7.6 (a) For the FCC crystal structure, the planar density for the (110) plane is given in Equation 3.11 as

\[
PD_{110}^{(FCC)} = \frac{1}{4R^2 \sqrt{2}} = \frac{0.177}{R^2}
\]

Furthermore, the planar densities of the (100) and (111) planes are calculated in Homework Problem 3.53, which are as follows:

\[
PD_{100}^{(FCC)} = \frac{1}{4R^2} = \frac{0.25}{R^2}
\]

\[
PD_{111}^{(FCC)} = \frac{1}{2R^2 \sqrt{3}} = \frac{0.29}{R^2}
\]

(b) For the BCC crystal structure, the planar densities of the (100) and (110) planes were determined in Homework Problem 3.54, which are as follows:

\[
PD_{100}^{(BCC)} = \frac{3}{16R^2} = \frac{0.19}{R^2}
\]

\[
PD_{110}^{(BCC)} = \frac{3}{8R^2 \sqrt{2}} = \frac{0.27}{R^2}
\]

Below is a BCC unit cell, within which is shown a (111) plane.

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The centers of the three corner atoms, denoted by A, B, and C lie on this plane. Furthermore, the (111) plane does not pass through the center of atom D, which is located at the unit cell center. The atomic packing of this plane is presented in the following figure; the corresponding atom positions from the Figure (a) are also noted.

Inasmuch as this plane does not pass through the center of atom D, it is not included in the atom count. One sixth of each of the three atoms labeled A, B, and C is associated with this plane, which gives an equivalence of one-half atom.

In Figure (b) the triangle with A, B, and C at its corners is an equilateral triangle. And, from Figure (b), the area of this triangle is \( \frac{xy}{2} \). The triangle edge length, \( x \), is equal to the length of a face diagonal, as indicated in Figure (a). And its length is related to the unit cell edge length, \( a \), as

\[
x^2 = a^2 + a^2 = 2a^2
\]

or

\[
x = a\sqrt{2}
\]

For BCC, \( a = \frac{4R}{\sqrt{3}} \) (Equation 3.3), and, therefore,

\[
x = \frac{4R\sqrt{2}}{\sqrt{3}}
\]

Also, from Figure (b), with respect to the length \( y \) we may write

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7.23 (a) Perhaps the easiest way to solve for $\sigma_0$ and $k_y$ in Equation 7.7 is to pick two values each of $\sigma_y$ and $d^{1/2}$ from Figure 7.15, and then solve two simultaneous equations, which may be set up. For example

<table>
<thead>
<tr>
<th>$d^{1/2}$ (mm) $-1/2$</th>
<th>$\sigma_y$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>75</td>
</tr>
<tr>
<td>12</td>
<td>175</td>
</tr>
</tbody>
</table>

The two equations are thus

$$75 = \sigma_0 + 4k_y$$
$$175 = \sigma_0 + 12k_y$$

Solution of these equations yield the values of

$$k_y = 12.5 \text{ MPa (mm)}^{1/2} \left[1810 \text{ psi (mm)}^{1/2}\right]$$

$$\sigma_0 = 25 \text{ MPa (3630 psi)}$$

(b) When $d = 2.0 \times 10^{-3}$ mm, $d^{1/2} = 22.4$ mm$^{-1/2}$, and, using Equation 7.7,

$$\sigma_y = \sigma_0 + k_y d^{1/2}$$

$$= (25 \text{ MPa}) + \left[12.5 \text{ MPa (mm)}^{1/2} \left(22.4 \text{ mm}^{-1/2}\right)\right] = 305 \text{ MPa (44,200 psi)}$$
7.13 We are asked to compute the critical resolved shear stress for Zn. As stipulated in the problem, $\phi = 65^\circ$, while possible values for $\lambda$ are $30^\circ$, $48^\circ$, and $78^\circ$.

(a) Slip will occur along that direction for which $(\cos \phi \cos \lambda)$ is a maximum, or, in this case, for the largest $\cos \lambda$. Cosines for the possible $\lambda$ values are given below.

- $\cos(30^\circ) = 0.87$
- $\cos(48^\circ) = 0.67$
- $\cos(78^\circ) = 0.21$

Thus, the slip direction is at an angle of $30^\circ$ with the tensile axis.

(b) From Equation 7.4, the critical resolved shear stress is just

$$\tau_{crss} = \sigma_y (\cos \phi \cos \lambda)_{\text{max}}$$

$$= (2.5 \text{ MPa}) \left[\cos(65^\circ) \cos(30^\circ)\right] = 0.90 \text{ MPa} \ (130 \text{ psi})$$
7.7 Below is shown the atomic packing for a BCC \{110\}-type plane. The arrows indicate two different \langle111\rangle type directions.