and otherwise treated the same as section titles; however, subsection titles are
generally inset and/or italicized. Note: the use of subsection and sub-subsection titles in this Introduction section.

I.3.2. Technical Symbols and Equations

Any abbreviations or technical symbols used in the text should be explicitly written out fully and the abbreviation or symbol provided in parentheses upon the first usage of the term being abbreviated or represented by a symbol. This includes units for data measurements when they are defined. Note: how the terms in Equation (1) are defined as area of a circle \( A \) and radius of a circle \( r \) in this sentence even though their meanings are likely obvious to most engineers.

Equations, when presented, should be constructed using Microsoft Equation Editor or another related equation writing tool. All equations should be numbered using parentheses and should be referenced in the body of the text as Equation (#), as in Equation (1) on this page. Terms and symbols in an equation should be defined and their symbols noted in parentheses in the order in which they appear in the equation, as was the case for Equation (1) in the previous paragraph. Definition of terms and their symbols/abbreviations may appear before or IMMEDIATELY after an equation is presented. Equations should be separated clearly from the rest of the text by a clear space above and below the equation and centered, as demonstrated with Equation (1).

\[
A = \pi r^2 \quad (1)
\]

I.3.3. Citing References

In citing sources, references in the text should be presented as bracketed numbers (e.g., [1]) and should appear in or at the end of relevant sentences. For example, in the INTRODUCTION section of this manual, a reference is made to a study conducted on how much time engineers spend writing and reading technical documents. The results suggest that engineers spend over half of their working time writing or editing technical documents [1]. Note: how the reference appeared at the end of the previous sentence when the results of the study were stated. A reference may also be used as a noun in a sentence, as in the following sentence. As illustrated in the results of [1], engineers’ spend a majority of their working time preparing and editing technical documents. References can also appear in the middle of a sentence after the name of an author is mentioned, as in the following sentence. According to a study performed by Spretnak [1] at the University of California, Berkley, engineers’ spent a large portion of their time preparing technical documents. All references should appear in a bibliography (References section) at the end of the report in a numbered list; see the Notes on References section of this document. The list is usually ordered by alphanumeric order of the first word(s) in the citations. Use an accepted form of citation when citing references in the References section list, such as MLA, APA, or a well-respected engineering journal (e.g., IEEE Transactions).
I.3.4. Figures and Tables

Regarding figures, several Notes should be made. The title of a figure is presented below the figure. This may eliminate the need of a title for a chart or graph in the figure. If the title for the graph is kept, the title of the figure should NOT be the same as the title of the graph. Figure titles are generally prefaced with a figure designation, such Fig.1. When referenced in the text, figures should be referred to by their figure designations. For example, it is acceptable to refer to Fig.1 using “Fig.1”, but it is not acceptable to refer to Fig.1 using the title of the figure or graph such as “see Example figure” or “see Performance in Identifying Test Images.” When beginning a sentence with the figure designation, the word “Figure” should be used, as in the following sentence. Figure 1 presents the results of the normal controls and traumatic brain injury groups for old and new images during a visual working memory test.

The figure designation, title, and a description of the figure make up a figure caption. The figure designation should be bolded. The figure caption should appear below the figure in a reduced font. The font style of the caption should be the same as the rest of the report. The labels and title(s) of graph(s) in a figure need not be the same font style or font size as the rest of the text; however, they should be clearly legible. This is convenient when using a figure presented previously in another technical document. If using a previously published figure, a citation for the figure should be given in the caption and main body text. Often it is useful to insert a textbox in which the figure and its caption are placed. If the text wrapping of the textbox is set to square, the textbox will be free to float regardless of the text.

Whenever possible, figures should appear at the top left or right corners of a page/section, as in the case of Fig.1. If a figure is wide enough, it may be centered at the top of a page/section. Any symbols or abbreviations in figures should be written out fully and the abbreviations/symbols defined in the figure’s caption. If more than one set of data points are presented in a graph in the figure, a legend should be included. Trend lines or best-fit curves count as additional data points and should be indicated in the legend.
Table 1. Example Table
Cognitive Tests and Other Evaluations Used to Make Diagnoses of MCI/AD

<table>
<thead>
<tr>
<th>General Cognitive Measures:</th>
<th>Baseline Only:</th>
</tr>
</thead>
<tbody>
<tr>
<td>CDR</td>
<td>National Adult Reading Test</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Memory Domain Measures:</th>
<th>Medical Evaluation:</th>
</tr>
</thead>
<tbody>
<tr>
<td>WMS Logical Memory I &amp; II</td>
<td>Physical Exam</td>
</tr>
<tr>
<td>California Verbal Learning Test</td>
<td>Neurological Exam</td>
</tr>
<tr>
<td>[Other memory measures]</td>
<td>Medical History</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Attention/Executive Domain Measures:</th>
<th>Psychiatric Evaluation:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trail Making Tests A &amp; B</td>
<td>NPI-Q</td>
</tr>
<tr>
<td>WAIS-R Digit Span &amp; Digit Symbol</td>
<td>GDS</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Language Domain Measures:</th>
<th>Functional Ability Measures:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Animal &amp; Vegetable Fluency</td>
<td>FAQ</td>
</tr>
<tr>
<td>Boston Naming</td>
<td></td>
</tr>
</tbody>
</table>

MCI = Mild Cognitive Impairment  
AD = Alzheimer’s Disease  
CDR = Clinical Dementia Rating  
FFQ = Food Frequency Questionnaire  
NPI-Q = Neuropsychiatric Inventory Questionnaire  
GDS = Geriatric Depression Scale  
FAQ = Function Assessment Questionnaire

Formatting requirements for tables are markedly different than those used for figures; however, a few points are common to both. Tables, like figures, should be referenced using a designation such as Table 1. Also, like figures, tables should never be referenced by their titles and should appear in the top left or right corners of a page/section. If a table is wide enough, it may be centered rather than aligned to the left or right.

Unlike figures, a table’s designation and title should appear above the table in bold font. Column and row titles or other titles in the table should also be bolded. The data and non-title text in the table should not be bolded. The caption for a table should appear beneath the table and should include abbreviation and symbol definitions for ALL symbols and abbreviations in the table. The caption should be in a reduced font with the same font style as the rest of the text. All other text in the table, including the table title, generally is the same font style and size as the rest of the text. If necessary, the text in the table may be a reduced font (matching the caption) to conserve space.
II. Procedure/Methods

Often referred to as the Methods section, the Procedure section of a technical lab report should detail steps involved in the experiment. Documenting the procedure used is important for the recipient of the report to be able to understand what was done and so the experiment can be replicated at a later date. Lab procedures have long been written in a first-person narrative fashion. As such, one’s audience will likely expect for the procedure to be written in a narrative fashion, and so it should be. However, the contemporary standard is to write a technical report in third-person past-tense and to avoid using first- or second-person pronouns whenever possible.

Determining the correct degree of detail for the procedure in laboratory experiments is difficult. Ultimately, the detail provided should be sufficient for a reader to replicate the experiment without adding a great deal of length to the report. Any variables which may affect the outcome of the experiment should be noted. For example, students should attempt to report as much identifying information as possible regarding the equipment used, including: make, model, serial number, station number in the lab and where lab is located (e.g., Station No. 5, MABE 345 lab, Rm. 610 Dougherty Engineering Bldg., University of Tennessee, Knoxville). It is acceptable to provide these details in a table in appendix at the end of the report. Be sure to cite such appendices as is appropriate using appendix designations such as Appendix A. See Appendix A for details on formatting for appendices. Other variables which may affect the outcome of the experiment include difficulties with equipment operation, ambient temperature, unusual and suspected outliers in recorded data points, etc. All such anomalies should be clearly indicated in the report.

III. Results

Next to the Abstract, the Results section is perhaps the most important portion of the lab report and the most likely portion to be read in detail. In some formats, the Results and Discussion sections appear as a single Results and Discussion section. When they appear separately, it is important to remember not to discuss implications of results in the Results section. Such observations and inferences should be left for the Discussion section of the report. In this section, the results of the experiment should be presented briefly in text, figures, and tables. Again, the text should describe the results objectively without discussing the implications of the results.

IV. Discussion

This section should discuss observations of results and discuss their potential implications. Generally, the last paragraph(s) of this section form(s) a conclusion section and its content is...
given the most weight. Therefore, be sure that the last paragraph(s) in this section address(es) the objectives detailed in the **Introduction** section. A conclusion should be made on whether the experiment was successful or not and justification should be given. For students performing experiments in this manual, possible real-life applications of the concepts/techniques of the experiments should be mentioned.

Finally, an important Note: on how one should expect the recipient of the technical report to read the report. Firstly, the recipient will read the abstract, in great detail. Next, the recipient will move to the **Results** section of the report to peruse figures and tables displaying results. He/she will likely not read the details of the results right away. After glancing through the results, the recipient will move to the **Discussion** section. He/she may start reading the **Discussion** section near the end of the report, trying to extract the conclusions mentioned in the abstract and the justifications for those conclusions. Finally, after having completed this brief and partial reading, the recipient will either put the report down or move to the **Introduction** or **Procedure** sections. If a lab report is well written, the recipient may not be tempted to read the **Introduction** or **Procedure**, but content him/herself with the understanding gained through the Abstract, Results, and Discussion sections. The reading style just described is typical of most practiced readers of technical documents. Students should be aware of this reading style when preparing their reports.

**V. Notes on References**

The reference section should simply present a list of citations for references used. The numbering of the list should be presented in brackets, as the sources are cited in the text. A reduced font size for the list may be used to conserve space. The list is usually ordered by alphanumeric order of the first word(s) in the citations. Use an accepted form of citation when citing references in the **References** section list, such as MLA, APA, or a well-respected engineering journal (e.g., *IEEE Transactions*). Note: this section, **Notes on References**, does not appear in a typical technical document and so should be neglected when using this template for writing a technical report.

**VI. References**

Appendix A. Formatting Guidelines for Appendices

Appendices may be added to the back of a lab report. Appendices should have designations and titles, and should be referenced in the regular text using their designation as figures and tables are referenced. For appendices, however, the designations should be presented in bold font, as is done with section titles. An example of a reference to this appendix is given in the second paragraph of the Procedure section. Appropriate designations for appendices include formats such as “Appendix A”, “Apndx. A”, etc. Appendix titles should always appear at the top of the page, as though separate appendices are separate documents. The font style and size of an appendix’s designation and title should be the same as for a section of the report. The body text of an appendix should be the same as that of the lab report itself. All rules for figures and tables and other general formatting guidelines should apply to appendices as well.

Students writing lab reports on the experiments in this manual should include their signed original data sheets as an appendix.
EXPERIMENT No. 1: INSTRUMENT SPECIFICATIONS

I. Objectives

In this experiment, a weighing device utilizing a linear variable differential transformer (LVDT) will be evaluated and its specifications established. The device can be modeled as a zeroth-order system and can be calibrated using a static calibration procedure. The objectives of this experiment are (1) to calibrate the weighing device and (2) to determine instrument specifications based on experimental measurements. Specifically, the following device specifications will be determined: (1) static sensitivity, (2) hysteresis error, (3) repeatability error, (4) linearity error, (5) zero error, and (6) overall instrument error.

II. Equipment

In this experiment, the weight of various small masses will be measured using a weighing device specifically designed for this experiment. A schematic of the weighing device is presented in Fig.1.1. It is composed of a horizontal, aluminum, cantilevered beam and an LVDT which is used to measure the vertical deflection of the beam caused by the mass being measured. A mounting for an optional micrometer is also included at the top of the LVDT. The micrometer can be used to calibrate the device. The mass to be measured is placed so that it is centered on a black square located at the end of the beam. Note: the LVDT measures the deflection of the

Fig.1.1. Weighing Device Dimensions.

Fig.1.2. Schematics of Linear Variable Differential Transformer (LVDT). (a) general arrangement; (b) circuit diagram. \( V_0 \) = constant supplied AC voltage; \( V_1 \) = voltage of first secondary coil (1st coil); \( V_2 \) = voltage of second secondary coil (2nd coil); \( V_0 = V_1 - V_2 \).
beam at a different point on the beam than where the mass is centered. In Fig.1.1., the distance from the clamped end of the cantilevered beam to the center of the black square (center of the mass to be weighed) is designated as ; distance to the point where the deflection of the beam is measured by the LVDT is designated as . The width and thickness of the beam are designated by and , respectively.

An LVDT is an alternating current (AC) transformer with coils placed around a magnetic core; see Fig.1.2. The coils consist of a primary coil and two secondary coils which are separated from the magnetic core by an insulating material. A constant AC voltage is applied to the primary coil. The first and second secondary coils are located above and below the primary coil, respectively. As the magnetic core moves up or down, it effects the amount of voltage produced in the secondary coils. The voltage difference between the two secondary coils is proportional to the change in linear displacement of the magnetic core. Thus, the voltage difference in the secondary coils can be used to determine the direction and magnitude of the displacement of the magnetic core and, therefore, the vertical displacement of the cantilevered beam. Manufacturer’s specifications for the LVDT are provided in Table 1.1.

A schematic of the entire measurement system is provided in Fig.1.3. Software called LVDT.VEE is used to retrieve the voltage difference from the LVDT via an analog/digital converter (ADC). Identifying equipment information for the ADC and PC used should be recorded. Note: brackets ([ ] ) in Table 1.1 indicate where missing equipment information should be recorded by students.

### III. Theory

![Fig.1.3. Schematic of Measurement System for Experiment No. 1.](image-url)
III.1. Weighing Device

From beam theory, the true value of the deflection ($y$) at the point along the beam (where the LVDT is located) due to a mass ($m$) placed at the point (center of the black square at the end of the beam) can be determined using Equations (1.1) and (1.2), where deflection is positive downward, $g = 9.81 \text{ m/s}^2$, is the Young’s Modulus of the beam material (for aluminum, $E = 10^6 \text{ Pa}$), and $I$ is the second moment of inertia for the cross-section of the beam about the neutral axis (axis of zero stress, assumed to be at a thickness of $t/2$):

III.2. Dynamic and Static Calibration

Calibration of a measurement system is the process of measuring the output of the system when a known input is applied. When a system has inertial or time-dependent behavior, dynamic calibration is performed. Inertial or other time-dependent characteristics result in a delay in a system’s response (lag time) and may result in oscillatory behavior which is dependent on the frequency of the input. Generally, any system which can be modeled as being governed by at least a first-order differential equation of motion requires dynamic calibration. Such a system is calibrated for different frequency inputs by applying sinusoidal inputs with known amplitudes and frequencies and recording the response of the system.

Any system which can be modeled as having no time delay or oscillatory response is considered a zeroth-order system and static calibration is employed when calibrating the system. In static calibration, known constant inputs are applied and the output of the system is recorded. The slope of the best-fit output vs. input curve for a zeroth-order system is known as static sensitivity and is typically denoted as $K$; see Fig.1.4. Ideally, a zeroth-order system has linear input-output behavior for some operating range. For this range, $K$ is constant. Due to the relatively fast response time of the LVDT in the measurement system used in this experiment, the weighing device can be modeled as a zeroth-order system. In this experiment, weights of known mass will be weighed using the measuring device and the static sensitivity and instrument specification errors for the system will be determined.

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Fig.1.4. Static Sensitivity for Linear Best-Fit Calibration Curve. $K = \Delta y/\Delta x$ = static sensitivity; $y_L$ = best-fit linear calibration curve.
III.3. Instrument Specification Errors

III.3.1. Types of Error

Ideally, a measurement system should be calibrated to provide accurate and precise measurements. Accuracy can be defined as the ability of a measurement system to indicate the true value. Precision is defined as the spread of measurements taken for the same input. If a system is capable of measuring an input within a small range of its true value, then the system is both accurate and precise. It is possible for a measurement system to record various outputs for the same input. In such a case, if the recorded values average closely to the input’s true value, then the system is accurate but imprecise. If the recorded values do not average close to the input’s true value but are closely distributed relative to each other, then the system is precise but inaccurate. See Fig.1.5 for an illustration of the differences between precision and accuracy.

If a system is imprecise or inaccurate then the system’s measurements are subject to error. Here, error is defined as the difference between the true value of an input and the observed measurement value; see Equation (1.3), where is error, is the true input value, and is the measured value. All measurement systems have some error. Errors can be divided into two primary groups: (1) precision errors and (2) bias errors. Precision error is characterized by residuals with a random distribution. Here, a residual is defined as the difference between observed and expected measurements. Precision errors are caused by extraneous variables—random factors which cannot be controlled and/or identified and influence system output. If a measurement system has only precision error, then repeated measurements of the same input value should average to the true input value. In other words, the residuals should average to zero. Thus, precision errors cause a system to be imprecise but not inaccurate. Bias errors, on the other hand, cause a measurement system to be inaccurate, but not imprecise. Thus, bias errors are generally defined as constant offsets in the measurements recorded (e.g., the average values of residuals). If a system has residuals which display a random distribution about an average value...
other than zero, the bias error can be approximated using the average residual value and the precision error can be estimated using the standard deviation of the residuals; see Fig.1.5.

For a zeroth-order system, there are several common errors which can be observed and quantified during an experiment. In this experiment, recorded measurements will be used to calculate common errors in the weighing device previously described. To begin, a description of common instrument errors is provided.

### III.3.2. Zero Error

Zero error is a vertical shift in the calibration curve of a device observed after the device is used. Before using any device, it should first be calibrated. After it is calibrated, the device should be zeroed (given no input) and the output of the device should be recorded. At the end of an experiment, after using the device to record desired measurements, the device should again be zeroed and the output recorded. The zero error is then defined as the absolute difference in the two recorded zeroed measurements; see Equation (1.4), where \( e \) is zero error, \( y_{01} \) is the zeroed measurement recorded before the experiment, and \( y_{02} \) is the zeroed measurement recorded after the experiment.

### III.3.3. Linearity Error

Linearity error is defined as the absolute maximum deviation of observed measurements from the linear best-fit calibration curve. See Fig.1.6 and Equation (1.5), where \( \Delta e \) is linearity error, \( y_i \) is the observed measurement for input \( x_i \), and \( y_i^{L} \) is the expected measurement for input \( x_i \) as predicted by the linear best-fit calibration curve. When more than one measurement is taken at each input \( x_i \), in Equation (1.5) should be replaced by the average readings as defined by Equation (1.6), where \( y_{i1} \) are measurements for inputs \( x_i \), \( = 1 \) is an input value index, and \( = 1 \) is a measurement recording index such that measurements were recorded for input value \( x_i \).

![Fig.1.6. Observed and Expected Measurements Using Linear Calibration Curve.](image)

- \( y_{i1}^{L} \) = recorded output for input \( x_i \);
- \( y_{L} \) = best-fit linear calibration curve.
III.3.4. Hysteresis Error

A measurement system may output different values for the same input depending on whether the input is preceded by a previous input of larger or smaller value. In other words, a system may demonstrate directionally dependent behavior. When an input is preceded by an input of larger value, this is known as downscaling; when preceded by an input of smaller value, this is known as upscaling. The difference in observed measurement values for the same input between upscaling and downscaling is known as hysteresis. Hysteresis error is defined as the absolute maximum observed difference in upscaling and downscaling measurements; see Fig. 1.7 and Equation (1.7), where is hysteresis error, is the observed measurement for input when upscaling, and is the observed measurement for input when downscaling. When more than one measurement is taken during upscaling or downscaling for inputs , the average upscaling and downscaling measurements for each input should be used.

III.3.5. Repeatability Error

Often when repeated measurements are taken for the same input value, different values are observed. Differences in repeated measurements not due to hysteresis are known as repeatability error. Repeatability error is a precision error and therefore has the property of having a random distribution. Repeatability error is defined mathematically in Equation (1.8), where is repeatability error and is the sample standard deviation of observed measurements . is defined in Equation (1.9).
III.3.6. Overall Instrument Error

Overall instrument error is an estimate of the overall error in a measurement system due to the individual errors previously described (and other errors if known—e.g., resolution error). It is defined mathematically as the root sum of squares of known instrument errors; see Equation (1.10), where $OIE$ is the overall instrument error.

$$OIE = \sqrt{\varepsilon_2^2 + \varepsilon_L^2 + \varepsilon_H^2 + \varepsilon_R^2 + \cdots} \quad (1.10)$$

III.3.7. Full Scale Output

Often it is useful to report errors in terms of %FSO, where FSO denotes the full scale output. FSO is defined as the range for which the calibration of the device is valid. For example, a device may have a linear calibration curve over the range of 10 to 100 but exhibit nonlinear behavior outside of this range. Assume it is desired to estimate overall instrument error using the definitions given previously for common specification errors over the range in which the system’s behavior is linear. The FSO for the system would then be $100 - 10 = 90$ and errors can be expressed as a percentage of this value.

IV. Pre-lab Questions

1. What is the difference between static and dynamic calibration? When is it appropriate to perform static calibration? When is it necessary to perform dynamic calibration?
2. What are the differences between bias and precision error? What are a few characteristics of each?
3. What is zero error?
4. What is hysteresis?
5. What is repeatability error?
6. A system operates between the range $\pm 0.2$, has a linear calibration curve for the range 0.0 to 2.0 of the form $y^2 = 200x + 15$, and has a linearity error of 0.01. Express the linearity error in terms of %FSO.

V. Procedure

1. Use a ruler to measure the following: the distance from the clamped end of the beam to the point where the LVDT contacts the beam ($x$); the distance from the clamped end of the beam to the center of the black square at the end of the beam ($a$); and the width of the beam ($b$). Using a set of calipers, take four measurements of the thickness of the beam ($t_1, t_2, t_3, t_4$) roughly equidistant along its length and compute the average thickness $\bar{t}$. Record these measurements in the BEAM DIMENSIONS table of the datasheet. Note: in calculations involving the thickness of the beam use $t = \bar{t}$.
2. Using the values now recorded in the BEAM DIMENSIONS table and Equation (1.2), compute the second moment of inertia of the cross-section of the beam ($I$). Record this value in the BEAM DIMENSIONS table of the data sheet.
3. Check that the system is connected and set up correctly before beginning the experiment. A schematic of the setup is presented in Fig. 1.3. Students should ask their lab instructor for assistance if needed.
4. Start LVDT.VEE.
5. Remove any weights from the weighing device. Mount the micrometer and turn it until the LVDT just touches the cantilevered beam without causing deflection. Record the voltage value displayed by LVDT.VEE. Record this value in the CALIBRATION OF LVDT table of the data sheet for a deflection ($\nu$) of 0.00 in. and for $y_0$.
6. Turn the micrometer to deflect the beam 0.05, 0.10, 0.15, and 0.20 in. and record the LVDT.VEE voltage reading at each deflection in the CALIBRATION OF LVDT table of the data sheet. If unclear, students should check with their lab instructor to be sure of how many turns of the specific micrometer being used accounts for a deflection of 0.05 in.
7. Using Microsoft Excel or a similar program, students should plot the LVDT.VEE readings vs. deflection for the calibration data now recorded. Students should then add a best-fit linear trend line for the data. The trend line should be of the form $y^L = K\nu + y_0$, where $K$ is the slope of trend line, $\nu$ is the deflection in inches, and $y_0$ is the LVDT.VEE voltage reading without deflection (0.00 in.). Students should ask their lab instructors for assistance in adding the trend line if needed.
8. Record the value for the slope $K$ of the trend line in the CALIBRATION OF LVDT table of the data sheet.
9. The calibration performed in step 7 is valid for deflections of 0.00—0.20 in. The corresponding expected output values for the weighing device are $y_0$ and $0.20K + y_0$. Thus for this calibration FSO = 0.20$K$. This value should be recorded in the CALIBRATION OF LVDT table of the data sheet.
10. Remove the micrometer from its mounting and record the voltage reading in the LVDT.VEE display as the initial zero reading in the ZERO ERROR table of the data sheet.
11. For masses ($m_i$, $i = 1\ldots5$) of $m_1 = 0$, $m_2 = 100$, $m_3 = 200$, $m_4 = 300$, and $m_5 = 400$ g do the following:
   a. Using Equation (1.1) and the recorded data in the BEAM DIMENSIONS table of the data sheet compute the theoretical deflection ($\nu_i$) of the beam at point $x$ due to mass $m_i$. Note: 1 in. = 25.4 mm. Record the value in the EXPERIMENT MEASUREMENTS table of the data sheet.
   b. Using the value determined in step 11.a for the deflection ($\nu_i$), compute the expected LVDT reading ($y^L_i$) and record the value in EXPERIMENT MEASUREMENTS table of the data sheet.
   c. Place the mass on the black square at the free end of the cantilevered beam.
   d. Take five ($j = 1\ldots5$) upscaling measurements for the given mass $m_i$. To induce upscaling, gently lift up on the end of the beam and then gently bring the beam back down before recording the LVDT.VEE voltage reading. The reading should be recorded in the corresponding $y_{ij}$ cell of the EXPERIMENT MEASUREMENTS table of the data sheet.
   e. Average the five $y_{ij}$ values for $j = 1\ldots5$ taken in step 11.d to obtain the average upscaling reading ($y^{\text{up}}_i$). Record this value in the EXPERIMENT MEASUREMENTS table of the data sheet.
f. Take five \((j = 6…10)\) downscaling measurements for the given mass \(m_i\). To induce downscaling, gently press down on the end of the beam and then gently bring the beam back up before recording the LVDT.VEE voltage reading. The reading should be recorded in the corresponding \(y_{ij}\) cell of the EXPERIMENT MEASUREMENTS table of the data sheet.

g. Average the five \(y_{ij}\) values for \(j = 6…10\) taken in step 11.f to obtain the average downscaling reading \((\overline{y}_i^{\text{down}})\). Record this value in the EXPERIMENT MEASUREMENTS table of the data sheet.

h. Calculate the hysteresis for mass \(m_i\) using the average upscaling and downscaling readings. Specifically compute \(\varepsilon_{Hi} = |\overline{y}_i^{\text{up}} - \overline{y}_i^{\text{down}}|\). Record this value in the EXPERIMENT MEASUREMENTS table of the data sheet.

i. Take five \((j = 11…15)\) oscillatory measurements for the given mass \(m_i\). To induce oscillatory behavior, gently flick the end of the beam and wait for the beam to settle before recording the LVDT.VEE voltage reading. The reading should be recorded in the corresponding \(y_{ij}\) cell of the EXPERIMENT MEASUREMENTS table of the data sheet.

j. Average the five \(y_{ij}\) values for \(j = 11…15\) taken in step 11.i to obtain the average oscillatory reading \((\overline{y}_i^{\text{osc}})\). Record this value in the EXPERIMENT MEASUREMENTS table of the data sheet.

k. Average the fifteen \(y_{ij}\) values for \(j = 1…15\) taken in steps 11.d, 11.f, and 11.i to obtain the overall average reading \((\overline{y}_i)\) using Equation (1.6). Record this value in the EXPERIMENT MEASUREMENTS table of the data sheet.

l. Using the average measurement value \((\overline{y}_i)\), compute the linearity error for mass \(m_i\). Specifically, compute \(\varepsilon_{Li} = |\overline{y}_i - y_i|\). Record this value in the EXPERIMENT MEASUREMENTS table of the data sheet.

m. Using the fifteen \(y_{ij}\) values for \(j = 1…15\) taken in steps 11.d, 11.f, and 11.i, the overall average reading \((\overline{y}_i)\) determined in step 11.k, and Equation (1.9), determine the standard deviation \(s_{\overline{y}_i}\) of the measurements for mass \(m_i\). Record this value in the EXPERIMENT MEASUREMENTS table of the data sheet.

12. After recording all measurements taken in step 11, remove all masses from the end of the beam and take the final zero reading. Record this value in the ZERO ERROR table of the data sheet.

13. Determine the zero error as the absolute difference between the initial and final zero readings recorded. Record this value in the ZERO ERROR table of the data sheet.

14. Determine the hysteresis error as the maximum value of the \(\varepsilon_{Hi}\) row of the EXPERIMENT MEASUREMENTS table of the data sheet. Record this value in the HYSTERESIS ERROR table of the data sheet.

15. Determine the linearity error as the maximum value of the \(\varepsilon_{Li}\) row of the EXPERIMENT MEASUREMENTS table of the data sheet. Record this value in the LINEARITY ERROR table of the data sheet.

16. Determine the repeatability error as two times the maximum value of the \(s_{y_i}\) row of the EXPERIMENT MEASUREMENTS table of the data sheet. Record this value in the REPEATABILITY ERROR table of the data sheet.

17. Convert errors to %FSO and record these values in the appropriate error tables of the data sheet.
18. Turn off all equipment and tidy the lab station.
19. Record appropriate equipment identification information.

VI. Post-lab Questions

1. Plot the calibration data in the CALIBRATION OF LVDT table of the data sheet and add the best-fit linear trend line. Does the instrument actually follow a linear input-output relationship or is the relationship nonlinear?
2. Plot the average upscaling ($\bar{y}_i^{up}$) and downscaling ($\bar{y}_i^{down}$) values on the same graph as prepared in question 1 and indicate the hysteresis (difference between upscaling and downscaling) for each mass.
3. Plot the measurements $y_{ij}$ vs. the masses $m_i$ where $y_{ij}$ values are distributed along a vertical line for each mass. Comment on the distribution of the measurements for each mass value. Are there any changes in the standard deviations $s_{y_i}$?
4. Plot the average measurements $\bar{y}_i$ for each mass and the best-fit linear calibration curve $y^L$ on the same graph. Indicate linearity error for each mass. Is the linearity error significant (consider in terms of %FSO)?
5. Compute the overall instrument error in terms of %FSO using Equation (1.10). Based on this value, does the device have a significant error over the calibrated range? Is the device accurate and precise enough to use for indicating small changes in mass?
## EXPERIMENT MEASUREMENTS

<table>
<thead>
<tr>
<th>( m_i )</th>
<th>( m_1 = 0 ) g</th>
<th>( m_2 = 100 ) g</th>
<th>( m_3 = 200 ) g</th>
<th>( m_4 = 300 ) g</th>
<th>( m_5 = 400 ) g</th>
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<tbody>
<tr>
<td>( y_i^L ) (mV)</td>
<td>( y_i ) (mV)</td>
<td>( y_i ) (mV)</td>
<td>( y_i ) (mV)</td>
<td>( y_i ) (mV)</td>
<td>( y_i ) (mV)</td>
</tr>
</tbody>
</table>

### ZERO ERROR

- **Initial zero reading (mV)**
- **Final zero reading (mV)**
- **Zero error (mV)**
- **Zero error (%FSO)**

### LINEARITY ERROR

- **Linearity error (mV)**
- **Linearity error (%FSO)**

### REPEATABILITY ERROR

- **Repeatability error (mV)**
- **Repeatability error (%FSO)**

### HYSTERESIS ERROR

- **Hysteresis error (mV)**
- **Hysteresis error (%FSO)**

### CALIBRATION OF LVDT

<table>
<thead>
<tr>
<th>( v ) (in.)</th>
<th>LVDT.VEE (mV)</th>
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</thead>
<tbody>
<tr>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>0.10</td>
<td></td>
</tr>
<tr>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td>0.20</td>
<td></td>
</tr>
</tbody>
</table>

\[ y^L = Ky + y_0 \]

- **\( y_0 \) (mV)**
- **\( K \) (mV/in.)**
- **FSO (mV)**

---

**Lab Instructor’s signature:** ______________________  **date:** ____/____/____

**Name:** ____________________  **Lab Partner:** __________________ ________

---

**Experiment No. 1 Data Sheet**
EXPERIMENT No. 2: SIGNAL ANALYSIS

I. Objectives

In this experiment, simulated and real digital samples will be analyzed in the time and frequency domains. The objectives of this experiment are (1) to illustrate concepts of sampling theory and (2) to familiarize students with frequency analysis of digital signals. Practical applications of frequency domain analysis will be illustrated in the use of frequency components to code numbers on touch-tone telephones and the use of frequency of vibration in determining the length of an aluminum rod.

II. Equipment

II.1. FFTDEMO

In Part 1 of this experiment, digital sample data for cosine and square waves with various frequency components will be artificially simulated using software called FFTDEMO. FFTDEMO creates time and frequency domain plots of cosine and square waves with amplitudes of 100 units for each constituent frequency component. Here, the term constituent frequency component implies the total waveform being the sum of constituent waveforms which differ in frequency. For example, the total waveform \( x(t) = x_1(t) + x_2(t) \), where \( x_1(t) = 100\cos(2\pi f_1 t) \) and \( x_2(t) = 100\cos(2\pi f_2 t) \) are the constituent frequency components and \( f_1 \neq f_2 \). The software allows the user to choose the waveform types (cosine or square) and the frequencies of constituent frequency components. The program models digital sampling with a set number of samples (\( N = 512 \)) and choice of sampling frequency (\( f_s \)). The choices of frequencies and sampling frequency will be used to illustrate concepts of resolution.

II.2. SCOPE

In Part 2 of this experiment, a signal generator will be used to generate sinusoidal signals of various frequencies. Equipment identification information for the generator used is presented in Table 2.1. Note: brackets ([ ]) in Table 2.1 indicate missing information which should be recorded by students; OR indicates that more than one make/model of a device will be available to students and that students should Note: which make/model was used.

Software called SCOPE will be used to record digital samples of the analog signals generated by the signal generator. These samples will be gathered via an ADC, the information for which is provided in Table 2.1. See Fig.2.1(a) for a schematic of the setup for Part 2. Unlike FFTDEMO, SCOPE samples real signals via the ADC and allows for choice of the number of samples (count) and the sampling frequency (rate). SCOPE displays time domain and frequency domain representations of the signal being sampled. Auto-scaling and manual scaling adjustments for both axes are possible. Additionally, SCOPE has arrowhead icons on each graph which can be used to quickly determine the difference in x- and y-axis coordinates of two points in either time or frequency domain plots. This feature is useful in measuring amplitudes and time delays. Students should ask their lab instructor for help in adjusting axes or using the arrowhead feature.
of SCOPE if needed. Analysis of signals in SCOPE will illustrate concepts of Nyquist sampling theory.

In Part 3 of this experiment, a sound level meter will be used to measure the sounds of a touch-tone telephone when various numbers are pressed; see Fig.2.1(b) for a schematic of the setup for Part 3. Equipment information for the sound level meters is presented in Table 2.1. As much identifying information as possible should be recorded for the touch-tone telephone used. SCOPE will be used to examine the frequency components of the tones used to code for various numbers on the touch-tone telephone.

In Part 4, SCOPE will be used to record readings from an accelerometer placed on the end of a long aluminum rod when the aluminum rod is struck by a small hammer at its opposite end. Students will use the fundamental frequency of the vibration of the rod to estimate its length. Note: brackets ([ ]) in Table 2.1 indicate missing equipment information for the accelerometer used which should be recorded.

III. Theory

III.1. Sampling Theory

When recording digital samples with the intent of analyzing information concerning the frequency behavior of the signal two parameters are crucial: (1) the rate at which samples are recorded and (2) the number of samples taken.

The rate at which samples are recorded is known as the sampling frequency \( f_s \). Choice of sampling frequency directly determines the resolution of the digital sample in the time domain. Specifically, the time between samples (time domain resolution) is equal to the reciprocal of the sampling frequency. For example, if a signal is sampled at 50 Hz, the resolution for the digital sample in the time domain is 0.2 s; see Equation (2.1), where \( res_{time} \) is resolution in the time domain. Resolution in the frequency domain is dependent on both the sampling frequency and the number of samples taken.

Table 2.1 Equipment Information for Experiment No. 2

<table>
<thead>
<tr>
<th>Signal Generator</th>
<th>Make</th>
<th>Gwinstek</th>
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<tbody>
<tr>
<td></td>
<td>Model</td>
<td>GFG-8020H</td>
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<tr>
<td></td>
<td>Serial</td>
<td>[on bottom beneath barcode] OR</td>
</tr>
<tr>
<td></td>
<td>Make</td>
<td>Goldstar</td>
</tr>
<tr>
<td></td>
<td>Model</td>
<td>FG-2002C</td>
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<table>
<thead>
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<th>ADC</th>
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<th>Personal Measurement Device™</th>
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<tr>
<td></td>
<td>Model</td>
<td>PMD-1208FS</td>
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<table>
<thead>
<tr>
<th>Touch-tone telephone</th>
<th>Make</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Model</td>
<td>[to be recorded]</td>
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<table>
<thead>
<tr>
<th>Sound Level Meter</th>
<th>Make</th>
<th>RadioShack®</th>
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<td></td>
<td>Model</td>
<td>Sound Level Meter OR</td>
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<tr>
<td></td>
<td>Make</td>
<td>Nady Audio</td>
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<td></td>
<td>Model</td>
<td>ASM-2 Analog SPL Meter</td>
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<table>
<thead>
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<th>Accelerometer</th>
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<tr>
<td></td>
<td>Model</td>
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<table>
<thead>
<tr>
<th>PC</th>
<th>Make</th>
<th>Dell</th>
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<tbody>
<tr>
<td></td>
<td>Model</td>
<td>Optiplex GX620</td>
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<tr>
<td></td>
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<td>[station #]</td>
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<tr>
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<td>MABE345 Lab</td>
<td></td>
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<tr>
<td></td>
<td>Rm. 610 Dougherty Engr. Bldg.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>University of Tennessee, Knoxville</td>
<td></td>
</tr>
</tbody>
</table>
Specifically, the minimum difference between two frequencies which can be distinguished in a digital sample (frequency domain resolution) is equal to the sampling frequency divided by the number of samples taken; see Equation (2.2), where resolution in the frequency domain and is the number of samples taken. For example, if a signal is sampled at 1000 Hz, and 100 samples are taken, the frequency resolution of the digital sample is 10 Hz. This means that if the frequency of the signal being measured is 45 Hz, the digital sample may appear to have a frequency of 40 or 50 Hz instead of the true frequency of 45 Hz. This due to the fact that with a sampling frequency of 1000 Hz and 100 samples, only 0.10 s of data is recorded. A 0.10 s sample may contain four or five complete cycles of the 45 Hz signal, thus resulting in the false conclusion that the signal has a either a 40 Hz or 50 Hz frequency. Careful attention should be paid to the number of samples taken relative to the sampling frequency used to ensure that enough data is collected to accurately determine the frequency of the signal being measured.

![Fig.2.1. Schematics of Measurement Systems for Experiment No. 2. (a) measurement system for Part 2; (b) measurement system for Part 3; (c) measurement system for Part 4.](image-url)
An important Note: on colloquialisms in the English language regarding the use of the word “resolution” is mentioned here. In scientific language, a low resolution value indicates that a plot or image has a very small degree of difference between points in a given axis. For an image, a low resolution value would indicate a very clear and detailed image. This is contradictory to the colloquial use of the term resolution. In colloquial English, to describe an image as having “low resolution” generally implies a fuzzy, low quality picture. In actuality, such an image would have a high resolution value for at least one of its two axes causing the “fuzziness” of the image. Students should keep in mind the difference between the scientific definition of resolution and the colloquial meaning when performing this experiment and analyzing the results.

In addition to issues of resolution, choice of sampling frequency is also important in preventing the recording of alias (false) frequencies. Aliasing refers to the phenomenon by which a high frequency signal, when sampled at too low a rate, may result in a digital sample with a much lower frequency. This issue is different from frequency domain resolution. In the example used previously, the 40 Hz or 50 Hz of the digital sample is not a result of aliasing, but rather the result of poor resolution caused by the small number of samples taken. Aliasing may occur regardless of the number of samples taken. For example, say a 100 Hz signal is sampled at 100 Hz. The measurements taken would always be taken at the same point in the signal’s cycle, resulting in the recording of a constant value. The same situation would also occur if the signal being measured had a frequency of 200 Hz, 300 Hz, 400 Hz, etc.

In order to accurately measure a given frequency, the sampling frequency used must be at least twice that of the frequency being measured, otherwise, aliasing occurs. The minimum frequency which can be accurately measured using a given sampling frequency is known as the Nyquist sampling frequency \( f_N \) and is equal to half the sampling frequency used; see Equation (2.3). The false (alias) frequency \( f_a \) which will appear in a digital sample of a signal with a given

![Fig.2.2. Nyquist Folding Diagram.](image)

\( = \) signal frequency; \( = \) Nyquist frequency; \( = \) alias frequency.

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frequency ($f$) sampled at a given sampling frequency ($f_s$) can be predicted using a Nyquist folding diagram as presented in Fig. 2.2. To use a Nyquist folding diagram, first determine what the given frequency’s value is in terms of the Nyquist frequency. Next, follow the diagram along the folding line until this value is reached. Finally, draw a line straight down. The frequency at which this vertical line intersects the horizontal line at the bottom of the Nyquist folding diagram is the alias frequency. For example, if a 120 Hz signal is sampled at 80 Hz, then the Nyquist frequency would be 40 Hz and the alias frequency would also be 40 Hz. Note: that an alias frequency can never be greater than the Nyquist frequency.

$$f_N = \frac{f_s}{2} \quad (2.3)$$

### III.2. Fourier Series

In analyzing the frequency of a signal it is often useful to represent the signal using a Fourier series. Fourier series is a series expansion of a function using sine and cosine as the trial functions of the expansion. In other words, the Fourier series ($F(x)$) of a given function ($f(x)$) is an approximation of the function made up of an infinite sum of sine and cosine functions; see Equation (2.4). Formulas for the coefficients in Equation (2.4) are presented as Equations (2.5) and (2.6), where $T$ is the period of the signal being approximated. When the values for the coefficients are plotted vs. frequency ($f = n/T$ Hz), the resulting plot is known as a frequency spectrum or frequency domain representation of the signal. This is often useful in distinguishing frequencies of constituent frequency components of signals which may have complex time domain behavior.

$$f(x) \approx F(x) = \sum_{n=0}^{\infty} \left[ A_n \cos \left( \frac{2\pi n}{T} x \right) + B_n \sin \left( \frac{2\pi n}{T} x \right) \right] \quad (2.4)$$

$$A_{n>0} = \frac{1}{T} \int_{x_1}^{x_1+T} f(x) \, dx$$

$$B_{n>0} = \frac{2}{T} \int_{x_1}^{x_1+T} f(x) \cos \left( \frac{2\pi n}{T} x \right) \, dx$$

In order for a function to be accurately represented using a Fourier series two criteria must first be met: (1) the function must have a constant average value and (2) the function must be periodic. Often it is desirable to approximate an aperiodic function using a Fourier series. Because an aperiodic function automatically fails the second criteria, a typical Fourier series cannot be used. However, over a given range $x_1$ to $x_2$ a valid Fourier series approximation may
be determined using $T = x_2 - x_1$. It is important to Note: that such a Fourier series is only valid for the range $x_1$ to $x_2$.

### III.3 Harmonics

Occasionally the nonzero coefficients in a Fourier series expansion of a signal correspond to frequencies of $f_n = (2n + 1)k/T$ or $(n + 1)k/T$, where $n = 0 \ldots \infty$ and $k$ is a constant. In such a case, the lowest frequency ($n = 0$) is generally termed the fundamental frequency of the signal. Subsequent frequencies ($n > 0$) are then termed harmonics of order $n$. For example, a 16 Hz square wave has Fourier series coefficients for frequencies $f_n = (2n + 1)k/T$, where $n = 0 \ldots \infty$, $k = 1$, and $T = 1/16$. Thus, a 16 Hz square wave has a fundamental frequency of 16 Hz, a 1st order harmonic frequency of 48 Hz, a 2nd order harmonic of 80 Hz, etc.

The fundamental frequency of a system is an important feature which is dependent on many other characteristics of a system. If the relationship between a system’s fundamental frequency and another of the system’s properties is known, the fundamental frequency may be used to derive an indirect measurement of the other system property. For example, the fundamental frequency of vibration of an aluminum rod is related to the rod’s length as presented in Equation (2.7), where $L$ is the length of the rod, $K$ is the bulk modulus of the rod, $\rho$ is the density of the rod, and $f_P$ is the fundamental frequency of vibration of the rod. A similar technique is used in practice to locate breaks in telephone cables.

\[
L = \frac{1}{2f_P} \sqrt{\frac{K}{\rho}} \quad (2.7)
\]

### III.4. Even and Odd Functions

A function is said to be an even function if $f(x) = f(-x)$, or in other words is symmetric about the vertical axis. Conversely a function is said to an odd function if $f(x) = -f(-x)$, or in other words is symmetric about the function $f(x) = x$ or the function $f(x) = -x$. Examples of even functions include constants, cosine, and functions of even power. Examples of odd functions include sine and functions of odd power. Most functions are a combination of even and odd functions. Such functions are neither even nor odd. The sum of even functions results in an even function. Similarly, the sum of odd functions results in an odd function.

Identifying whether a function is even or odd can be very useful in determining a Fourier series expansion of the function. For example, if the function is even, this implies that it must be the sum of only even functions (cosines). Thus, all of the coefficients for the sine terms of the function’s Fourier series expansion are zero ($B_n = 0$). Likewise, if a function is known to be odd, then the coefficients for the cosine terms in its Fourier expansion are zero ($A_n = 0$). It is important to Note: that these shortcuts in determining a function’s Fourier series’ coefficients is only valid for Fourier series approximations over the range $\pm x$. For example, the Fourier series expansion of the even function $f(x) = x^2$ would have $B_n = 0$ sine coefficients ONLY if the approximation were for a range symmetric about the vertical axis, such as $[-2,2]$ or more generally $[-x,x]$. 18
III.5. Frequency Conversions

A few important equations for converting between linear frequency \((f, \text{ units } = \text{Hz})\) and circular frequency \((\omega, \text{ units } = \text{rad/s})\) are provided below. Be sure to watch units when performing analyses.

\[
f = \frac{1}{T} \quad (2.8)
\]

\[
\omega = 2\pi f \quad (2.9)
\]

IV. Pre-lab Questions

1. Would a resolution value of 2 Hz or 4 Hz result in a frequency spectrum with greater accuracy in determining the frequencies of a sampled signal’s constituent frequency components?

2. If a complex periodic signal has the form \(f(t) = A_1 \cos(\omega_1 t) + A_2 \cos(\omega_2 t)\), where \(\omega_1 \neq \omega_2\), what are the coordinates (Hz, amplitude) of the peaks in the frequency domain?

3. Are the following functions even, odd, or neither? Explain.
   a. \(f(x) = x\)
   b. \(f(x) = x^2\)
   c. \(f(x) = x^3\)
   d. \(f(x) = a + \sin(\omega x)\)

4. If a signal with a frequency of 30 Hz is sampled at 50 Hz, does aliasing occur? If so, what is the alias frequency?

5. What are the first three nonzero coefficients in the Fourier series expansion for \(f(x) = 2 - x^2\) over the range \([-2, 2]\)?

V. Procedure

Table 2.2 Details for FFTDEMO Case Studies

<table>
<thead>
<tr>
<th>Case</th>
<th>(f_s) (Hz)</th>
<th>Waveform</th>
<th>(f) (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>512</td>
<td>C</td>
<td>205</td>
</tr>
<tr>
<td>2</td>
<td>1024</td>
<td>C</td>
<td>205</td>
</tr>
<tr>
<td>3</td>
<td>4096</td>
<td>C</td>
<td>205</td>
</tr>
<tr>
<td>4</td>
<td>1024</td>
<td>C</td>
<td>50, 205, 605</td>
</tr>
<tr>
<td>5</td>
<td>1024</td>
<td>S</td>
<td>16</td>
</tr>
</tbody>
</table>

\(f_s = \text{sampling frequency}\)

\(f = \text{frequency/ies}\)

\(C = \text{cosine}\)

\(S = \text{square wave}\)
V.1. Part 1: Resolution

1. FFTDEMO will be used to examine four cases of cosine and square waves with various constituent frequency components in both the time and frequency domains using different sampling frequencies. Pertinent information for each case is given in Table 2.2.

2. Start FFTDEMO.

3. For Cases 1 through 5, perform the following steps for entering the required information to generate simulated digital sample data.
   a. Once FFTDEMO is opened a prompt should appear requesting the desired “frequency span”. Here, “frequency span” means the sampling frequency. Enter the sampling frequency as indicated in Table 2.2 and press ENTER. For example, for Case 1 enter 512 and press ENTER.
   b. After entering the sampling frequency, a new prompt should appear requesting a choice of waveform type. Type C for cosine or S for square wave as indicated in Table 2.2 and press ENTER. For example, for Case 1 press C to choose a cosine waveform and press ENTER.
   c. After entering the choice of waveform, a prompt will appear requesting the frequency of the first constituent frequency component. Enter the first frequency indicated in Table 2.2 and press ENTER. For example, for Case 1 enter 205 and press ENTER.
   d. After entering the frequency of the first constituent frequency component, a prompt will appear requesting the frequency of the second constituent frequency component. For all cases except Case 4, there is only one frequency component. For these cases, enter no new information, but simply press ENTER. For Case 4, enter the second frequency indicated in Table 2.2 and press ENTER. A prompt will appear requesting the frequency of the third frequency component. Enter the third frequency indicated in Table 2.2 and press ENTER twice.
   e. Note: that one can press ESC at any time to return to the beginning. Press Q or Q and ENTER to exit FFTDEMO altogether.

4. After entering the required information, a time domain plot of the generated sample data should now be displayed. Note: that the arrow keys can be used to scroll a blinking cursor to the left or right. Note: the time and amplitude values at the position of the blinking cursor are presented above the plot frame.

5. For Cases 1-3:
   a. Using the arrow keys to scroll left and right, determine the amplitude \( A \) of the waveform shown. Record this value in the FFTDEMO table of the data sheet.
   b. Using the arrow keys to scroll left and right, determine the time between peaks \( \Delta t \) of the waveform shown. Record this value in the FFTDEMO table of the data sheet.
   c. Determine the frequency of the digital sample shown in the time domain plot by taking the reciprocal of the time between peaks. Specifically, compute \( f = 1/\Delta t \). Record this value in the FFTDEMO table of the data sheet.

6. Press F to switch from the time domain plot to a frequency domain plot. As with the time domain plot, a blinking cursor is available which can be scrolled to the left or right using the arrow keys. Note: the frequency and amplitude values at the position of the blinking cursor are presented above the plot frame.
7. Note: the resolution in the frequency domain plot is presented on the screen. Record this value in the FFTDEMO table of the data sheet.
8. For Cases 1-3, use the arrow keys to scroll left and right and determine the amplitude \( A \) and frequency \( f \) of the largest peak in the frequency domain plot. Record these values in the FFTDEMO table of the data sheet.
9. For Case 4, use the arrow keys to scroll left and right and determine the amplitudes \( A_1 \) and frequencies \( f_1 \) of the three largest peaks in the frequency domain plot. Record these values in the FFTDEMO table of the data sheet.
10. For Case 5, use the arrow keys to scroll left and right and determine the amplitudes \( A_i \) and frequencies \( f_i \) of the five largest peaks in the frequency domain plot. Record these values in the FFTDEMO table of the data sheet as the fundamental frequency and first four harmonics in order of largest to smallest amplitude.
11. Having completed steps 1-10, exit FFTDEMO by pressing Q or Q and ENTER.

V.2. Part 2: Sampling Theory

1. Check that the system is connected and set up correctly before beginning this part of the experiment. A schematic of the setup is presented in Fig.2.1(a). Students should ask their lab instructor for assistance if needed.
2. Turn on the signal generator. Turn the amplitude dial to set amplitude at about 80% of maximum. Set the waveform type to sinusoidal.
3. Start SCOPE. Set the sampling frequency (rate) to 1000 and the number of samples (count) to 100.
4. For frequencies \( f_i \), \( i = 1 \ldots 9 \) of \( f_1 = 50 \text{ Hz}, f_2 = 500 \text{ Hz}, f_3 = 600 \text{ Hz}, f_4 = 700 \text{ Hz}, f_5 = 800 \text{ Hz}, f_6 = 900 \text{ Hz}, f_7 = 1000 \text{ Hz}, f_8 = 1000 \text{ Hz}, \) and \( f_9 = 1500 \text{ Hz} \) do the following:
   a. Set the signal generator to the current frequency \( f_i \).
   b. Click RUN in SCOPE to take a sample of the signal. Adjust the time and frequency domain plots’ axes as desired.
   c. Use the arrowhead feature in SCOPE to determine the frequency at which the largest peak occurs in the frequency domain. Record this value as the measured frequency \( f_m \) in the NYQUIST SAMPLING table of the data sheet.
   d. Determine the frequency predicted to be measured \( f_p \) based on Nyquist sampling theory by using the Nyquist folding diagram presented in Fig.2.2. Record this value in the NYQUIST SAMPLING table of the data sheet.

V.3. Part 3: Frequency Spectrum

1. Check that the system is connected and set up correctly before beginning this part of the experiment. A schematic of the setup is presented in Fig.2.1(b). Students should ask their lab instructor for assistance if needed.
2. Start SCOPE. Set the sampling frequency (rate) and number of samples (count) both to 10,000.
3. For the numbers \( n = 0 \ldots 9 \), to the following:
   a. Press and hold the number \( n \) key on the touch-tone telephone. While holding down the key, place the sound level meter close to the earpiece of the telephone. (Make sure the sound level meter is turned on.)
b. Click RUN in SCOPE to take a sample. Adjust the frequency domain plot’s axes as desired.

c. Two large amplitude peaks should be obvious in the frequency domain plot. Use the arrowhead feature in SCOPE to determine the frequencies at which these two largest peaks occur. Note: that a peak at 60 Hz should be ignored, as such a peak is likely due to background noise.

d. Record the values for the frequencies of the two largest peaks \( f_1 \) and \( f_2 \) in the appropriate row of the TELEPHONE TONES table.

V.4. Part 4: Length of a Rod

1. Check that the system is connected and set up correctly before beginning this part of the experiment. A schematic of the setup is presented in Fig.2.1(c). Students should ask their lab instructor for assistance if needed.

2. Use a meter stick to measure the length of the rod \( L_m \) and record the value in the ROD LENGTH table of the data sheet.

3. Start SCOPE. Set the sampling frequency (rate) to 2000 and the number of samples (count) to 1000.

4. Strike the end of the aluminum rod opposite the accelerometer with the hammer. Wait a few seconds and then click RUN in SCOPE to take a sample. Adjust the frequency domain plot’s axes as desired.

5. Use the arrowhead feature in SCOPE to determine the fundamental frequency of vibration of the beam and record the value in the ROD LENGTH table of the data sheet.

6. Using the fundamental frequency determined in step 4 and Equation (2.7), determine the predicted length of the rod \( L_p \) and record the value in the ROD LENGTH table of the data sheet. Note: for aluminum \( K = 6,894.757 \text{ Pa} \) and \( \rho = 2800 \text{ kg/m}^3 \).

7. Turn off all equipment and tidy the lab station.

8. Record appropriate equipment identification information.

VI. Post-lab Questions

1. Describe the effect of the choice of sampling frequency on the frequency domain resolutions for Cases 1-5 in Part 1.

2. How did the high resolution value affect the measured amplitudes and frequencies of the constituent components in Case 3 in Part 1?

3. How did the measured frequencies compare to the frequencies expected to be measured based on Nyquist sampling theory in Part 2?

4. Create a matrix for the frequencies of the two frequency components for the numbers on the touch-tone telephone recorded in Part 3, where each column and row corresponds to different frequency values.

5. How did the measured length of the aluminum rod and the predicted length of the rod in Part 4 compare? Calculate the percent difference in the lengths with respect to the measured length.
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Lab Instructor’s signature: _____________________  date: ____/____/____
EXPERIMENT No. 3: DYNAMICS OF INSTRUMENTS—I

I. Objectives

In this experiment, characteristics of simple first-order systems will be explored. The objective of this experiment is to examine first-order system behavior in simple low- and high-pass RC circuits and a thermocouple. Special attention will be paid to attenuation characteristics of three filters (two low-pass RC circuits and one high-pass RC circuit). The time constants for each circuit and for the thermocouple’s response will also be determined.

II. Equipment

Part 1 of this experiment will involve the construction of first-order RC circuits to act as filters for analog signals generated with a signal generator. The signal generator used is the same as used in EXPERIMENT No. 2. The output from the signal generator will serve as input for a circuit block. One resistor and one capacitor will be attached to the circuit block in order to complete the block’s internal circuit and create a simple RC circuit. Two different resistors (5 kΩ and 10 kΩ ± 1%) and one capacitor (1 µF ± 0.1%) will be used to vary the characteristics of the RC circuit. The output of the circuit block will then be passed through an ADC before being recorded by a PC and analyzed using the SCOPE software described in EXPERIMENT No. 2. See Fig. 3.1(a) for a schematic of the measurement system setup for Part 1. See Fig. 3.2 for diagrams of the circuit block’s internal circuit and the arrangement of resistor and capacitor for low- and high-pass RC circuits.

In Part 2 of this experiment, the response time of a thermocouple will be examined; see Fig. 3.1(b) for a schematic of the thermocouple measurement system. Output from the thermocouple will be viewed using SCOPE. The time constant of the thermocouple will be determined for two cases: (1) thermocouple placed in ice bath when previously having been measuring ambient air temperature; and (2) thermocouple removed from ice bath to again measure ambient air temperature. Students will be able to observe the obvious difference in the thermocouple’s response time for the two cases due to the difference in heat transfer coefficients.
of air and water. See Table 3.1 for specific information on thermocouple and other equipment used in this experiment. Note: brackets ([ ] ) in Table 3.1 indicate information which students will need to record; OR indicates that more than one model is available—students should record the information for the model used.

III. Theory

III.1. First-Order Systems

First-order systems are systems which have a relationship between input and output which can be modeled as being governed by a first-order differential equation. Such systems include storage or dissipative elements which cause a delay in the response of the system to changes in input. First-order systems’ responses are also dependent on the frequency content of the input. The general formula for a first-order system’s governing equation of motion relating output ($y(t)$) to input ($F(t)$) is presented in Equation (3.1), where $a_0$ and $a_1$ are constants, $\tau$ is the time constant of the system, and $K$ is the static sensitivity of the system (for a zeroth-order approximation of the system). $K$ is defined by Equation (3.2) and $\tau$ is defined by Equation (3.3). The time constant of a first-order system is a crucial parameter. It is a measurement of the speed of the system’s response to a change in input. Specifically, the time constant is the time required for the system to achieve 63.2% of the change in amplitude due to a change in input relative to the system’s steady-state response. The smaller the time constant is, the faster the system’s response; see Fig.3.3.

$$F(t) = a_1 \dot{y}(t) + a_0 y(t)$$  \hspace{1cm} (3.1)

$$\tau \dot{y}(t) + y(t) = K F(t)$$

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Because the response of a first-order system involves a time delay and is dependent on the frequency of the input, dynamic calibration must be performed. In dynamic calibration, inputs of known amplitude and frequency are applied to the system and the response is recorded. From the recorded response, the static sensitivity and time constant of the system can be determined.

### III.2. First-Order Step Response

The response of a first-order system to a step input is presented in Equation (3.4), where \( U \) is the amplitude of the input, \( \Theta(t) \) is the Heavyside step function, \( V_{in} \) is the initial output value before the input is applied, and \( V_{out} = \) (Equation (3.5)) is the steady-state response of the system. The second part of Equation (3.4) is known as the transient response, because it decays exponentially. The rate of decay of the transient response is directly proportional to the value of the time constant.
III.3. First-Order Sinusoidal Response

Consider a sinusoidal input of the form presented in Equation (3.6), where $a$ is the amplitude of the input, $\omega_0$ is a sinusoidal function (sine or cosine), $f_0$ is the frequency of the input signal, and $\phi$ is the phase of the input signal. The response of a first-order system to such an input is presented in Equation (3.7). The second part of Equation (3.7) is the steady-state solution; see Equation (3.8). For a sinusoidal input, the steady-state solution is also a sinusoidal function (usually of the same form as the input—sine or cosine) with the same frequency as the input. The steady-state response will have different amplitude and a delay ($\tau$) relative to the input signal, both of which are dependent on the frequency of the input and the time constant of the system; see Equations (3.9) and (3.10). The ratio of the steady-state output amplitude to the input amplitude is known as the magnitude ratio ($\eta$). The time delay of the system is often known as the phase of the system. This phase should not be confused with the phase of the input signal.

Fig. 3.3. Illustration of the Effect of Time Constant Value on the Response of a First-Order System.
III.4. First-Order RC Filter Circuits

III.4.1. Basic Filter Designs

Often it is desirable to remove certain frequency components from an input signal by applying a filter. Four basic filter designs (low-pass, high-pass, bandpass, and notch) are presented in Fig.3.4. In each case, the magnitude ratio \( M \) of the system’s response vs. input frequency \( f \) is plotted. Plots such as these are often referred to as Bode diagrams. In Bode diagrams, it is common practice to present the magnitude in units of decibels (dB) and the frequency in a log-scale.

Figure 3.4(a) shows a Bode diagram for a low-pass filter. Note: that for a low-pass filter frequencies less than the cutoff frequency \( f_c \) have a magnitude ratio approximately equal to one, indicating that the amplitude of the output (system’s steady-state response) is equal to the amplitude of the input. When the magnitude ratio is less than one, the decrease in the output’s amplitude relative to the input amplitude is known as attenuation. The difference in amplitudes of input and output signals is said to be significant enough to be termed true attenuation when the frequency is past the cutoff frequency. Cutoff frequency is defined as the frequency at which the output power is half that of the input, which occurs when \( M/K = 1/\sqrt{2} = 0.707 = -3 \text{ dB} \). The range of frequencies for which \( M/K > -3 \text{ dB} \) is termed the passband, as frequencies in this range “pass” through the system with little attenuation. The range of frequencies for which \( M < -3 \text{ dB} \) is termed the stopband, as frequencies in this range are severely attenuated or “stopped” by the system. The rate of attenuation vs. change in frequency in the stopband region is known as rolloff. The greater the rolloff, the better the filter performs.

Figure 3.4(b) presents the Bode diagram for a high-pass filter. High-pass filters attenuate frequencies below the cutoff frequency. This filter is often used when low frequency components are considered to be due to noise or are otherwise deemed unfavorable. Figure 3.4(c) presents the Bode diagram for a bandpass filter. A bandpass filter is essentially the combination of low- and high-pass filters. Such filters have two cutoff frequencies, and only frequencies between the two cutoff frequencies are not attenuated. Bandpass filters are generally used when only a limited range of frequencies are of interest. Finally, Fig.3.4(d) presents the Bode diagram for a notch filter. A notch filter is similar in design to a bandpass filter, having two cutoff frequencies. Unlike a bandpass filter, a notch filter attenuates frequencies between its two cutoff frequencies. Notch filters are often used to remove specific undesirable frequencies which may be corrupting the input signal. For example, notch filters are often used to remove frequencies around 60 Hz, which often appear due to the frequency of power supplies for electrical equipment. A large 60 Hz frequency component is a common cause of corrupt data.

In this experiment, simple RC circuits will be used to construct first-order low- and high-pass filter circuits.

III.4.2. First-Order RC Low-Pass Filter

A first-order RC low-pass filter circuit for voltage has a circuit diagram as presented in Fig.3.1(b), where the resistor is in series with the supply voltage and the capacitor is in parallel.
By analyzing the circuit, one can determine that the equation of motion relating the output voltage to the input voltage is a first-order differential equation. Specifically, the equation of motion is presented in Equation (3.11). Relating Equation (3.11) to the general formula for a first-order system’s equation of motion, one can determine the time constant, of the system; see Equation (3.12). Having determined the time constant, one can then determine the magnitude ratio and phase for inputs of various frequency using Equations (3.9) and (3.10), where $\omega = 1$.

III.4.3. First-Order RC High-Pass Filter

A first-order RC high-pass filter circuit for voltage has a circuit diagram as presented in Fig.3.1(d), where the resister is in parallel with the supply voltage and the capacitor is in series. By analyzing the circuit, one can determine that the equation of motion relating the output voltage to the input voltage is a first-order differential equation; see Equation (3.13). The time constant for the system can easily be seen to be the same as for the low-pass filter RC circuit; see Equation (3.12). Assuming an input voltage of the form $v(t) = \cos(\omega t)$, the derivative of the input voltage $v'(t) = -\omega \sin(\omega t)$, where $\omega = 1$. Neglecting the phase change, the static sensitivity $M_{df} = 1$ and the amplitude of the output decreases with lower input frequencies.
III.5. Thermocouple Time Constant

A thermocouple is a temperature measuring device usually comprised of two wires of dissimilar metals which differ in their thermal conductivity properties. The wires are jointed at one end, usually by soldering. When the ends of the wires are exposed to different temperatures an electromagnetic force (EMF) is produced which is proportional to the difference in the two temperatures.

From the First Law of Thermodynamics, one can derive Equation (3.14), where $\dot{Q}$ is the heat energy added to the system (thermocouple), $m$ is mass, $C$ is heat capacity, and $T(t)$ is temperature. If the primary form of heat transfer is via convection (as is the case for a thermocouple in a fluid), then the heat energy added to the system is proportional to the product of (1) the difference in temperature of the thermocouple ($T_m(t)$) and the temperature of the medium being measured ($T_m(t)$) and (2) the surface area ($S$) of the thermocouple exposed to the medium. The proportionality constant is the heat transfer (convection) coefficient ($h$) between the thermocouple and medium materials; see Equation (3.15).

\[ \dot{Q} = mC\dot{T}(t) \quad (3.14) \]
\[ \dot{Q} = hS[T_m(t) - T(t)] \quad (3.15) \]

Setting the right hand sides of Equations (3.14) and (3.15) equal to each other, one can rewrite the relationship to arrive at a first-order differential equation; see Equation (3.16). If one considers the temperature of the medium being measured ($T_m(t)$) as the input and the temperature of the thermocouple ($T(t)$) as the output, then by comparing Equation (3.16) to the general formula for a first-order system given by Equation (3.1), one can determine the time constant of the thermocouple; see Equation (3.17). Note: that the time constant of the thermocouple is indirectly correlated with the convection coefficient ($h$). Thus, the smaller the convection coefficient, the larger the time constant and the slower the response of the thermocouple will be. This fact will be illustrated in Part 2 of this experiment, when the time constant for a thermocouple will be determined for two cases: (1) measuring the temperature of water (high convection coefficient); and (2) measuring the ambient air temperature (low convection coefficient).

\[ \frac{mC}{hS} \dot{T}(t) + T(t) = T_m(t) \quad (3.16) \]
\[ \tau = \frac{mC}{hS} \quad (3.17) \]

IV. Pre-lab Questions

1. What information does the time constant of a first-order system provide about the system’s response to any given input?
2. What is the steady-state response of a system to a step input of 2 units if the system can be modeled as having the first-order equation of motion given below?

\[ 200\dot{y}(t) + y(t) = 10F(t) \]

3. Suppose a system has the equation of motion given below.
   a. Does the system act as a low- or high-pass filter? Explain.
   b. What is the cutoff frequency? (When does the magnitude ratio \( M/K = 1/\sqrt{2} \)?)

\[ 50\dot{y}(t) + y(t) = \dot{F}(t) \]

4. A Pt-Pt/13% RD (Type R) thermocouple has an approximately spherical junction between its two constituent wires with a diameter of 0.5 mm. It is used to measure the temperature of gasses in a combustion tunnel. When the flame is ignited, it produces an approximate step increase in the gas temperature of 800 K. The average heat transfer coefficient on the surface of the thermocouple is 500 W·°C/m². The gas temperature before ignition is 320 K. The density of Pt is 21,450 kg/m³. The Heat capacity of Pt is 134 J·°C/kg. Answer the following:
   a. What is the time constant of the thermocouple?
   b. If the same thermocouple was used in an aqueous environment in which the heat transfer coefficient is 6000 W·°C/m², what would be the thermocouple time constant?
   c. Sketch the thermocouple response vs. time for cases (a) and (b).

V. Procedure

i. Before beginning this experiment, first acquire a glass of ice water. Set the glass aside for now. It will be used in Part 2 of this experiment.

V.1. Part 1: Time Constants and Attenuation

1. Check that the system is connected and set up correctly before beginning this part of the experiment. A schematic of the setup is presented in Fig.3.1(a). Students should ask their lab instructor for assistance if needed.
2. Turn on the signal generator. Turn the amplitude dial to set amplitude at about 80% of maximum. Set the waveform type to square wave. Set the frequency to 5 Hz.
3. Start SCOPE. Set the sampling frequency (rate) to 10,000 and the number of samples (count) to 1000.
4. For each of the three RC filter arrangements (Filter No. 1, Filter No. 2, and Filter No. 3) for the circuit block presented in Fig.3.2(b)-(d), do the following:
   a. Plug in the correct resistor and the capacitor in the positions indicated in the corresponding subfigure of Fig.3.2.
   b. Click RUN in SCOPE to take a sample. Repeat until a sample similar to the example first-order response plots presented in Fig.3.3 is captured.
c. Use the arrowhead feature of SCOPE to determine the amplitude ($A$) of the response (peak value – minimum value). Record this value in the TIME CONSTANTS table of the data sheet.

d. Multiply the amplitude value by 0.632 and record this value in the TIME CONSTANTS table of the data sheet.

e. Determine the time it takes for the system to reach 0.632A from its starting value (minimum value). Record this value as the measured time constant ($\tau_m$) in the TIME CONSTANTS table of the data sheet.

f. Compute the theoretical time constant ($\tau_{th}$) using Equation (3.12). Record this value in the TIME CONSTANTS table of the data sheet.

g. Determine the minimum and maximum theoretical value of the time constant given uncertainties of ± 1% for resistors and ± 0.1% for the capacitor.

5. Change the waveform type of the signal generator to sinusoidal.

6. For frequencies ($f_i$, $i = 1…8$) of $f_1 = 5$ Hz, $f_2 = 10$ Hz, $f_3 = 20$ Hz, $f_4 = 40$ Hz, $f_5 = 70$ Hz, $f_6 = 100$ Hz, $f_7 = 150$ Hz, and $f_8 = 200$ Hz do the following:
   a. Arrange the circuit block for Filter No. 1 as illustrated in Fig.3.2(b).
   b. Click RUN in SCOPE to take a sample.
   c. Use the arrowhead feature of SCOPE to determine the amplitude ($A$) of the response (peak value – minimum value). Record this value in corresponding cell of the ATTENUATION table of the data sheet.
   d. Repeat steps 6.a—6.c for Filer No. 2 (illustrated in Fig.3.2(c)) and Filter No. 3 (illustrated in Fig.3.2(d)).

7. For each frequency/filter combination, determine the magnitude ratio by dividing the amplitudes determined in step 6 by the input amplitude. For Filter No. 1 and Filter No. 2 (low-pass filters), assume the input amplitude is equal to the amplitude measured at 5 Hz. For Filter No. 3 (high-pass filter), assume the input amplitude is equal to the amplitude measured at 200 Hz.

V.2. Part 2: Thermocouple Response

1. Check that the system is connected and set up correctly before beginning this part of the experiment. A schematic of the setup is presented in Fig.3.2(b). Students should ask their lab instructor for assistance if needed.

2. Set the thermocouple to report temperature readings in °C.

3. Start SCOPE. Set the sampling frequency (rate) to 1500 and the number of samples (count) to 4500. Note: this will result in a only three-second long sample.

4. Record the ambient air temperature of the room ($T_{amb}$) using the thermocouple. Record this value in the THERMOCOUPLE table of the data sheet.

5. Determine the absolute difference in temperature ($\Delta T$) between the ambient air temperature ($T_{amb}$) determined in step 4 and the ice bath (0°C). Record this value in the THERMOCOUPLE table of the data sheet.

6. Multiply the temperature difference ($\Delta T$) determined in step 5 by 0.632 and record this value in the THERMOCOUPLE table of the data sheet.

7. Acquire the glass of ice water prepared before beginning this experiment. This will serve as the ice bath for this experiment.
8. Click RUN in SCOPE to take a sample and IMMEDIATELY place the thermocouple probe in the ice bath. The graph of the sample in the time domain should appear similar to the plot presented Fig.3.3 (although inverted). Repeat until a good sample is acquired. Note: that it may take a few minutes for the thermocouple to warm back up between attempts.

9. Export the recorded data to a spreadsheet program such as Microsoft Excel. Students should ask their lab instructor for assistance in exporting data if necessary. Students should save this data to a flash drive for later use.

10. Use the spreadsheet program to plot the thermocouple temperature vs. time. Determine the time required for the thermocouple to achieve a change in temperature equal to 0.632ΔT. Record the time required as the time constant (τ) in the Amb. → Ice row of the THERMOCOUPLE table of the data sheet.

11. Place the thermocouple probe back in the ice bath if it is not still there.

12. In SCOPE, set the sampling frequency (rate) to 100 and the number of samples (count) to 12,000. Note: this will result in a two-minute long sample.

13. Click RUN in SCOPE and remove the thermocouple probe from the ice bath. The sample will take a couple of minutes to complete running.

14. Export the recorded data to a spreadsheet program such as Microsoft Excel. Students should save this data to a flash drive for later use.

15. Use the spreadsheet program to plot the thermocouple temperature vs. time. Determine the time required for the thermocouple to achieve a change in temperature equal to 0.632ΔT. Record the time required as the time constant (τ) in the Ice → Amb. row of the THERMOCOUPLE table of the data sheet.

16. Dump out the glass of ice water. Turn off all equipment and tidy the lab station.

17. Record appropriate equipment identification information.

VI. Post-lab Questions

1. How do the measured time constants in Part 1 compare to the theoretical time constants? Do they fall within the range of minimum and maximum theoretical values for the time constant given the uncertainties in resistors and the capacitor? If not, suggest reasons why this might be the case.

2. For each of the three RC filters explored in this experiment, plot magnitude ratio vs. frequency using the data in the ATTENUATION table of the data sheet. Plot the magnitude in a decibel scale (M [dB] = 20 log10 M). Determine the cutoff frequency (frequency where M/K = -3 dB) for each filter. Interpolate if needed. Note: for low-pass filter K = 1; for high-pass filter K = ω.

3. Plot the thermocouple temperature vs. time for both cooling (Amb. → Ice) and warming (Ice → Amb.) on the same graph. Explain why the response rate for the cooling is faster than the response rate for warming. (Hint: consider the issue of convection coefficient mentioned in the Theory section.)
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* $= A_{in}$

## THERMOCOUPLE

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<th>$T_{amb}$ (°C)</th>
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EXPERIMENT No. 4: DYNAMICS OF INSTRUMENTS—II

I. Objectives

In this experiment, characteristics of a simple second-order system will be explored. The system of interest is a cantilevered beam, the frequency of vibration of which can be controlled using a solenoid actuator and measured using an LVDT. The objectives of this experiment are (1) to calibrate and (2) to examine the dynamic characteristics of the second-order cantilevered beam system.

II. Equipment

The vibrating beam assembly used in this experiment utilizes a solenoid actuator to induce oscillations in a cantilevered beam. The frequency of the oscillations can be controlled manually by turning a knob on the assembly’s dial face. The assembly is also equipped with an LVDT for measuring the displacement of the beam. Displacement measurements are passed through an ADC before being recorded and viewed using the software SCOPE, as described in EXPERIMENT No. 2; see Fig.4.1 for a schematic of the entire measurement system. The assembly will be calibrated using software called CALBEAM, similar to the LVDT.VEE software used in EXPERIMENT No. 1. Equipment information for the vibrating beam assembly, ADC, and PC is presented in Table 4.1. Note: brackets ([ ] ) in Table 4.1 indicate missing information which should be recorded by students.

![Schematic of Measurement System for Experiment No. 4.](image)

**Table 4.1 Equipment Information for Experiment No. 4**

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<tr>
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<td>Personal Measurement Device™</td>
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<tr>
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<tr>
<td>University of Tennessee, Knoxville</td>
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</table>
III. Theory

III.1. Second-Order Systems

Second-order systems are systems which have a relationship between input and output which can be modeled as being governed by a second-order differential equation. Such systems have inertial qualities, and include instruments such as accelerometers, and microphones (diaphragm pressure transducers). Like first-order systems, second-order systems’ responses are also dependent on the frequency content on the input. The general formula for a second-order system’s governing equation of motion relating output \( y \) to input \( f \) is presented in Equation (4.1), where \( K \), \( \omega_n \), and \( \zeta \) are constants, \( K \) is the static sensitivity of the system (for a zeroth-order approximation of the system), \( \omega_n \) is the natural frequency of the system, and \( \zeta \) is the damping ratio of the system.

Because the response of a second-order system involves a time delay and is dependent on the frequency of the input, dynamic calibration must be performed. In dynamic calibration, inputs of known amplitude and frequency are applied to the system and the response is recorded. From the recorded response, the static sensitivity, natural frequency, and damping ratio of the system can be determined.

III.2 Natural Frequency and Damping Ratio

The natural frequency of a second-order system is often referred to as the resonance frequency—the frequency at which the magnitude ratio is maximized—because the natural frequency is an
accurate approximation of the resonance frequency; see Fig. 4.2(b) for an example of a second-order system’s Bode diagram.

The damping ratio is a measure of the decay in oscillatory behavior of the system. If the damping ratio $\zeta > 1$, the system is said to be overdamped. When a system is overdamped, its response is has no oscillatory behavior. The larger the overdamping, the slower the system’s response will be. If the damping ratio $\zeta = 1$, the system is said to be critically damped. A critically damped system approaches its steady-state response at the fastest possible rate without oscillations. If the damping ratio $\zeta < 1$, the system is said to be underdamped. An underdamped system will have oscillatory behavior in its response. The lower the damping ratio, the larger the amplitude of the oscillations will be, and slower they will decay. If the damping ratio $\zeta = 0$, the system is said to be undamped. An undamped system will experience oscillations in its response which will not (theoretically) ever decay; see Fig. 4.2(a) for an illustration of the effect of damping ratio on the step response of a second-order system.

III.3. Second-Order Total Response

The total response for a second-order system is presented in Equation (4.4), where $y_p(t)$ is the steady-state or particular solution, and $s(t)$ is a conditional function, the solution for which is dependent on the damping ratio of the system. The total solution can be divided into two components in two different ways: (1) as the sum of the particular (steady-state) solution ($y_p(t)$) and the homogenous solution ($y_h(t)$); or (2) as the sum of the free solution ($y_{free}(t)$) and the forced solution ($y_{forced}(t)$); see Equations (4.5) and (4.6). Solutions for the conditional function $s(t)$ and necessary equations for coefficients are presented in Equations (4.7), (4.8), (4.9), (4.10), and (4.11), where $\omega_d$ is the damped frequency of the system and $\omega_o$ is the overdamped frequency of the system.

\[
y(t) = y_p(t) + e^{-\zeta\omega_n t} s(t) \quad (4.4)
\]

\[
y(t) = y_p(t) + y_h(t) \quad (4.5)
\]

\[
y(t) = y_{free}(t) + y_{forced}(t) \quad (4.6)
\]

\[
s(t) = \begin{cases} 
C \cos(\omega_n t) + \frac{B}{\omega_n} \sin(\omega_n t), & \zeta = 0 \text{ (undamped)} \\
C \cos(\omega_d t) + \frac{B}{\omega_d} \sin(\omega_d t), & 0 < \zeta < 1 \text{ (underdamped)} \\
C + B t, & \zeta = 1 \text{ (critically damped)} \\
C \cosh(\omega_o t) + \frac{B}{\omega_o} \sinh(\omega_o t), & \zeta = 0 \text{ (overdamped)}
\end{cases} \quad (4.7)
\]

\[
C = y(0) - y_p(0) \quad (4.8)
\]

\[
B = \dot{y}(0) - \dot{y}_p(0) + \zeta \omega_n C \quad (4.9)
\]
III.4. Second-Order Particular Solutions

III.4.1. Step Response

The particular (steady-state) solution for step input of amplitude $A$ for a second-order system is the same as for a first-order system, regardless of the system’s natural frequency or damping ratio; see Equation (4.12).

$$y_p = KA \quad (4.12)$$

III.4.2. Sinusoidal Response

Consider a sinusoidal input of the form presented in Equation (4.13), where $A$ is the amplitude of the input, $\text{trig}$ is a sinusoidal function (sine or cosine), $\omega$ is the frequency of the input signal, and $\varphi$ is the phase of the input signal. The steady-state response of a second-order system to such an input is the same as the steady-state response of a first-order system; see Equation (4.13), where $M$ is the system’s magnitude ratio and $\theta$ is the delay or phase of the system. The magnitude ratio and phase for a second-order system have different definitions than for a first-order system. Formulas for second-order magnitude and phase are presented in Equations (4.15) and (4.16), where $r$ is the frequency ratio and is equal to the input frequency divided by the natural frequency of the system; see Equation (4.17).

$$F(t) = A \text{trig}(\omega t \pm \varphi) \quad (4.13)$$

$$y_p(t) = AM \text{trig}(\omega t \pm \varphi - \theta) \quad (4.14)$$

$$M = \frac{K}{\sqrt{(1 - r^2)^2 + (2\zeta r)^2}} \quad (4.15)$$

$$\theta = \tan^{-1}\left(\frac{2\zeta r}{1 - r^2}\right) \quad (4.16)$$

$$r = \frac{\omega}{\omega_n} \quad (4.17)$$

As mentioned previously, the natural frequency of a second-order system is a close approximation for the resonance frequency of the system—the input frequency which will maximize the magnitude ratio. Technically, the resonance frequency ($\omega_r$) is defined as in Equation (4.18). It follows from the definition of the resonance frequency given in Equation (4.18) that a peak in the magnitude ratio does not exist when $\zeta > 1/\sqrt{2} \approx 0.707$. 

$$\omega_r = \omega_n \sqrt{1 - \zeta^2} \quad (4.10)$$

$$\omega_o = \omega_n \sqrt{\zeta^2 - 1} \quad (4.11)$$
\[ \omega_r = \omega_n \sqrt{1 - 2\zeta^2} \quad (4.18) \]

III.5. Second-Order Homogeneous, Free, and Forced Solutions

III.5. Homogenous Solution

An indirect method for solving for the homogeneous solution for a second-order system is presented in Equation (4.19), where the homogeneous solution is equal to the total response minus the particular (steady-state) solution. There is not direct a method for solving for the homogeneous solution, as the total solution and particular solution are needed to determine the homogeneous solution.

\[ y_h(t) = y(t) - y_p(t) = e^{-\zeta \omega_n t} s(t) \quad (4.19) \]

III.5. Free Solution

In solving for the free solution, one should set the particular (steady-state) solution equal to zero. This then implies that \( y_p(0) = 0 \) and \( \dot{y}_p(0) = 0 \). Note: that the initial conditions for the total solution (\( y(0) \) and \( \dot{y}(0) \)) are still needed to solve for the coefficients \( C \) and \( B \) in the conditional function \( s(t) \).

III.5. Forced Solution

The forced solution is equal to the total response minus the free response, thus one can solve for the forced solution indirectly using Equation (4.20) if the total solution and free solution are known. In solving for the forced solution directly, one should set the input equal to zero. This then implies that \( y(0) = 0 \) and \( \dot{y}(0) = 0 \). Note: that the initial conditions for the particular solution (\( y_p(0) \) and \( \dot{y}_p(0) \)) are still needed to solve for the coefficients \( C \) and \( B \) in the conditional function \( s(t) \).

\[ y_{\text{forced}}(t) = y(t) - y_{\text{free}}(t) \quad (4.20) \]

III.6. Log Decrement Method

The damping ratio for a second-order system can be determined even if the natural frequency of the system is unknown. One method used to achieve this is known as the log decrement method. The method is only applicable if the system is underdamped or undamped and oscillations are present in the system’s response. The log decrement (\( \delta \)) is equal to the natural log of the ratio of concurrent amplitudes in the system’s transient response; see Equation (4.21), where \( A_i \) are the amplitudes of the transient oscillations, \( i = 1 \ldots N-1 \), and \( N \) is the number of transient oscillations. Theoretically, the log decrement is the same regardless of the choice of \( i \). The damping ratio can then be determined using Equation (4.22), which relates the log decrement to the damping ratio of the system.
\[ \delta = \ln \left( \frac{A_i}{A_{i+1}} \right) \quad (4.21) \]

\[ \zeta = \frac{\delta}{\sqrt{\delta^2 - 4\pi^2}} \quad (4.22) \]

### III.7. Damping Ratio from Simultaneous Equations

In addition to the log decrement method, the damping ratio for a second-order system can also be determined using simultaneous equations. Specifically, the slope \( m \) of the natural log of the amplitudes of the transient response vs. time curve is related to the damping ratio and natural frequency of the system; see Equation (4.23). Assuming the natural frequency of the system is unknown, one can measure the time between oscillation peaks \( \Delta t \) to determine the damped frequency \( \omega_d \) of the system; see Equation (4.24). The damped frequency is then related to the natural frequency and the damping ratio as presented in Equation (4.10). Solving Equations (4.23), (4.24), and (4.10) simultaneously for the damping ratio yields Equation (4.25). As with the log decrement method, the method of simultaneous equations requires that the system be underdamped or undamped.

\[ m = -\zeta \omega_n \quad (4.23) \]

\[ \omega_d = \frac{2\pi}{\Delta t} \quad (4.24) \]

\[ \zeta = \frac{m}{\sqrt{\omega_d^2 + m^2}} \quad (4.25) \]

### IV. Pre-lab Questions

1. The following equation describes the behavior of a second-order system.

\[ \ddot{y}(t) + \dot{y}(t) + 100y(t) = 30 \sin(20t + 0.25) \]

   a. Determine the natural frequency and damping ratio of the system.
   b. Determine the frequency ratio, magnitude ratio, and phase of the system.
   c. Determine the resonance frequency of the system. Does the magnitude ratio of the system have a peak at any given frequency?
   d. Determine the steady-state response of the system.

2. The following equation describes the behavior of a second-order system, where \( u(t) \) is the Heavyside step function.

\[ 4\ddot{y}(t) + \dot{y}(t) + 200y(t) = 25u(t) \]

\[ y(0) = \dot{y}(0) = 0 \]
a. Determine the natural frequency and damping ratio of the system.
b. Find the particular solution.
c. Find the homogeneous solution.
d. Find the free solution
e. Find the forced solution.

V. Procedure

V.1. Part 1: Natural Frequency and Damping Ratio

1. Check that the system is connected and set up correctly before beginning. A schematic of the setup is presented in Fig. 4.1. Use a lead wire to short across the vibrating beam assembly’s actuator and another lead wire to short across the assembly’s induction coil (velocity transducer). Students should ask their lab instructor for assistance if needed.
2. Start CALBEAM. This software will be used to calibrate the vibrating beam system’s LVDT.
3. Mount the micrometer and turn it until the LVDT just touches the cantilevered beam without causing deflection. Record the voltage value displayed by CALBEAM in the CALIBRATION OF LVDT table of the data sheet for a deflection (ν) of 0.00 mm and for y₀.
4. Turn the micrometer to deflect the beam 1.00, 2.00, 3.00, and 4.00 mm and record the CALBEAM voltage reading at each deflection in the CALIBRATION OF LVDT table of the data sheet. If unclear, students should check with their lab instructor to be sure of how many turns of the specific micrometer being used accounts for a deflection of 1.00 mm.
5. Using Microsoft Excel or a similar program, students should plot the CALBEAM readings vs. deflection for the calibration data now recorded. Students should then add a best-fit linear trend line for the data. The trend line should be of the form y = Ky₀ + y₀, where K is the slope of trend line, ν is the deflection in mm, and y₀ is the CALBEAM reading without deflection (0.00 mm). Students should ask their lab instructor for assistance in adding the trend line if needed.
6. Record the value for the slope K of the trend line in the CALIBRATION OF LVDT table of the data sheet.
7. Remove the micrometer from its mounting, loosen the set screw, twist out and install the cap with the pin all the way in the mounting hole. Secure finger tight with the set screw.
8. Start SCOPE. Set the sampling frequency (rate) to 512 and the number of samples (count) to 1024.
9. Pluck the beam, wait about one second, and then click RUN in SCOPE to take a sample. Adjust axes as desired.
10. Measure the amplitudes (Aₜ) of eight consecutive peaks using the arrowhead feature in SCOPE and record these values in the LOG DECREMENT TABLE of the data sheet.
11. Take the natural logs of the amplitudes measured in step 10 and record these values in the ln(Aₜ) column of the LOG DECREMENT table of the data sheet.
12. Use the arrowhead feature in SCOPE to measure the differences in time between peaks (∆t or T) and record these values in the LOG DECREMENT table of the data sheet.
13. Use the arrowhead feature to determine the time each peak occurs (\(t_i\)) and record these values in the LOG DECREMENT table of the data sheet.
14. Average the differences in time between peaks (\(\bar{\Delta t}\)) to determine the damped period (\(T_d\)) of the system. Record this value in the LOG DECREMENT table of the data sheet.
15. Take the reciprocal of the damped period (\(T_d\)) and multiply by \(2\pi\) to determine the damped frequency (\(\omega_d\)). Record this value in the LOG DECREMENT table of the data sheet.
16. Use Equation (4.21) to determine the log decrement (\(\delta\)) for each pairing of peaks. Record these values in the LOG DECREMENT table of the data sheet.
17. Use Equation (4.22) and the values for the log decrement determined in step 16 to determine the damping ratio (\(\zeta\)) based on data from each consecutive pair of peaks. Record these values in the LOG DECREMENT table of the data sheet.
18. Average the values of the damping ratio determined in step 17 and record this value in the LOG DECREMENT table of the data sheet as \(\zeta\).
19. Using Microsoft Excel or a similar program, students should plot \(\ln(A_i)\) vs. \(t_i\) values in the LOG DECREMENT table of the data sheet. Students should then add a best-fit linear trend line for the data. The trend line should be of the form \(y^L = mt + b\), where \(m\) is the slope of trend line and \(b\) is the y-axis intercept. Students should ask their lab instructors for assistance in adding the trend line if needed.
20. Record the value for the slope (\(m\)) of the \(\ln(A_i)\) vs. \(t_i\) curve determined in step 19 in the LOG DECREMENT table of the data sheet.
21. Using the value for the damped frequency (\(\omega_d\)) determined in step 15, the slope (\(m\)) determined in step 19, and Equation (4.25), compute the damping ratio. Record this value as \(\zeta^*\) in the LOG DECREMENT table of the data sheet.
22. Compute the average of the two damping ratio values determined in steps 18 and 21. Record this value as \(\zeta_{avg}\) in the LOG DECREMENT table of the data sheet.
23. Use the average damping ratio value determined in step 22 and Equation (4.10) to solve for the natural frequency of the system (\(\omega_n\)). Record this value in the LOG DECREMENT table of the data sheet.

V.2. Part 2: Magnitude Ratio

1. Connect two lead wires from the actuator to the variable frequency output (VFO) exciter. Remove lead wire shorting across the actuator.
2. Set the VFO amplitude at the red mark. (The red mark should already be present.) Do not change this amplitude setting.
3. In SCOPE, set the sampling frequency (rate) to 512 and the number of samples (count) to 512.
4. For frequencies \((f_i, \ i = 1\ldots11)\) of \(f_1 = 3\ \text{Hz}, f_2 = 4\ \text{Hz}, f_3 = 5\ \text{Hz}, f_4 = 6\ \text{Hz}, f_5 = 7\ \text{Hz}, f_6 = 8\ \text{Hz}, f_7 = 10\ \text{Hz}, f_8 = 12\ \text{Hz}, f_9 = 15\ \text{Hz}, f_{10} = 20\ \text{Hz},\) and \(f_{11} = 30\ \text{Hz}\) do the following:
   a. Set the VFO exciter frequency to the current frequency \(f_i\).
   b. Wait until the vibrating beam settles and click RUN in SCOPE to take a sample.
   c. Using the arrowhead feature in SCOPE, measure the amplitude (A) of the oscillations and the time between peaks (\(\Delta t\)). Record these values in the appropriate row of the MAGNITUDE RATIO table of the data sheet.
d. Take the reciprocal of the time between peaks (\(\Delta t\)) and multiply by \(2\pi\) to obtain the measured frequency (\(\omega_{\text{meas}}\)).

e. Using the values for the damping ratio (\(\zeta_{\text{avg}}\)) and the natural frequency (\(\omega_n\)) determined in Part 1 and Equation (4.15), compute the theoretical magnitude ratio (\(M_{\text{th}}\)), where \(r = 2\pi f_i/\omega_n\). Record this value in the MAGNITUDE RATIO table of the data sheet.

f. If \(f_i > 3\), divide the amplitude value recorded for \(f_i\) by the amplitude value recorded for 3 Hz. Record this value as the measured magnitude ratio (\(M_{\text{meas}}\)) in the MAGNITUDE RATIO table of the data sheet.

g. Determine the absolute percent difference between the measured and theoretical magnitude ratio values compared to the theoretical value. Specifically, compute \(\%\ \text{diff} = 100|\frac{M_{\text{meas}} - M_{\text{th}}}{M_{\text{th}}}|\). Record this value in the MAGNITUDE RATIO table of the data sheet.

5. Turn off all equipment and tidy the lab station.

6. Record appropriate equipment identification information.

VI. Post-lab Questions

1. Using the values for the natural frequency and the damping ratio determined in Part 1 and Equation (4.18), compute the resonance frequency of the vibrating beam assembly.

2. Plot the magnitude ratio values determine in Part 2 vs. frequency. Does the system have a peak in its Bode diagram? If so, at approximately what frequency does the peak occur?

3. Compare the answers to questions 1 and 2.

4. Compare the values obtained for the damping ratio using the log decrement method (\(\bar{\zeta}\)) and the method of simultaneous equations (\(\zeta^*\)) in Part 1.

5. Describe physically what happened to the behavior of the beam when frequencies close to 9 Hz were used in Part 2. Why did this behavior occur? (Hint: consider the resonance frequency values computed in questions 1 and 2 and the Bode diagram plotted in question 2.)
### LOG DECREMENT

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<th>$\xi$</th>
<th>$t_i$ (s)</th>
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$$\xi = m = \frac{\omega_d}{\omega_n} = \frac{\omega_d}{\omega_n}$$

### MAGNITUDE RATIO

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### CALIBRATION OF LVDT

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</tbody>
</table>

$$y = K v + y_0$$

$y_0$ (mV)

$K$ (mV/mm)

Lab Instructor’s signature: _____________________ date: ____/____/____
EXPERIMENT No. 5: UNCERTAINTY ANALYSIS

I. Objectives

In this experiment, the volumetric flow rate of a faulty fan assembly will be measured and its 95% uncertainty will be computed. The fan assembly is designed such that the fan blades are not balanced and thus the fan wobbles as it spins. Additionally, errors in the pressure transducer, thermometer, ambient air pressure gauge, and other devices used in measuring the volumetric flow rate will be considered. The objectives of this experiment are (1) to gain a theoretical understanding of uncertainty analysis and (2) to observe how small errors in equipment can propagate via uncertainty analysis.

II. Equipment

II.1 Fan Assembly

In this experiment, the volumetric flow rate for a fan is determined by measuring other variables, including air temperature and pressure drop in a pitot tube, and using analytical equations. The fan assembly consists of a motor, the fan, a pitot tube placed above the fan in the path of the air flow, and a pressure transducer. The pitot tube is attached to a stand with a swiveling base which allows for the mouth of the pitot tube to be placed at various radial distances from the center of the fan. A guide needle extending from the stand is used along with demarcations on the top of the fan motor to set the desired radial location of the mouth of the pitot tube; see Fig.5.1 for a schematic of the fan assembly. See Table 5.1 for information on the pressure transducer used.

II.2 FAN-PITOT.VEE

The output from the pressure transducer is passed through an ADC before serving as input for software called FAN-PITOT.VEE; see Fig.5.1 for a schematic of the entire measurement system. In addition to the pressure differential measured by the pressure transducer, air density is also an input to FAN-PITOT.VEE. The software takes 200 samples when started and reports the mean and standard deviation of the pressure differential in units of inH₂O and the velocity of the air in units of ft/s. The velocity of the air is determined using the mean pressure differential as the change in pressure in the pitot tube.
III. Theory

III.1. Uncertainty

III.1.1. Concept of Uncertainty

In EXPERIMENT No. 1 the concept of error and types of error (bias and precision error) were introduced. Error was defined as the difference between the true value of the variable being measured and the measured value of the variable. While this definition may suffice during calibration, when the true value of the input being used to calibrate a device is known, this definition of error is not applicable in determining the error in measurements taken during an experiment, in which case the true value of the variable being measured is unknown. Since true error of an instrument’s measurements cannot be determined when the input being measured is unknown, the error is estimated. The estimated or probable error in measurements taken of unknown inputs is known as uncertainty. When measurements of unknown inputs are taken (which is the case the majority of the time in practice) it is important to report the probable error or uncertainty in the measurements in addition to the values recorded.

III.1.2. Kline-McClintock Method

When a variable of interest is not directly measured, but is rather determined using measurements of other related variables in an analytical equation, the uncertainties in the individual related variables propagate and may lead to a large uncertainty in the variable of interest. For example, say that it is desired to measure the variable \( y \), and that \( y \) cannot be measured directly. Further assume that the variable of interest \( y \) is related to other variables \( x_i, i = 1 \ldots N \) by an analytical function. In other words, assume \( y = f(x_1, x_2, \ldots, x_N) \). The uncertainty in the variable of interest due to the uncertainties in the measurements of the related variables may be estimated using the Kline-McClintock method, the formula for which is present in Equation (5.1), where \( u_y \) is the uncertainty in the variable of interest \( y \), and \( u_{x_i} \) is the uncertainty in the measurement of the individual variable \( x_i \).

\[
 u_y = \sqrt{\sum_{i=1}^{N} \left( \frac{\partial y}{\partial x_i} u_{x_i} \right)^2} \tag{5.1}
\]
To illustrate this concept, assume it is desirable to indirectly measure the variable $z = xy$ by measuring the variables $x$ and $y$. Further assume that the uncertainties in measurements of $x$ and $y$ are known to be equal to $u_x$ and $u_y$, respectively. The uncertainty in the desired variable can then be solved using the general formula given in Equation (5.1). The resulting solution for the uncertainty ($u_z$) is presented as Equation (5.2).

$$u_z = \sqrt{(yu_x)^2 + (xu_y)^2} \quad (5.2)$$

### III.2. Design Stage Uncertainty

Before any measurements are actually taken with a device, the uncertainty of the device can be estimated from the resolution of the device and manufacturer’s specified errors. Uncertainty predicted from manufacturer’s specifications alone, before measurements are taken, is known as design stage uncertainty. Design stage uncertainty does not include uncertainties of bias and precision errors caused by extraneous variables during an experiment. The formula for design stage uncertainty ($u_d$) is given in Equation (5.3), where $\text{res}$ is the resolution of the measurement instrument and $OIE$ is the overall instrument error of the instrument defined in EXPERIMENT No. 1 by Equation (1.10). The value $\text{res}/2$ in Equation (5.3) is often termed resolution error ($\epsilon_{\text{res}}$). If the resolution error is included in the calculation of the overall instrument error then the design stage uncertainty is simply equal to the overall instrument error.

$$u_d = \sqrt{\left(\frac{\text{res}}{2}\right)^2 + OIE^2} \quad (5.3)$$

### III.3. Bias and Precision Uncertainty

Like instrument specification errors explored in EXPERIMENT No. 1, uncertainties can also be divided into bias and precision categories. The uncertainty ($u_x$) in the direct measurement of any variable ($x$) is due to a combination of the bias ($B_x$) and precision uncertainties ($P_x$) of the variable; see Equation (5.4). Typically, bias uncertainty is estimated to be equal to the design stage uncertainty of the instrument used to measure the variable and is reported with a certain percent confidence ($C\%$); see Equation (5.5). Precision uncertainty, however, is estimated using the standard deviation ($s_x$) of repeated measurements and the assumption that the repeated measurements constitute a representative sample of the distribution (assumed to be a normal distribution) of all possible values the repeated measurements could have taken; see Equation (5.6) and (5.7), where $N$ is the number of repeated measurements taken, $x_i$ are individual measurements, and $\bar{x}$ is the average measured value. This definition for the precision uncertainty in a measured variable only has 68% confidence. To obtain a precision uncertainty with a given percent confidence ($C\%$) a coefficient based on the student’s t-distribution must be used; see Equation (5.8), where $t_{N-1,c\%}$ is the student’s t-distribution coefficient for $N-1$ degrees of freedom and $C\%$ confidence. Values for the student’s t-distribution coefficient can be found in reference tables known as T tables commonly available online and in statistical reference textbooks.
The Kline-McClintock method can also be separated in terms of bias and precision uncertainties; see Equation (5.9), where $B_{x_i}$ is the bias uncertainty in the variable $x_i$, $P_{x_i}$ is the precision uncertainty in variable $x_i$, and $v$ is the degrees of freedom of variable $y$. The degrees of freedom ($v$) in Equation (5.9) is given by Equation (5.10), where $N_i$ is the number of measurements taken of variable $x_i$ and $s_{x_i}$ is the standard deviation of variable $x_i$. Using the definitions for bias and precision uncertainty given in Equations (5.5) and (5.6), Equation (5.9) can be expanded into Equation (5.11), where $u_{d,x_i}$ is the design stage uncertainty in the device used to measure the variable $x_i$.

\[
u = \left[ \frac{\sum_{i=1}^{N_i} \left( \frac{\partial y}{\partial x_i} s_{x_i} \right)^4}{\sum_{i=1}^{N_i} \frac{\partial y}{\partial x_i} \sqrt{N_i}} \right]^{1/4} \times \left( \frac{\partial y}{\partial x_i} s_{x_i} \sqrt{N_i} \right)^2 \right]^{2} (5.10)\]

\[
u = \left[ \frac{\sum_{i=1}^{N_i} \left( \frac{\partial y}{\partial x_i} s_{x_i} \right)^4}{\sum_{i=1}^{N_i} \frac{\partial y}{\partial x_i} \sqrt{N_i}} \right]^{1/4} \times \left( \frac{\partial y}{\partial x_i} s_{x_i} \sqrt{N_i} \right)^2 \right]^{2} (5.10)\]

\[
u = \left[ \frac{\sum_{i=1}^{N_i} \left( \frac{\partial y}{\partial x_i} s_{x_i} \right)^4}{\sum_{i=1}^{N_i} \frac{\partial y}{\partial x_i} \sqrt{N_i}} \right]^{1/4} \times \left( \frac{\partial y}{\partial x_i} s_{x_i} \sqrt{N_i} \right)^2 \right]^{2} (5.10)\]

\[
u = \left[ \frac{\sum_{i=1}^{N_i} \left( \frac{\partial y}{\partial x_i} s_{x_i} \right)^4}{\sum_{i=1}^{N_i} \frac{\partial y}{\partial x_i} \sqrt{N_i}} \right]^{1/4} \times \left( \frac{\partial y}{\partial x_i} s_{x_i} \sqrt{N_i} \right)^2 \right]^{2} (5.10)\]
III.4. Uncertainty in Volumetric Flow Rate Measurement

III.4.1 Analytical Equations

From fluid dynamics theory, the volumetric flow rate \( Q \) of air caused by the fan in the fan assembly is given by Equation (5.12), where \( v \) is the velocity of the air flow, \( r \) is the radial distance coordinate from the center of the fan of the mouth of the pitot tube, \( R \) is the radius of the fan, and \( \varphi \) is the counterclockwise angle coordinate of the mouth of the pitot tube. If one assumes that the velocity is not a function of the angle coordinate of the mouth of the pitot tube, then Equation (5.12) reduces to Equation (5.13). Unfortunately an algebraic function for the velocity does not exist as the velocity will need to be measured during the experiment. Therefore, one must iterate numerically using the trapezoidal rule to solve for the volumetric flow rate; see Equation (5.14), where the areas \( A_i \) are illustrated in Fig.5.2 and \( r_i \) corresponds to radial coordinate positions of the mouth of the pitot tube where measurements are taken. The definition for the trapezoidal areas \( A_i \) is given by Equation (5.15).

![Fig.5.2. Illustration of Numerical Integration for Determining Volumetric Flow Rate.](image)

Also from fluid dynamics, Bernoulli’s equation can be applied to positions (1) and (2) of the pitot tube as noted in Fig.5.1. The resulting equation is presented as Equation (5.16), where \( \rho \) is...
pressure, \( \rho \) is the density of the fluid (air), \( g = 9.81 \text{ m/s}^2 \), and \( h \) is the vertical position (height) of the positions on the pitot tube. The pitot tube is designed such that the pressure at position (1) in Fig. 5.1 is equal to the static (ambient) pressure, and the velocity of the air at position (2) is zero (stagnation). Thus, assuming the velocity at position (2) is zero one can simplify Equation (5.16) into Equation (5.17). The difference in height of the two positions (\( \Delta h = h_2 - h_1 \)) and the difference in pressure of the two positions (\( \Delta P = P_2 - P_1 \)) will be measured during the experiment. Assuming the velocity at position (1) is equal to the velocity at the mouth of the pitot tube, one can then solve for the velocity of the air flow; see Equation (5.18).

\[
\frac{P_2}{\rho g} + \frac{v_2^2}{2g} + h_2 = \frac{P_1}{\rho g} + \frac{v_1^2}{2g} + h_1 \quad (5.16)
\]

\[
\frac{\Delta P}{\rho g} + \Delta h = \frac{v_1^2}{2g} \quad (5.17)
\]

\[
v = \sqrt{\frac{2(\Delta P + \rho g \Delta h)}{\rho}} \quad (5.18)
\]

The density of the air is proportional to the ratio of air pressure (\( P \)) to temperature (\( T \)) with the proportionality constant being \( 1/R_{\text{air}} \), where \( R_{\text{air}} = 286.9 \text{ J/kg·K} \); see Equation (5.19). The room temperature can be assumed to be 25 °C and the barometric pressure to be 1.014 kPa. Thus the air density can be assumed to be 1.2217 kg/m\(^3\). This density value should be used as an input to FAN-PITOT.VEE. If desired, the actual room temperature and pressure can be measured during the experiment, though the uncertainties in these values will need to be determined based on the equipment used in order for them to be used in calculating the uncertainty in the volumetric flow rate.

\[
\rho = \frac{P}{R_{\text{air}} T} \quad (5.19)
\]

III.4.2 Uncertainty Analyses

Having derived the necessary analytical equations, the uncertainty in the volumetric flow rate can now be determined. First, the Kline-McClintock method should be applied to the definition of the volumetric flow rate given by Equation (5.14); see Equation (5.20). Given that all of the partial derivatives in Equation (5.20) are equal to one, Equation (5.20) can be simplified to Equation (5.21).

\[
u_Q = \sqrt{\sum_{i=1}^{7} \left( \frac{\partial Q}{\partial A_i} u_{A_i} \right)^2} \quad (5.20)
\]
Next, the uncertainties in the areas $A_i$ need to be determined. To begin, apply the Kline-McClintock method to the definition for the areas $A_i$ given by Equation (5.15); see Equation (5.22). The partial derivatives for Equation (5.22) are provided as Equations (5.23), (5.24), (5.25), and (5.26). To simplify the analyses, assume that the uncertainties in the radial coordinate positions are equal for all radial coordinate positions (that is, assume $u_{r_i} = u_r$ for all $i$). Equation (5.22) can then be reduced to Equation (5.27), where the coefficient $X$ is defined by Equation (5.28). Further assume that the uncertainty in the radial coordinate positions is due solely to the resolution of the ruler used in making the demarcations for the guide needle of the fan assembly. Assuming this is the case and that the ruler used had a resolution of 1 mm (95%), the uncertainty in the radial coordinate positions is given by Equation (5.29), where $u_{d,r}$ is the design stage uncertainty in the ruler used to make the demarcations. Note: the definition in Equation (5.29) assumes no instrument errors in the ruler ($OIE = 0$).

$$u_Q = \sqrt{\sum_{i=1}^{\gamma} u_{A_i}^2} \quad (5.21)$$

$$u_{A_i} = \sqrt{\left(\frac{\partial A_i}{\partial r_i} u_{r_i}\right)^2 + \left(\frac{\partial A_i}{\partial r_{i+1}} u_{r_{i+1}}\right)^2 + \left(\frac{\partial A_i}{\partial v_i} u_{v_i}\right)^2 + \left(\frac{\partial A_i}{\partial v_{i+1}} u_{v_{i+1}}\right)^2} \quad (5.22)$$

$$\frac{\partial A_i}{\partial r_i} = \pi (r_{i+1}v_i - 2r_i v_i - r_{i+1}v_{i+1}) \quad (5.23)$$

$$\frac{\partial A_i}{\partial r_{i+1}} = \pi (r_i v_i - r_i v_{i+1} + 2r_{i+1} v_{i+1}) \quad (5.24)$$

$$\frac{\partial A_i}{\partial v_i} = -r_i (r_i - r_{i+1}) \quad (5.25)$$

$$\frac{\partial A_i}{\partial v_{i+1}} = -r_{i+1} (r_i - r_{i+1}) \quad (5.26)$$

$$X = \pi^2 [(r_{i+1}v_i - 2r_i v_i - r_{i+1}v_{i+1})^2 + (r_i v_i - r_i v_{i+1} + 2r_{i+1} v_{i+1})^2] \quad (5.28)$$

$$u_r = u_{d,r} = \frac{res}{2} = 0.5 \text{ mm (95%)} \quad (5.29)$$
The next step in solving for the uncertainty in the volumetric flow rate is to solve for the uncertainties in the velocity measurements. To begin, apply the Kline-McClintock method to the definition for the velocity measurement given by Equation (5.18); see Equation (5.30). The partial derivatives for Equation (5.30) are provided as Equations (5.31), (5.32), and (5.33). The height difference ($\Delta h$) will be measured during the experiment using a ruler with a resolution of 1 mm (95%). Therefore, applying the same logic as was done for determining the uncertainty in the radial coordinate positions, the uncertainty in the height difference measurement can be shown to be 0.5 mm; see Equation (5.34), where $u_{d,\Delta h}$ is the design stage uncertainty in the ruler used to measure the height difference. Note: the definition in Equation (5.34) assumes no instrument errors in the ruler ($OIE = 0$).

\[
\begin{align*}
\frac{\partial v_i}{\partial \Delta P_i} &= \frac{1}{2\rho(\Delta P_i + \rho g \Delta h)} \\
\frac{\partial v_i}{\partial \Delta h} &= 2g \sqrt{\frac{\rho}{2(\Delta P_i + \rho g \Delta h)}} \\
\frac{\partial v_i}{\partial \rho} &= -\frac{\Delta P_i}{\rho} \sqrt{\frac{1}{2\rho(\Delta P_i + \rho g \Delta h)}} \\
u_{\Delta h} &= u_{d,\Delta h} = \frac{res}{2} = 0.5 \text{ mm (95%)}
\end{align*}
\]

Next, the uncertainty in the density measurement should be determined. This again requires the application of the Kline-McClintock method, this time to the definition of the density given in Equation (5.19); see Equation (5.35). The partial derivatives in Equation (5.35) are provided by Equations (5.36) and (5.37). The uncertainties in the temperature and pressure can be assumed to be due only to the resolution of the thermometer and barometer used to measure the air temperature and pressure. For the sake of illustrating the concepts in this experiment, one may assume that $u_T = 1 \text{ °C (95%)}$ and $u_P = 5 \text{ Pa (95%)}$. If the true temperature and pressure are measured during the experiment, the uncertainties in the measurements due to instrument specification errors and resolution (design stage uncertainty) should be determined using Equation (5.3).

\[
\begin{align*}
\frac{\partial v_i}{\partial \Delta P_i} &= \frac{1}{2\rho(\Delta P_i + \rho g \Delta h)} \\
\frac{\partial v_i}{\partial \Delta h} &= 2g \sqrt{\frac{\rho}{2(\Delta P_i + \rho g \Delta h)}} \\
\frac{\partial v_i}{\partial \rho} &= -\frac{\Delta P_i}{\rho} \sqrt{\frac{1}{2\rho(\Delta P_i + \rho g \Delta h)}} \\
u_{\Delta h} &= u_{d,\Delta h} = \frac{res}{2} = 0.5 \text{ mm (95%)}
\end{align*}
\]

\[
\begin{align*}
\frac{\partial v_i}{\partial \rho} &= \frac{1}{\sqrt{2\rho(\Delta P_i + \rho g \Delta h)}} \\
u_P &= \sqrt{\left(\frac{\partial \rho}{\partial T} u_T\right)^2 + \left(\frac{\partial \rho}{\partial P} u_P\right)^2}
\end{align*}
\]

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Finally, the uncertainty in the pressure differential needs to be determined. According to the specifications for the pressure transducer provided in Table 5.1, the transducer has linearity ($\varepsilon_L$), hysteresis ($\varepsilon_H$), and repeatability ($\varepsilon_R$) errors of 0.5 %FSO, 0.3 %FSO, and 0.5 %FSO, respectively. The full scale output for the transducer is also provided in Table 5.1 and is equal to 0.5 inH2O. All error values have a 95% confidence. These calibration errors cause a bias uncertainty in the pressure differential measurement. In addition to the bias uncertainty, the pressure differential also has precision uncertainty due to extraneous variables. As described previously, FAN-PITOT.VEE takes 200 samples of the pressure differential and reports the mean value and standard deviation of the measurements. Using the equations provided previously, the total uncertainty in the pressure differential can be determined by combining the bias and precision uncertainties; see Equations (5.38), (5.39), and (5.40).

\[
\frac{\partial P}{\partial T} = -\frac{P}{RT^2} \quad (5.36)
\]

\[
\frac{\partial P}{\partial T} = \frac{1}{RT} \quad (5.37)
\]

\[u_{\Delta P_i} = \sqrt{B_{\Delta P}^2 + \left(P_{\Delta P_i}^{95\%}\right)^2} \quad (95\%) \quad (5.38)
\]

\[B_{\Delta P} = u_{d,\Delta P} = 0.1E_{\text{transducer}} = \sqrt{\varepsilon_L^2 + \varepsilon_H^2 + \varepsilon_R^2} \quad (95\%) \quad (5.39)
\]

\[P_{\Delta P_i}^{95\%} = \tau_{199.95\%} \frac{s_{\Delta P_i}}{\sqrt{200-1}} \approx 0.1404s_{\Delta P_i} \quad (95\%) \quad (5.40)
\]

After having derived the above relationships and uncertainty values for directly measured variables, the uncertainty in the volumetric flow rate can be determined by working backwards, as will be done in this experiment. Be sure to WATCH UNITS!

**IV. Pre-lab Questions**

1. Two resistors, $R_1 = 200 \ \Omega$ and $R_2 = 50 \ \Omega$, are connected in a circuit and have uncertainties of $\pm 0.20$ and $\pm 0.50$, respectively. Calculate the uncertainty in the circuit’s equivalent resistance ($R_{eq}$) given the equations provided below if
   a. the resistors are in series
   b. the resistors are in parallel

\[R_{eq,\text{series}} = \sum_{i=1}^{N} R_i \quad \quad R_{eq,\text{parallel}} = \frac{\prod_{i=1}^{N} R_i}{\sum_{i=1}^{N} R_i}
\]

2. The area of a flat, rectangular parcel of land is computed from the measurement of the length of two adjacent sides, $X$ and $Y$. Measurements are made using a scaled chain accurate to within 0.4% (95%). The two sides are measured $N = 8$ times with the
following results. Determine the uncertainty in the area of the land parcel as a percentage with 95% confidence. (Hint: \( t_{7.95\%} = 1.895 \).)

\[
\begin{align*}
X &= 450 \text{ m} & Y &= 450 \text{ m} \\
\sigma_X &= 4.2 \text{ m} & \sigma_Y &= 4.2 \text{ m}
\end{align*}
\]

3. In a yogurt filling line, containers are filled with 1 kg of yogurt. The mass of the yogurt is measured while the filling nozzle is open. The nozzle has a flow rate of 0.5 kg/s. Based on statistical analysis of previous data, factors including viscosity and density of the yogurt introduce an uncertainty of 2% (95%) to the flow rate. The dispensing time is controlled mechanically through a cam system. This system has an uncertainty in the time during which the nozzle is open of 0.1 s based on calibration.
   a. Calculate the filling time of each container.
   b. Categorize the uncertainties in the flow rate and time as precision or bias. Explain.
   c. Calculate the uncertainty in the mass of the filled yogurt containers.
   d. It is desired to reduce the uncertainty in the mass of the yogurt. Should the engineers concentrate on reducing the uncertainty in the flow rate or time? Explain.

V. Procedure

When computing partials and uncertainties during this experiment, be sure always to use standard metric units. Standard metric units are as follows: length (m), force (N), mass (kg), pressure (Pa), density (kg/m\(^3\)), velocity (m/s).

1. Check that the system is connected and set up correctly before beginning. A schematic of the setup is presented in Fig.5.1. Students should ask their lab instructor for assistance if needed.
2. Using the equipment information provided in Table 5.1, convert the errors of the pressure transducer from %FSO to inH\(_2\)O. Record these values in the PRESSURE DIFFERENCE UNCERTAINTY table of the data sheet.
3. Convert the errors determined in step 2 to Pa, using the conversion 1 inH\(_2\)O = 249.089 Pa. Record these converted values in the PRESSURE DIFFERENCE UNCERTAINTY table of the data sheet.
4. Using values for the transducer errors determined in step 3 and Equation (5.39), compute the bias uncertainty \( B_{p_{\Delta p}} \) in the differential pressure measurement. Record this value in the PRESSURE DIFFERENCE UNCERTAINTY table of the data sheet.
5. If the actual room temperature and pressure are to be measured, do this now and determine the density of the air. The uncertainties in the air temperature and pressure measurements should also be determined at this point if not using the given values in this manual.
6. Use Equations (5.36) and (5.37) to solve for the partials \( \frac{\partial p}{\partial T} \) and \( \frac{\partial p}{\partial p} \). Record these values in the DENSITY UNCERTAINTY table of the data sheet.
7. Using the partials determined in step 6, compute the uncertainty in the density measurement using Equation (5.35). Record this value in the DENSITY UNCERTAINTY table of the data sheet.

8. Use a ruler to measure the vertical distance ($\Delta h$) between positions (1) and (2) of the pitot tube as shown in Fig.5.1. Record this value in the HEIGHT & RADIAL UNCERTAINTIES table of the data sheet.

9. Start FAN-PITOT.VEE. Set the sampling frequency (rate) to 100 and the number of samples (count) to 200.

10. Convert the air density (lab manual value is 1.2217 kg/m$^3$) to lbm/ft$^3$ using the conversion 1 kg/m$^3$ = 0.06243 lbm/ft$^3$. Enter this value as the air density in FAN-PITOT.VEE.

11. Turn on the fan assembly. Wait for the fan to achieve a steady rate of spin.

12. The radial position coordinate of the mouth of the pitot tube can be set by using the guide needle. The demarcations (0…7) for the guide needle are numbered in units of 7 mm, with mark 0 being the center of the fan ($r_1 = 0$ mm) and mark 7 being the edge of the fan ($r_7 = 49$ mm). For radial positions ($r_1, i = 1…8$) of $r_1 = 0$ mm, $r_2 = 7$ mm, $r_3 = 14$ mm, $r_4 = 21$ mm, $r_5 = 28$ mm, $r_6 = 35$ mm, $r_7 = 42$ mm, and $r_8 = 49$ mm do the following:

   a. Click START in FAN-PITOT.VEE to take samples of the pressure differential at the current radial position.
   
   b. Note: the pressure differential value ($\Delta P_i$) displayed as the mean sample value (“mean $vp$ (in. H2O)”) in the FAN-PITOT.VEE display. Units will be in inH$_2$O. Record this value in the PRESSURE DIFFERENCE UNCERTAINTY table of the data sheet.
   
   c. Convert the measurement for pressure differential recorded in step 12.b to units of Pa. Record this value in the PRESSURE DIFFERENCE UNCERTAINTY table of the data sheet.
   
   d. Note: the standard deviation of the pressure differential samples ($s_{\Delta P_i}$) displayed (“sdev of $vp$ (in. h2o)”) in the FAN-PITOT.VEE display. Units will be in inH$_2$O. Record this value in the PRESSURE DIFFERENCE UNCERTAINTY table of the data sheet.
   
   e. Convert the standard deviation for the pressure differential recorded in step 12.d to units of Pa. Record this value in the PRESSURE DIFFERENCE UNCERTAINTY table of the data sheet.
   
   f. Using the value for the standard deviation for the pressure differential determined in step 12.e and Equation (5.40), compute the precision uncertainty ($B_{\Delta P_i}^95\%$) in the pressure differential. Record this value in the PRESSURE DIFFERENCE UNCERTAINTY table of the data sheet.
   
   g. Using the values determined for the bias ($B_{\Delta P}$) and precision ($B_{\Delta P_i}^95\%$) uncertainties of the pressure differential and Equation (5.38), compute the uncertainty ($U_{\Delta P_i}$) in the differential pressure and record the value in the PRESSURE DIFFERENCE UNCERTAINTY table of the data sheet.
   
   h. Note the velocity ($v_i$) displayed (“velocity (ft/s)”) in the FAN-PITOT.VEE display. Units will be in ft/s. Record this value in the VELOCITY UNCERTAINTY table of the data sheet.
i. Convert the velocity recorded in step 12.h to units of m/s using the conversion 1 m = 3.28084 ft. Record this value in the VELOCITY UNCERTAINTY table of the data sheet.

j. Use Equations (5.31), (5.32), and (5.33) to solve for the partials $\frac{\partial v_i}{\partial \Delta h}$, $\frac{\partial v_i}{\partial \Delta f}$, and $\frac{\partial v_i}{\partial \rho}$, respectively. Record these values in the VELOCITY UNCERTAINTY table of the data sheet.

k. Using the values for the partials determined in step 12.j and Equation (5.30), compute the uncertainty ($u_{v_i}$) in the velocity measurement. Record this value in the VELOCITY UNCERTAINTY table of the data sheet.

13. Thus far the uncertainties in the velocity and pressure differential measurements have been determined for all radial positions. Using these values, for positions $i = 1 \ldots 7$ compute the partials $\frac{\partial A_i}{\partial v_i}$ and $\frac{\partial A_i}{\partial v_{i+1}}$ using Equations (5.25) and (5.26), respectively. Record these values in the VOLUMETRIC FLOW RATE UNCERTAINTY table of the data sheet.

14. For positions $i = 1 \ldots 7$ compute the coefficient $X$ using Equation (5.28). Record these values in the VOLUMETRIC FLOW RATE UNCERTAINTY table of the data sheet.

15. Using the partials and coefficients determined in steps 13 and 14 and Equation (5.27), compute the uncertainties ($u_{A_i}$) in the areas $A_i$ for positions $i = 1 \ldots 7$. Record these values in the VOLUMETRIC FLOW RATE UNCERTAINTY table of the data sheet.

16. Using Equation (5.15), compute the areas $A_i$ for positions $i = 1 \ldots 7$. Record these values in the VOLUMETRIC FLOW RATE UNCERTAINTY table of the data sheet.

17. Using the values for the areas $A_i$ determined in step 16 and Equation (5.14), compute the volumetric flow rate ($Q$). Record this value in the VOLUMETRIC FLOW RATE UNCERTAINTY table of the data sheet.

18. Using the uncertainties $u_{A_i}$ determined in step 15 and Equation (5.21), compute the uncertainty in the volumetric flow rate ($u_Q$). Record this value in the VOLUMETRIC FLOW RATE UNCERTAINTY table of the data sheet.

19. Convert the value for the uncertainty in the volumetric flow rate determined in step 18 to a percentage of the volumetric flow rate measurement determined in step 17. Record this value in the VOLUMETRIC FLOW RATE UNCERTAINTY table of the data sheet.

20. Turn off all equipment and tidy the lab station.

21. Record appropriate equipment identification information.

VI. Post-lab Questions

1. Reproduce the plot shown in Fig.5.2 using the data recorded in this lab. How does the measured data compare to the generalized plot presented in Fig.5.2?

2. Convert the uncertainties in velocity, and pressure to percentages of the measured values for velocity and pressure differential. What is the largest uncertainty (%) for velocity and pressure differential?

3. How do the uncertainties for the velocity and pressure differential (%) determined in question 2 compare to the uncertainty in the volumetric flow rate (%) determined during the experiment?
4. Based on the results of this experiment, briefly discuss the importance of uncertainty analysis and determining the probable error in measurements that are indirectly derived from directly measured variables.
### DENSITY UNCERTAINTY

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<th>$\Delta P_i$ (Pa)</th>
<th>$s_{\Delta P_i}$ (inH$_2$O)</th>
<th>$s_{\Delta P_i}$ (Pa)</th>
<th>$P_{\Delta P_i}^{95%}$ (Pa)</th>
<th>$u_{\Delta P_i}$ (Pa)</th>
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<tr>
<th>(r_i) (m)</th>
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Experiment No. 5 Data Sheet
EXPERIMENT No. 6: FLOW METERS

I. Objectives

In this experiment, various obstruction and insertion flow meters will be used to measure the volumetric flow rate of water through a specialized flow pump system. The meters which will be explored include Venturi and orifice obstruction meters, a rotameter, a paddlewheel meter, and a vortex shedding meter. The objectives of this experiment are (1) to gain a theoretical understanding of common flow meters and (2) to gain hands on experience in how these flow meters are used in practice.

II. Equipment

This experiment involves measuring the volumetric flow rate of water through a mobile flow pump system with various flow meters. A schematic of the system is presented in Fig.6.1. Refer to Fig.6.1 often when reading the description of the system below to reinforce understanding of the system’s design. Equipment information for the system and the meters it incorporates is presented in Table 6.1.

The main body of the system is a large hollow tank which holds several gallons of tap water. When turned on, a motorized pump housed inside the tank pumps water from the tank through a hose. The hose leads out from the pump and to a 180° loop consisting of two horizontal PVC pipe segments. A vortex shedding meter and a paddlewheel meter are connected to these horizontal pipes, one on each segment. Output frequencies from the paddlewheel and vortex shedding meters are recorded using frequency counters which record pulses and report frequency of pulses averaged over 10-second windows. Note: frequency counters are not shown in Fig.6.1.

From the 180° loop, another hose directs the flow of water to a larger diameter plastic pipe situated horizontally along the front of the tank lid. The hose attaches to the pipe on the left

<table>
<thead>
<tr>
<th>Table 6.1 Equipment Information for Experiment No. 6</th>
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side of the assembly. This large diameter horizontal pipe houses a Venturi and an orifice meter, connected in series. Since the flow meters are in series, the flow rate through each should be the same under steady-state conditions. On the right end of the horizontal section of pipe, an elbow joint is used to divert the pipe’s path to a vertical position. In this vertical section is a rotameter. Demarcations on the outside of this vertical section can be used to take rotameter readings. At the top of the vertical section of the pipe is attached a metal pipe assembly which diverts the flow of water to a downward vertical trajectory bank into the tank. An emergency control valve is a part of the metal pipe assembly and can be used to quickly stop flow through the pump system.

Inflow to the tank flows into a bucket which is attached to a lever extending from the left side of the tank. A fulcrum allows this lever to raise and lower the bucket. There is a valve in the bottom of the bucket which is connected to a rod (valve release bar). When the lever is held up, the bucket is in its lowest position and the rod touches the bottom of the tank pushing the valve open and allowing water to flow out of the bucket. When the lever is down, the valve closes, thereby preventing water from flowing out of the bucket and into the tank. The tank should have enough reserves of water such that this does not cause an interruption in the flow of water through the system. If left in this position, the bucket will eventually overfill and water will pour back into the tank.

Fig.6.1. Flow Pump System. Manometer indicators are noted by positions (1)-(4): (1) = pressure drop across Venturi meter; (2) = pressure drop in dilation from Venturi to orifice meter; (3) = pressure drop across orifice meter; (4) = pressure drop across elbow between orifice meter and rotameter. Note: frequency counters for vortex shedding meter and paddlewheel meter are not shown.
The lever system allows for the measurement of volumetric flow rate. The lever-bucket assembly has a fulcrum which creates a 3-to-1 lever arm. Attached at the end of the lever is a hook, upon which weights of 2 kg can be placed (up to 8 kg). Thus, if the lever arm and bucket balance when a 2 kg weight is added to the hook, the bucket holds 6 kg of water. A small metal bar is attached to the left side of the tank above the lever arm, which can slide forward or backward to interrupt the movement of the lever arm. The small bar is situated such that it can interrupt the lever arm’s movement when the lever-bucket assembly is horizontally level. In conjunction with a stop watch, this setup allows for the measurement of mass flow rate, which can be divided by the density of water to determine the volumetric flow rate. This measurement of volumetric flow rate will be the standard to which measurements from various flow meters used in this experiment will be compared.

A manometer is situated on a vertical board placed on the top of the tank. Attached to the manometer is a bicycle air pump which can be used to increase the air pressure in the manometer. Small tubes originating from either ends of the Venturi and orifice meters are part of the manometer and are placed on the board with a ruler between them such that the pressure drop across each meter can be measured in units of mmH₂O. Note: 1 mmH₂O = 9.80665 Pa.

Fig. 6.2. Common Flow Meters. (a) orifice meter; (b) Venturi meter; (c) vortex shedding meter; (d) rotameter.
III. Theory

III.1. Obstruction Meters

Flow meters which constrict or obstruct the flow of fluid through a pipe via a narrowing of the pipe’s channel are known as obstruction meters. Two of the most commonly used obstruction meters are orifice and Venturi meters.

An orifice meter is simply a shunt placed in series with a pipe which causes an instantaneous change in the diameter of the pipe; see Fig.6.2(a). The inlet diameter \( (d_1) \) for an orifice meter is equal to the diameter of the pipe; the obstruction or orifice diameter \( (d_o) \) is the diameter of the orifice in the shunt used to constrict the flow of fluid in the pipe. The immediate constriction of the pipe’s diameter causes the formation of eddies on either side of the orifice meter shunt. The recirculating eddies create a core flow of fluid which is forced through shunt orifice. After having passed through the orifice, the core flow begins to expand again to the full pipe diameter; however, the core flow expands again over a longer distance than it initial forms. Thus, the pressure in the pipe on the inflow side of the orifice meter is greater than the pressure on the outflow side. This pressure drop can be used to determine the volumetric flow rate of fluid through the pipe.

Venturi meters are similar in design to orifice meters in that they too use a constriction of the pipe’s diameter to create eddies, a core flow, and a pressure drop. However, unlike an orifice meter, a Venturi meter has a more gradual transition from inlet to orifice diameter and the expansion of the pipe’s diameter from the orifice diameter back to the inlet diameter’s size is achieved over a longer distance; see Fig.6.2(b). For this reason, Venturi meters are typically long devices. The more gradual the transition in the diameters of the Venturi meter, the longer the meter will be, the fewer eddies will be created, and the smaller the pressure drop will become. Unlike orifice meters, the inlet diameter for a Venturi meter may not be equal to the diameter of the pipe.

Venturi meters allow for more precise measurement of the pressure drop across an orifice and therefore greater accuracy in determining the volumetric flow rate of fluid in a pipe. Unfortunately, Venturi meters can be expensive to manufacture and the length of the meter may make it difficult to implement. Orifice meters, however, are cheap, easily made, and are easier to implement. One must consider these tradeoffs in choosing which meter is most appropriate for an individual application.

The pressure drop across an obstruction meter \((\Delta P)\) can be used to determine the volumetric flow rate \((Q)\) of fluid through a pipe by using Equation (6.1), where \(A_1\) is the cross-sectional area of the inlet to the obstruction meter, \(A_2\) is the cross-sectional area of the core flow after passing through the orifice of the obstruction meter, \(\rho\) is the density of the fluid, \(g = 9.81\text{m/s}^2\), and \(h_L\) is the head loss across the obstruction \((h_L = 0 \text{ if pipe is horizontal})\).
The inability of a fluid to expand immediately into an area due to fluid particle inertia is known as \textit{vena contracta}. \textit{Vena contracta} is the phenomena responsible of the formation of the recirculating eddies and the core flow, as well as the slow expansion of the core flow after passing through the orifice of an obstruction meter. Due to the slow re-expansion of the core flow, \( A_2 \) is generally unknown. Contraction coefficients \( (C_c) \) which relate \( A_2 \) to the cross-sectional area of the orifice \( (A_0) \) have been documented for several styles of meters under various flow conditions \( (C_c = A_2/A_0) \). Using the contraction coefficient, Equation (6.1) can be rewritten as Equation (6.2).

\[
Q = A_2 \frac{2(\Delta P) - \rho g h_i}{\sqrt{\rho \left[ 1 - \left( \frac{A_2}{A_1} \right)^2 \right]}} \quad \text{(6.1)}
\]

Frictional losses have also been documented for several types of meters under various flow conditions. Often a friction coefficient \( (C_f) \) and the contraction coefficient \( (C_c) \) are combined into a single discharge coefficient \( (C = C_f C_c) \). In this case, Equation (6.2) can be rewritten as Equation (6.3), where \( C_c \) is not used inside the radical.

\[
Q = C A_0 \frac{2(\Delta P) - \rho g h_i}{\sqrt{\rho \left[ 1 - \left( \frac{A_0}{A_1} \right)^2 \right]}} \quad \text{(6.2)}
\]

\[
Q = C A_0 \frac{2(\Delta P) - \rho g h_i}{\sqrt{\rho \left[ 1 - \left( \frac{A_0}{A_1} \right)^2 \right]}} \quad \text{(6.3)}
\]

Some references define a velocity approach factor \( (E) \) and a single flow coefficient \( (K_0) \); see Equations (6.4), (6.5), (6.6), and (6.7), where \( D \) is hydraulic diameter, \( A \) is cross-sectional area, and \( p \) is cross-sectional perimeter.

\[
K_0 = CE \quad \text{(6.4)}
\]

\[
E = \frac{1}{\sqrt{1 - \beta^4}} \quad \text{(6.5)}
\]

\[
\beta = \frac{D_0}{D_1} \quad \text{(6.6)}
\]

\[
D = \frac{A}{4p} \quad \text{(6.7)}
\]
Using the definitions for the velocity approach factor and flow coefficient, and assuming a horizontal pipe, Equation (6.3) can be reduced to Equation (6.8). In this experiment, the square root relationship between pressure drop and volumetric flow rate for both an orifice meter and a Venturi meter will be confirmed. The flow coefficient for each meter will also be estimated.

\[
Q = K_0A_0 \sqrt{\frac{2(\Delta P)}{\rho}} \quad (6.8)
\]

### III.2. Insertion Meters

Insertion meters are meters which involve inserting a device into the flow of a pipe and recording the change in the device’s position caused by the flow in the pipe. There are a wide variety of insertion meters which can be used to measure the volumetric flow rate in a pipe. In this experiment, three insertion meters will be used: (1) a vortex shedding meter, (2) a rotameter, and (3) a paddlewheel meter.

A vortex shedding meter uses a blunt-faced annulus which is inserted and suspended in a pipe such that the blunt end of the annulus faces the flow of fluid in the pipe; see Fig.6.2(c). As the fluid in the pipe flows around the annulus, eddies are generated which cause the annulus to be lifted up/down (forward/backward or left/right). The frequency \( f \) of the annulus’s shifting position is related to the velocity \( (v) \) of the fluid; see Equation (6.9), where \( S_t \) is the Strouhal number (a constant particular to the individual vortex meter used). Assuming steady-state conditions and that the pipe has a circular cross-section with diameter \( d_1 \), one can use Equation (6.10) to find the volumetric flow rate of the fluid. Note: frequency \( (f) \) in Equations (6.9) and (6.10) has units of Hz.

For a vortex meter to function properly, the diameter of the pipe must not be large, as this would require a larger annulus which would be more difficult to move and therefore less likely to move. Additionally, the turbulence of the flow in the pipe needs to be enough to actually cause the annulus to shift its position; a laminar flow may not create eddies strong enough to actually move the annulus.

\[
f = \frac{S_t v}{D} \quad (6.9)
\]

\[
Q = f \left( \frac{\pi d_1^3}{4S_t} \right) \quad (6.10)
\]

Like a vortex shedding meter, a rotameter also uses an annulus inserted in a pipe; however, unlike and vortex shedding meter, the annulus for a rotameter is not suspended at a fixed point in the pipe, but is rather free to move along the length of the pipe. Furthermore, a rotameter is always housed in a vertical section of piping (with upward flow), whereas a vortex meter is usually in a horizontal pipe; see Fig.6.2(d). A rotameter operates on the principle of drag force. The three primary forces acting on the rotameter annulus are weight (downward), buoyancy of the annulus (upward), and drag force (upward). The vertical position of the rotameter annulus is

70
thus dependent on the drag force exerted on the annulus by the upward flow of the fluid. Generally, the cross-section of the annulus is a shape such that the drag force on the annulus is directly proportional to the velocity of the fluid (e.g., circular). Thus, the velocity of the water can be determined based on the vertical position of the rotameter, given the buoyancy and drag coefficient of the rotameter are known. Assuming steady-state conditions, the volumetric flow rate can then be determined by multiplying the velocity (\(v\)) of the fluid by the cross-sectional area (\(A\)) of the pipe; see Equation (6.11).

\[ Q = A v \quad (6.11) \]

The final insertion meter used in this experiment is a paddlewheel meter. Like a traditional paddlewheel, a paddlewheel meter uses a paddlewheel which is rotated by fluid flowing past and over/under it. The frequency of rotation of the paddlewheel is directly proportional to the velocity of the fluid in the pipe. Again, assuming steady-state conditions, the volumetric flow rate of the fluid in the pipe can then be determined using Equation (6.11).

**IV. Pre-lab Questions**

1. Briefly describe the differences between an orifice meter and a Venturi meter. What are the dis/advantages of each?
2. What causes a vortex shedding meter’s annulus to vibrate? Under what conditions can a vortex shedding meter be used? (Hint: consider pipe diameter and turbulence.)
3. What are the primary forces that act on a rotameter annulus? How can the vertical position of a rotameter annulus be used to determine volumetric flow rate?
4. A vortex shedding meter is used to determine the volumetric flow rate in a 10 cm diameter pipe. If the frequency of oscillation of the annulus is 80 Hz, estimate the volumetric flow rate of the fluid. Use \(S_t = 0.12\).
5. A cast Venturi meter is to be used to meter the flow of water through a 10 cm diameter pipe. For a maximum differential pressure of 90 cmH\(_2\)O (8,826 Pa) and a nominal volumetric flow rate of 0.5 m\(^3\)/min, select a suitable throat size (\(d_0\)). Use a safety factor of 2 for the volumetric flow rate. Assume the pipe is horizontal, the density of the water \(\rho \approx 1000 \text{ kg/m}^3\), and the discharge coefficient \(C = 0.984\) for cast Venturi meters.

**V. Procedure**

1. Make sure that the flow pump system’s pump is plugged into a wall outlet. Check that the water level in the tank is adequate. Connect the frequency counters to the paddlewheel and vortex shedding meters. Set the frequency counters to report frequencies every 10 seconds. Students should ask their lab instructor for assistance if needed.
2. Pull/push the main control value to start the flow pump system. Adjust the flow rate of the water through the system using the flow control valve.
3. For rotameter scale readings \((R_i, i = 1...5)\) of \(R_1 = 5, R_2 = 10, R_3 = 15, R_4 = 20,\) and \(R_5 = 25\), do the following:
   a. Use the flow control valve to adjust the flow such that the rotameter is stable at the rotameter scale reading \(R_i\).
b. For the first rotamer scale reading \( R_1 = 5 \), use the air pump to increase the pressure in the manometer such that the water level in the tubes corresponding to the orifice and Venturi meters are about two inches from the bottom of the ruler.

c. Use ruler (3) as indicated in Fig.6.1 to measure the pressure drop across the orifice meter \((\Delta P_o)\) in units of mmH\(_2\)O and record this value in the data sheet.

d. Use ruler (1) as indicated in Fig.6.1 to measure the pressure drop across the Venturi meter \((\Delta P_v)\) in units of mmH\(_2\)O and record this value in the data sheet.

e. Convert the values for the pressure drops across the orifice and Venturi meters determined in steps 3.c and 3.d to units of Pa using the conversion \( 1 \text{ mmH}_2\text{O} = 9.80665 \text{ Pa} \). Record these values in the data sheet.

f. Using the values for the pressure drops across the orifice and Venturi meters determined in step 3.e and \( \rho =1000 \text{ kg/m}^3 \), compute \( \sqrt{2\Delta P_o/\rho} \) and \( \sqrt{2\Delta P_v/\rho} \). Record these values in the data sheet.

g. Record the frequencies of the paddlewheel \((f_p)\) and vortex shedding meter \((f_v)\) in the data sheet.

h. Lift up on the lever arm and hold the lever arm at its highest position for at least 15 seconds to empty the bucket.

i. After emptying the tank, release the lever arm and slide small metal bar into position so that it can interrupt the lever arm’s path. When the lever arm contacts the small metal bar (is horizontal), add weight(s) to the hook and start a stopwatch timer. Weights are available in the form of 2-kg wafers. For \( R_1 = 5 \), \( R_2 = 10 \), and \( R_3 = 15 \), use one wafer. For \( R_4 = 20 \) and \( R_5 = 25 \), use two wafers.

j. When the lever arm rises and contacts the small metal bar again, stop the stopwatch timer. Record the time on the stopwatch \((\Delta t)\) in the data sheet.

k. Multiply the mass of the weights used on the hook by a factor of 3 to determine the mass \((m)\) of water collected in the bucket. Record this value in the data sheet.

l. Divide the mass \((m)\) determined in step 3.k by the time \((\Delta t)\) recorded in step 3.j to determine the mass flow rate. Divide the mass flow rate by the density of the water \((\rho =1000 \text{ kg/m}^3)\) to determine the volumetric flow rate \((Q)\). Record this value in the data sheet.

4. Using Microsoft Excel or a similar program, students should plot \( Q \) vs \( \sqrt{2\Delta P_o/\rho} \). Students should then add a best-fit linear trend line for the data. The trend line should be of the form \( y^t = K_{o,0}x + y_0 \), where \( K_{o,0} \) is the slope of trend line and flow coefficient for the orifice meter, \( x = \sqrt{2\Delta P_o/\rho} \), and \( y_0 \) is the y-axis intercept. Students should ask their lab instructors for assistance in adding the trend line if needed.

5. Record the value for \( K_{o,0} \) determined in step 4 in the data sheet.

6. Using Microsoft Excel or a similar program, students should plot \( Q \) vs \( \sqrt{2\Delta P_v/\rho} \). Students should then add a best-fit linear trend line for the data. The trend line should be of the form \( y^t = K_{v,0}x + y_0 \), where \( K_{v,0} \) is the slope of trend line and flow coefficient for the Venturi meter, \( x = \sqrt{2\Delta P_v/\rho} \), and \( y_0 \) is the y-axis intercept. Students should ask their lab instructors for assistance in adding the trend line if needed.

7. Record the value for \( K_{v,0} \) determined in step 6 in the data sheet.

8. Turn off the flow pump system and the frequency counters. Tidy the lab station.

9. Record appropriate equipment identification information.
VI. Post-lab Questions

1. For the rotameter, plot a curve of volumetric flow rate \( Q \) vs. scale reading \( R_i \). What is the sensitivity of the rotameter (slope of the best-fit linear trend line)? What is the linearity error?

2. For the paddlewheel meter, determine the sensitivity (units of \( \text{m}^3/\text{s-Hz} = \text{m}^3/\text{count} \)). What is the linearity error?

3. Were the plots used to determine the flow coefficients of the orifice and Venturi meters linear? What is the linearity error for each of these plots? Do these plots verify a square-root relationship between pressure drop and volumetric flow rate?
<table>
<thead>
<tr>
<th>run</th>
<th>rotameter</th>
<th>m (kg)</th>
<th>Δt (s)</th>
<th>Q (m³/s)</th>
<th>ΔPₓ (mmH20)</th>
<th>ΔPᵧ (Pa)</th>
<th>( \sqrt{\frac{2\Delta P_x}{\rho}} )</th>
<th>ΔPᵧ (Pa)</th>
<th>( \sqrt{\frac{2\Delta P_y}{\rho}} )</th>
<th>( f_p ) (Hz)</th>
<th>( f_v ) (Hz)</th>
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<td></td>
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<td></td>
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</tbody>
</table>

\[ K_{0,0} = \quad K_{0,v} = \]

Lab Instructor’s signature: ___________________ date: ____/____/____

Experiment No. 6 Data Sheet
I. Objectives

In this experiment, various common temperature measuring devices will be explored. Devices which will be used include a thermocouple, a thermistor, a resistance temperature detector (RTD), an optical comparator pyrometer, and a total radiation pyrometer. The objectives of this experiment are (1) to gain a theoretical understanding for how common temperature measuring devices work and (2) to gain practical hands on experience in using common temperature measuring devices.

II. Equipment

In Part 1 of this experiment, the temperature of a small portable oven will be measured using a thermocouple, a thermistor, and an RTD; see Fig.7.1(a) for a schematic of the measurement system for Part 1. Software called TCAL will be used to analyze and display a voltage reading for the RTD and a degree reading for the thermocouple. The thermocouple probe is connected to a linearizer/amplifier circuit box which itself is connected to the ADC. A switch on the linearizer/amplifier allows for conversion between Celsius and Fahrenheit; set the switch to Fig.7.1. Schematic of Measurement Systems for Experiment No. 7. (a) measurement system for Part 1; (b) optical comparator pyrometer in measurement of light bulb filament’s temperature; (c) total radiation pyrometer in measurement of temperature of heat plate.
output degrees in Fahrenheit. The RTD is utilized in a circuit as shown in Fig. 7.2, in which the voltage drop across the RTD is output to the ADC. The thermistor uses a separate circuit with a multimeter to measure resistance.

In Part 2 of this experiment, the temperature of a tungsten filament in a heat lamp will be estimated using an optical comparator pyrometer; see Fig. 7.1(b). Also in Part 2, the emissivity of a heat plate will be estimated using a total radiation pyrometer; see Fig. 7.1(c). Identifying equipment information is provided in Table 7.1. Note: brackets ([ ]) in Table 7.1 indicate missing information which should be recorded by students.

<table>
<thead>
<tr>
<th>Table 7.1 Equipment Information for Experiment No. 7</th>
</tr>
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<td>Model</td>
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<td>Serial No.</td>
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<tr>
<td><strong>Thermocouple</strong></td>
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<tr>
<td><strong>Optical Comparator Pyrometer</strong></td>
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<td>Serial No.</td>
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<tr>
<td><strong>Total Radiation Pyrometer</strong></td>
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<td>Model</td>
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<tr>
<td>Serial No.</td>
</tr>
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<td><strong>Thermistor</strong></td>
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<td>Model</td>
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<tr>
<td>Serial No.</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>
II.1. Optical Comparator Pyrometer

In Part 2 of this experiment, an optical comparator pyrometer will be used to estimate the temperature of the tungsten filament of a heat lamp; see Fig.7.1(b). The optical pyrometer used consists of an eyepiece and a box-shaped device. A trigger on the eyepiece turns the pyrometer on. When the trigger is squeezed, a horizontal red line will appear in the visual field of the eyepiece. The color of the red line can be adjusted by turning the knob on the side of the box-shaped device. There is a scale on the top right of the box-shaped device which displays corresponding temperature in Celsius. There are three scales on the display: low, high, and extra high. Which scale the device uses is set by turning the scale adjustment knob located on the front of the eyepiece.

To use the device, look through the eyepiece and pull the trigger. Aim the eyepiece such that the filament appears in the eyepiece display. Note: the range of the eyepiece display is very small, so it may be difficult to get the filament to appear in the eyepiece display. Pull the trigger to turn on the pyrometer and display the horizontal red line in the eyepiece display. Use the adjustment knob on the side of the box-shaped device to adjust the color of the horizontal line until it matches the color of the filament. Read the temperature in the appropriate scale of the display on the top of the box-shaped device. If the filament is not visible through the eyepiece or the color of the horizontal line cannot be made to match the color of the filament in the eyepiece display, try changing the setting of the scale.

II.2. Total Radiation Pyrometer

Also in Part 2 of this experiment, the emissivity of a heat plate will be estimated using a total radiation pyrometer; see Fig.7.1(c). The total radiation pyrometer is designed to have a gun-based shape. To turn on the pyrometer, pull the trigger of the gun. A laser pointer will appear which indicates where the pyrometer is aimed. WARNING: THE LASER LIGHT CAN CAUSE BLINDNESS, SO NEVER LOOK AT THE BEAM! REFLECTIONS FROM RINGS, WATCHES, ETC. CAN ALSO CAUSE BLINDNESS, SO REMOVE ALL JEWELRY AND EYEGLASSES BEFORE USE!

The temperature measured by the device is indicated in the device’s display, located above the handle. Also indicated on the display is the emissivity ( ) setting of the device, which is used by the device in determining the temperature of the object. The emissivity of the device can be adjusted using up and down arrows located just below the display.
III. Theory

III.1. Thermocouple

III.1.1. Seebeck Effect

A thermocouple is likely the most commonly used temperature measuring device. A thermocouple consists of two electrical conductors of dissimilar materials with at least one electrical connection; see Fig.7.3. If the temperatures of the junction sites of the conductors are not equal, an electromagnetic field (EMF) is generated which is proportional to the difference in the temperatures. The generation of the EMF is the result of a flow of thermal energy and electricity which occurs via the flow of free electrons in the conductors. For an open circuit as shown in Fig.7.3 with no current flow, the EMF is proportional to the difference in temperatures of the junctions of the conductors with a proportionality constant, which is dependent on the choice of materials of the conductors. This phenomenon is known as the Seebeck Effect.

![Fig.7.3. Thermocouple Arrangement.](image)

III.1.2. Fundamental Laws of Thermocouples

The behavior of thermocouples is governed by three fundamental laws:

The Law of Homogeneous Materials states that a thermoelectric current cannot be maintained in a circuit of homogeneous material by application of heat alone. For thermocouples, this law stipulates that two dissimilar materials are required to generate the EMF via the Seebeck Effect.

The Law of Intermediate Materials states that the algebraic sum of the thermoelectric forces in a circuit is zero if all the circuit is at the same temperature. Thus, one can use extension wires for the thermocouple if they are at the same temperature and not induce a change in the EMF. This fact is commonly used in practice because extension wires can greatly increase convenience of use in certain situations.

The Law of Successive (Intermediate) Temperatures states that for two dissimilar conductors, the EMF produced for a temperature differential \( \delta T \) is equal to the sum of the EMFs produced for temperature differentials \( \delta T_1 \) and \( \delta T_2 \), where \( \delta T < \delta T_1 < \delta T_2 \).
III.2. Thermistor

A thermistor is an electrical resistance temperature measuring device which generally consists of a ceramic-like semiconductor often encased in glass. Thermistors exhibit large resistance changes with temperature and their behavior is governed by an exponential function relating resistance and temperature; see Equation (7.1) where $R_0$ is the resistance of the thermistor at reference temperature $T_0$, $R$ is the resistance of the thermistor at the current temperature $T$, and $\beta$ is a characteristic coefficient of the thermistor. The characteristic coefficient ($\beta$) is dependent on the material, construction of the thermistor, and is a function of temperature; however, it is usually modeled as being constant over a specific range of temperature. The characteristic coefficient differs for every thermistor (even those of the same make and model) and thus each thermistor must be calibrated individually. In Part 1 of this experiment, data will be collected in order to determine the characteristic coefficient of the thermistor.

\[
\frac{R}{R_0} = e^{\beta \left(\frac{T}{T_0} - 1\right)} \quad (7.1)
\]

III.3. RTD

Like a thermistor, an RTD is an electrical resistance temperature measuring device which utilizes the fact that the electrical resistance of conductors/semiconductors varies which changes in temperature. Conductors in an electrical resistance temperature measuring device must be isolated from potentially corrosive environments which might alter the resistivity of the conductor. The conductor (wire) must also be mounted such that expansion/contraction due to temperature changes does not induce mechanical strain. For an RTD, over a specific range of temperatures, the relationship between a resistance ratio and change in temperature is approximately linear; see Equation (7.2), where $R_0$ is the resistance of the RTD at reference temperature $T_0$, $R$ is the resistance of the RTD at the current temperature $T$, $\alpha$ is the temperature coefficient of resistivity of the conductor, and $r$ is the resistance of lead wires. Specifically, the RTD in this experiment is part of the circuit shown in Fig.7.2.

\[
\frac{R + r}{R_0 + r} = 1 + \alpha(T - T_0) \quad (7.2)
\]

RTDs are commonly used in circuits known as Wheatstone Bridges; see Fig.7.4. A Wheatstone bridge is said to be balanced if its output voltage is zero. When balanced, the resistances of the bridge hold to a specific relationship given by Equation (7.3), where the resistor indicated by $R_4$ is located where the RTD is in Fig.7.4.

\[
\frac{R_1}{R_2} = \frac{R_3}{R_4} \quad (7.3)
\]

III.4. Radiation Pyrometers

Radiation pyrometers are temperature measuring devices which measure the (heat) radiation emitted by an object in order to estimate the temperature of the object. There are two types of
radiation pyrometers: (1) those which use the wavelength of radiation (color type or comparator type) and (2) those which measure the total radiation emitted by an object. Radiation pyrometers are useful in measuring the temperature of objects which are: (1) too hot to be measured using a device which requires direct contact with the object, (2) radioactive, or (3) cannot be reached and therefore cannot be measured using a device which requires direct contact with the object.

IV. Pre-lab Questions

1. A J-type thermocouple referenced to 0˚C develops an output EMF of 1.2 mV. What is the temperature sensed by the thermocouple? Use Table 7.2.

2. A J-type thermocouple with an ice-reference junction showed a reading of 5.052 mV. After the experiment, it was found that the actual temperature of the reference junction is 15˚C. Determine the actual value of the measured temperature using the Table 7.2. (Hint: use the Law of Successive (Intermediate) Temperatures.)

3. A thermistor is placed in a 100˚C environment and its resistance measures as 10kΩ. The thermistor’s characteristic coefficient is 4210K. If the thermistor is used to measure a particular temperature and its resistance is measured as 500Ω, determine the temperature being measured.

<table>
<thead>
<tr>
<th>Temperature (˚C)</th>
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<th>±1</th>
<th>±2</th>
<th>±3</th>
<th>±4</th>
<th>±5</th>
<th>±6</th>
<th>±7</th>
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<tr>
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<td>0.000</td>
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<td>10</td>
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<td>1.745</td>
<td>1.797</td>
<td>1.849</td>
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<td>5.595</td>
<td>5.650</td>
<td>5.705</td>
<td>5.759</td>
</tr>
</tbody>
</table>

Fig. 7.4. Circuit Diagram for Pre-lab Question 4.
4. A nominally 100-Ω RTD (0˚C) with \( \alpha = 0.004213/˚C \) is inserted into the circuit shown in Fig.7.4. \( R_3 \) is adjusted to give a zero-voltage output at 0˚C. The supply voltage is 3 V. What will be the output voltage at 250˚C? Assume \( R_1 = R_2 \). (Hint: use Kirchhoff’s Voltage Law and Voltage Divider Rule to create a system of equations which can be used in conjunction with Equation (7.2) to solve for output voltage.)

V. Procedure

Before beginning the experiment, obtain a glass of ice water and set it aside. It will be used later in Part 1 of the experiment.

Note: Part 2 may be conducted while waiting for oven to adjust between temperature settings in Part 1.

V.1. Part 1: Common Temperature Measuring Devices

1. Check that the system is connected and set up correctly before beginning the experiment. A schematic of the setup is presented in Fig.7.1. Students should ask their lab instructor for assistance if needed.
2. Start TCAL. Make sure that system is operating correctly and readings for the thermocouple and RTD are being displayed.
3. Place the thermistor probe into the glass of ice water. Note the resistance reading on the multimeter display and record this value in the OVEN TEMPERATURE MEASUREMENTS table of the data sheet.
4. For temperatures (\( T_i, i = 1\ldots5 \)) of \( T_1 = 100 \text{ °F}, T_2 = 137.5 \text{ °F}, T_3 = 175 \text{ °F}, T_4 = 212.5 \text{ °F}, \) and \( T_5 = 250 \text{ °F} \), do the following:
   a. Set the oven temperature to the current temperature \( T_i \)
   b. Wait for the oven to reach the desired temperature. Note: oven will overshoot and then slowly cool back down before finally reaching the desired temperature.
   c. When desired temperature is reached, note the thermocouple (TC), RTD (RTD), and thermistor (R) readings and record these values in the OVEN TEMPERATURE MEASUREMENTS table of the data sheet.
5. Turn off the oven and multimeter. Close TCAL and tidy lab station.

V.2. Part 2: Radiation Pyrometers

1. Turn on the heat lamp and give the lamp 10 minutes to warm up.
2. Using the comparator pyrometer, take readings for the temperature of the heat lamp filament using the high (\( T_{High} \)) and low (\( T_{Low} \)) settings. Record these values in the PYROMETER MEASUREMENTS table of the data sheet.
3. Average the values obtained in step 2 to obtain an estimate of the filament’s temperature (\( \bar{T} \)). Record this value in the PYROMETER MEASUREMENTS table of the data sheet.
4. Turn on the hot plate and set the temperature setting to 3. Wait 10 minutes for the hot plate to warm up.
5. Note the hot plate’s temperature as measured by the thermocouple. Record this value in the PYROMETER MEASUREMENTS table of the data sheet.
6. Standing approximately 3 feet from the hot plate, use the total radiation pyrometer to estimate the emissivity of the hot plate by adjusting the emissivity setting until the temperature displayed by the pyrometer matches the temperature obtained in step 5. Record the emissivity ($\varepsilon$) of the hot plate in the PYROMETER MEASUREMENTS table of the data sheet.

7. Turn off the heat lamp, hot plate, and multimeter used by thermocouple. Wait 10 minutes for devices to cool. Tidy the lab station.

8. Record appropriate equipment identification information.

VI. Post-lab Questions

1. Determine the static sensitivity of the RTD by plotting voltage vs. temperature. Determine the linearity error in % FSO.
2. For the thermocouple, plot a required instrument correction vs. measured temperature curve. Specifically, plot actual-measured temperature vs. measured temperature. Comment on any observable trends in this correction curve.
3. Through a linearization of Equation (7.1), determine the characteristic coefficient of the thermistor using the resistance of the ice bath as the reference resistance. (Hint: be sure to convert temperature units to Kelvin.)
# OVEN TEMPERATURE MEASUREMENTS

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<th>$T_i$ (°F)</th>
<th>$T_i$ (°C)</th>
<th>$T_C$ (°C)</th>
<th>$RTD$ (mV)</th>
<th>$R$ (kΩ)</th>
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# PYROMETER MEASUREMENTS

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<thead>
<tr>
<th>$T_{High}$ (°C)</th>
<th>$T_{Low}$ (°C)</th>
<th>Plate temp (°F)</th>
<th>$\epsilon$</th>
<th>$T'$ (°C)</th>
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I. Objectives

In this experiment, strain gauges arranged in a Wheatstone bridge circuit will be used to measure the strain in a cantilevered beam undergoing bending. The objectives of this experiment are (1) to gain a theoretical understanding of how strain gauges can be used to measure strain in a cantilevered beam, (2) to learn how gauges can be arranged in a Wheatstone bridge to compensate for temperature strain, and (3) to gain practical hands-on experience using strain gauges and a strain indicator.

II. Equipment

In this experiment, the strain in a cantilevered beam will be measured using strain gauges arranged in a Wheatstone bridge circuit. The cantilevered beam is part of a cantilevered beam assembly designed specifically for this lab; see Fig.8.1(a). The assembly is comprised of a flat, cantilevered aluminum beam, a mounting bracket which holds the beam in place, a micrometer which can be used to induce deflection (and therefore strain) in the beam, and four strain gauges arranged in a Wheatstone bridge circuit. The four strain gauges are mounted near the fixed end of the beam, two on the top and two on the back. On each side, one is oriented transversely and one is oriented longitudinally. The strain gauges are very small, so the arrangement of the gauges may not be obvious without the aid of a magnifying glass. Connecting wires from the strain gauges connect to plugs on either side of the mounting bracket. The plugs allow for connections to be made in order to create circuits with the strain gauges in series or parallel. The plugs are labeled using A, B, and numerals. From the various plugs, the circuit wires are gathered into a connection wire assembly cord. The leads at the end of the connection wire assembly will be attached to a strain indicator in order to create quarter and full bridge circuits in measuring strain.

Most of the cantilevered beam assemblies are vertically oriented, as shown in Fig.8.1(a); however, a few are horizontally oriented (not shown in Fig.8.1(a)). In addition to a difference in orientation, these assemblies also have a slightly different wiring arrangement and therefore require different connections to be made for circuits to match those of the vertical beams. Necessary connections for quarter (single gauge) and full bridge circuits are provided in Table 8.2.

Fig.8.1(b). The plugs P, S, and D are used to connect strain gauges into a Wheatstone bridge configuration. The bridge switch is used to select which type of bridge circuit is being read by the strain indicator. The bridge switch should be set to HALF/QUARTER for half or quarter bridges and FULL for full bridge arrangements. The gage factor dial can be used to set the gauge factor for the strain gauges. The use of the indicator needle, sensitivity, zero adjustment, sign change, strain display, and balance wheel will be discussed in the Procedure section as they are used. Identifying equipment information for the cantilevered beam assemblies and the strain indicators are provided in Table 8.1. Note: missing information enclosed in brackets ([ ] ) should be recorded by students.
Table 8.1 Equipment Information for Experiment No. 8

<table>
<thead>
<tr>
<th>Strain Indicator</th>
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<tbody>
<tr>
<td>Make: Budd Co. Instrument Division</td>
</tr>
<tr>
<td>Model: P-350</td>
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<tr>
<td>Serial No.: [bottom left corner on dial face]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cantilevered Beam Assembly</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lab station [station #]: MABE345 Lab</td>
</tr>
<tr>
<td>Rm. 610 Dougherty Engr. Bldg.</td>
</tr>
<tr>
<td>University of Tennessee, Knoxville</td>
</tr>
</tbody>
</table>

Table 8.2 Connections for Forming Wheatstone Bridge Circuits

<table>
<thead>
<tr>
<th>Quarter Bridge (Single Gauge)</th>
<th>Full Bridge (Four Gauges)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical Beams</td>
<td>Horizontal Beams</td>
</tr>
<tr>
<td>G - P1</td>
<td>G - P1</td>
</tr>
<tr>
<td>W - S1</td>
<td>Bk - S1</td>
</tr>
<tr>
<td>S1 - D120</td>
<td>S1 - D120</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Bk = black, O = orange, G/B1 - P1, Bk/Y - P1
Bl = blue, R = red, O/R - P2, R/Br - P2
Br = brown, Y = yellow, Y/Bk - S1, W/Bl - S1
G = green, W = white, W/Br - S2, G/O - S2

Fig 8.1. Device Schematics for Experiment No. 8. (a) arrangement of strain gauges; (b) schematic of cantilevered beam assembly; (c) schematic of strain indicator dial face.
III. Theory

III.1. Theoretical Strain for Beam

From beam theory, the theoretical longitudinal strain ($\varepsilon_{th}$) in the beam at the point where the strain gauges are located due to deflection of the beam by the micrometer can be theoretically determined using Equation (8.1), where $\nu$ is the deflection of the beam at the point where the micrometer touches the beam, $L$ is the distance between the fixed end of the beam and the point where the micrometer deflects the beam, $x$ is the distance between the fixed end of the beam and the point where the strain gauges are located, and $t$ is the thickness of the beam. For the horizontal beams, effects due to gravity can be neglected.

$$\varepsilon_{th} = \frac{3\nu(L-x)t}{2L^3} \quad (8.1)$$

III.2. Strain Gauges

Strain gauges are variable resistors which utilize the fact that axial strain in an electrically conducting material (wire) induces a change in the resistance of the material. Specifically, the change in resistance ($\Delta R$) is proportional to the axial strain ($\varepsilon$) in the material, where the proportionality constant is known as the gauge factor ($GF$); see Equations (8.2) and (8.3), where $R$ is the resistance of the material without strain (nominal strain), $\nu$ is the Poisson ratio of the material, $\pi_1$ is the piezoresistance coefficient of the material, and $E$ is the material’s Young’s Modulus.

$$\Delta R = GF(R\varepsilon) \quad (8.2)$$

$$GF = 1 + 2\nu + \pi_1E \quad (8.3)$$

III.3. Wheatstone Bridge

Wheatstone bridge circuits are commonly used in strain gauges because they allow for accurate measurements of small changes in resistances of gauges, they are easy to implement, and they allow for compensation for temperature-induced strain and an increased resolution. A circuit diagram for a Wheatstone bridge circuit is provided in Fig.8.2.

A full bridge arrangement means that all of the resistors in the bridge are strain gauges. A half bridge arrangement means that two of the four resistors in the bridge are strain gauges and a quarter bridge arrangement means that only one of the resistors in the bridge is a strain gauge.

A Wheatstone bridge is said to be balanced when the output voltage ($V_{out}$) is equal to zero. This then implies that a Wheatstone bridge is balance when the ratio relationship between the resistors in the circuit presented in Equation (8.4) is true.
A Wheatstone bridge is said to be unbalanced when the output voltage does not equal zero. Assuming that all of the resistances in the bridge are strain gauges (full bridge arrangement), then the change in output voltage is given by Equation (8.5), where \( \Delta V \) is the change in output voltage (relative to the balanced output of zero), \( V_s \) is the supply voltage, \( R_i \) are the nominal resistances of the strain gauges, \( g_i \) are the gauge factors of the strain gauges, and \( \varepsilon_i \) are the strains experienced by the individual gauges. If the strain gauges are identical (same gauge factors and same nominal resistances), then Equation (8.5) can be reduced to Equation (8.6).

The difference in output from a multiple-gauge (e.g., full bridge) versus a single-gauge Wheatstone bridge is generally quantified in terms of the bridge constant \( K \) of the circuit; see Equation (8.7). Note: the definition in Equation (8.7) assumes identical gauges. The bridge constant will be experimentally determined for the Wheatstone bridge circuit in this experiment. For the given the arrangement of the strain gauges, the theoretical bridge constant \( K \) is given by Equation (8.8), where \( \varepsilon_b \) is the longitudinal (axial) strain in the beam induced by bending, \( \varepsilon_t \) is the axial strain induced by temperature (thermal expansion of material), and \( \nu \) is Poisson’s ratio for the material (\( \nu = 0.33 \) for aluminum). Note: temperature induced strain \( \varepsilon_t \) is positive on top and bottom of beam while bending induced longitudinal strain \( \varepsilon_b \) is positive (tension) on top of beam and negative (compression) on bottom of beam. One may notice in Equation (8.8) that strain due to thermal expansion is negated in measuring longitudinal bending strain using a full bridge arrangement but temperature strain is not compensated for when using a single gauge (quarter bridge arrangement). If there is no temperature strain, then the theoretical bridge constant \( K = 2.66 \).
\[ \kappa = \left( \frac{\Delta V_{out}}{E_{in}} \right)_{full} = \frac{\varepsilon_1 - \varepsilon_2 - \varepsilon_3 + \varepsilon_4}{\varepsilon_1} \quad (8.7) \]

\[ \kappa_{th} = \frac{(\varepsilon_a + \varepsilon_r) - (-\varepsilon_a + \varepsilon_r) - \nu(-\varepsilon_a + \varepsilon_r) + \nu(\varepsilon_a + \varepsilon_r)}{\varepsilon_a + \varepsilon_r} = \frac{2(1 + \nu)\varepsilon_a}{\varepsilon_a + \varepsilon_r} \quad (8.8) \]

IV. Pre-lab Questions

1. A strain gauge (\(\varepsilon_1\)) has a nominal resistance of 150 \(\Omega\) and a gauge factor of 3.2. If it is used in a quarter bridge with the other resistors being 150 \(\Omega\) and the supply voltage being 3 V, what is the strain if the change in output voltage is 13 mV?
2. For question 1, the systematic uncertainties in the variables are given below. Estimate the systematic uncertainty in the strain measurement. (Hint: \(\Delta V_{out} = \Delta V_{out,final} - \Delta V_{out,bal}\))

   - Strain gauge nominal resistance: 0.12%
   - Strain gauge factor: 0.4%
   - Bridge resistors: 0.1%
   - Supply voltage: 0.3%
   - Output voltage: 0.5%

V. Procedure

V.1. Quarter Bridge

1. Use a ruler to measure the distance from the clamped end of the beam to the point where the micrometer contacts the beam (\(L\)) and the distance from the clamped end of the beam to the point where the strain gauges are located (\(x\)). Using a set of calipers, take four measurements of the thickness of the beam \((t_1, t_2, t_3, t_4)\) roughly equidistant along its length and compute the average thickness \(\bar{t}\). Record these measurements in the BEAM DIMENSIONS table of the datasheet. Note: in calculations involving the thickness of the beam use \(t = \bar{t}\).
2. Form the connections listed in Table 8.2 in order to create a quarter bridge (single gauge) Wheatstone bridge circuit. Flip the bridge switch to HALF/QUARTER BRIDGE. Turn the power on and adjust the sensitivity to maximum. If the indicator needle is pegged to either side, turn the balance knob in the direction opposite the indicator needle. If the indicator needle does not reach zero, change the sign and try again. The indicated strain in the strain display will not be zero for zero strain; however, a whole number can be set for zero strain by using the zero adjustment. Lock the zero adjustment after use and record the zero strain value (\(\varepsilon_{1,0}\)) in the STRAIN MEASUREMENTS table of the data sheet.
3. Turn the micrometer such that it just touches the beam without causing deflection (change in strain).
4. For deflections ($v_i, i = 1...6$) of $v_1 = 0.050$ in., $v_2 = 0.100$ in., $v_3 = 0.150$ in., $v_4 = 0.200$ in., $v_5 = 0.250$ in., and $v_6 = 0.300$ in., do the following:
   a. Adjust the balance such that the indicator needle reads zero. Record the indicated strain value ($\varepsilon_{1,0}$) in the STRAIN MEASUREMENTS table of the data sheet.
   b. Subtract the zero strain value from the strain value obtain in step 4.a to obtain the measured strain ($\varepsilon_{1,m} = \varepsilon_1 - \varepsilon_{1,0}$). Record this value in the STRAIN MEASUREMENTS table of the data sheet.
   c. For $v_6 = 0.300$ in., hold a heat lamp near the point on the beam where the strain gauges are located. Wait two minutes and then adjust the balance (if necessary) to zero the indicator needle. Record the indicated strain value and the measured strain value in the $\Delta\varepsilon$ row of the STRAIN MEASUREMENTS table of the data sheet.
   d. Using Equation (8.1) and the beam dimensions measured in step 1, determine the theoretical strain ($\varepsilon_{th}$) and record this value in the STRAIN MEASUREMENTS table of the data sheet.

V.2. Full Bridge

1. Form the connections listed in Table 8.2 in order to create a full bridge Wheatstone bridge circuit. Flip the bridge switch to FULL BRIDGE. Keep the sensitivity at maximum. Retract the micrometer for zero deflection. Use the balance and zero adjustment as before to determine the zero strain for the full bridge arrangement ($\varepsilon_{F,0}$). Record this value in the STRAIN MEASUREMENTS table of the data sheet.
2. Turn the micrometer such that it just touches the beam without causing deflection (change in strain).
3. For deflections ($v_i, i = 1...6$) of $v_1 = 0.050$ in., $v_2 = 0.100$ in., $v_3 = 0.150$ in., $v_4 = 0.200$ in., $v_5 = 0.250$ in., and $v_6 = 0.300$ in., do the following:
   a. Adjust the balance such that the indicator needle reads zero. Record the indicated strain value ($\varepsilon_F$) in the STRAIN MEASUREMENTS table of the data sheet.
   b. Subtract the zero strain value from the strain value obtain in step 3.a to obtain the measured strain ($\varepsilon_{F,m} = \varepsilon_F - \varepsilon_{F,0}$). Record this value in the STRAIN MEASUREMENTS table of the data sheet.
   c. For $v_6 = 0.300$ in., hold a heat lamp near the point on the beam where the strain gauges are located. Wait two minutes and then adjust the balance (if necessary) to zero the indicator needle. Record the indicated strain value and the measured strain value in the $\Delta\varepsilon$ row of the STRAIN MEASUREMENTS table of the data sheet.
4. Turn off strain indicator. Disconnect strain gauge from cantilevered beam assembly. Tidy lab station.
5. For deflections ($v_i, i = 1...6$) of $v_1 = 0.050$ in., $v_2 = 0.100$ in., $v_3 = 0.150$ in., $v_4 = 0.200$ in., $v_5 = 0.250$ in., and $v_6 = 0.300$ in., determine the bridge constant ($\kappa$) by dividing the measured strain from the full bridge by the measured strained from the single gauge. Record these values in the STRAIN MEASUREMENTS table of the data sheet.
6. Average the bridge constant values determined in step 5 and record the average value in the STRAIN MEASUREMENTS table of the data sheet.
7. For deflections ($v_i, i = 1...6$) of $v_1 = 0.050$ in., $v_2 = 0.100$ in., $v_3 = 0.150$ in., $v_4 = 0.200$ in., $v_5 = 0.250$ in., and $v_6 = 0.300$ in., determine the percent difference (% diff $\kappa$) between the observed bridge constant and the theoretical bridge constant. Record these values in the STRAIN MEASUREMENTS table of the data sheet.

8. From Equation (8.8), the theoretical value of the temperature induced strain can be computed as $\varepsilon_{r,th} = \frac{2(1 + v) - \kappa_{th}}{\kappa_{th}} \varepsilon_{th}/\kappa_{th}$, where $\varepsilon_{th}$ is the theoretical strain at a deflection of 0.300 in. Record this value in the STRAIN MEASUREMENTS table of the data sheet.

VI. Post-lab Questions

1. Plot a curve of measured strain vs. deflection. On the same axes, plot the theoretical strain values. Compare the two plots.

2. Estimate Poisson’s ratio for the beam from Equation (8.8) using measured rather than theoretical values. Specifically use the average bridge constant value and the values for measured single gauge strain at 0.300 in. deflection. How does this measured Poisson’s ratio compare to the publish value of 0.33?
BEAM DIMENSIONS

| L | x | t₁ | t₂ | t₃ | t₄ | t |

STRAIN MEASUREMENTS

<table>
<thead>
<tr>
<th>$\varepsilon_{1,0}$</th>
<th>$\varepsilon_{F,0}$</th>
<th>$\kappa_{th}$</th>
<th>2.66</th>
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<tbody>
<tr>
<td>$v_i$</td>
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<tr>
<td>$\Delta T$</td>
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</tr>
</tbody>
</table>

$\bar{\kappa}$

$\varepsilon_{\tau,th}$

Lab Instructor’s signature: ______________________ date: ____/____/____