5.7 We are asked to determine the position at which the nitrogen concentration is 0.5 kg/m³. This problem is solved by using Equation 5.3 in the form

\[ J = -D \frac{C_A - C_B}{x_A - x_B} \]

If we take \( C_A \) to be the point at which the concentration of nitrogen is 2 kg/m³, then it becomes necessary to solve for \( x_B \), as

\[ x_B = x_A + D \left[ \frac{C_A - C_B}{J} \right] \]

Assume \( x_A \) is zero at the surface, in which case

\[ x_B = 0 + (1.2 \times 10^{-10} \text{ m}^2/\text{s}) \left[ \frac{2 \text{ kg/m}^3 - 0.5 \text{ kg/m}^3}{1.0 \times 10^{-7} \text{ kg/m}^2 \cdot \text{s}} \right] \]

\[ = 1.8 \times 10^{-3} \text{ m} = 1.8 \text{ mm} \]
5.23 This problem asks us to determine the values of $Q_d$ and $D_0$ for the diffusion of Au in Ag from the plot of log $D$ versus $1/T$. According to Equation 5.9b the slope of this plot is equal to $-\frac{Q_d}{2.3R}$ (rather than $-\frac{Q_d}{R}$ since we are using log $D$ rather than ln $D$) and the intercept at $1/T = 0$ gives the value of log $D_0$. The slope is equal to

$$\text{slope} = \frac{\Delta (\log D)}{\Delta \left(\frac{1}{T}\right)} = \frac{\log D_1 - \log D_2}{\frac{1}{T_1} - \frac{1}{T_2}}$$

Taking $1/T_1$ and $1/T_2$ as $1.0 \times 10^{-3}$ and $0.90 \times 10^{-3}$ K$^{-1}$, respectively, then the corresponding values of log $D_1$ and log $D_2$ are $-14.68$ and $-13.57$. Therefore,

$$Q_d = -2.3 R \text{ (slope)}$$

$$Q_d = -2.3 R \frac{\log D_1 - \log D_2}{\frac{1}{T_1} - \frac{1}{T_2}}$$

$$= -(2.3)(8.31 \text{ J/mol - K}) \left[ \frac{-14.68 - (-13.57)}{(1.0 \times 10^{-3} - 0.90 \times 10^{-3}) \text{ K}^{-1}} \right]$$

$$= 212,200 \text{ J/mol}$$

Rather than trying to make a graphical extrapolation to determine $D_0$, a more accurate value is obtained analytically using Equation 5.9b taking a specific value of both $D$ and $T$ (from $1/T$) from the plot given in the problem; for example, $D = 1.0 \times 10^{-14}$ m$^2$/s at $T = 1064$ K ($1/T = 0.94 \times 10^{-3}$ K$^{-1}$). Therefore

$$D_0 = D \exp \left(\frac{Q_d}{RT}\right)$$

$$= (1.0 \times 10^{-14} \text{ m}^2/\text{s}) \exp \left[ \frac{212,200 \text{ J/mol}}{(8.31 \text{ J/mol - K})(1064 \text{ K})} \right]$$

$$= 2.65 \times 10^{-4} \text{ m}^2/\text{s}$$

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\[ = 3.8 \times 10^{-12} \text{ m}^2/\text{s} \]

Note: this problem may also be solved using the “Diffusion” module in the VMSE software. Open the “Diffusion” module, click on the “D0 and Qd from Experimental Data” submodule, and then do the following:

1. In the left-hand window that appears, enter the two temperatures from the table in the book (converted from degrees Celsius to Kelvins) (viz. “873” (600°C) and “973” (700°C), in the first two boxes under the column labeled “T (K)”. Next, enter the corresponding diffusion coefficient values (viz. “5.5e-14” and “3.9e-13”).

3. Next, at the bottom of this window, click the “Add Curve” button.

4. A log D versus 1/T plot then appears, with a line for the temperature dependence for this diffusion system. At the top of this window are give values for \( D_0 \) and \( Q_d \); for this specific problem these values are 1.04 \( \times \) 10\(^{-5} \) m\(^2\)/s and 138 kJ/mol, respectively.

5. To solve the (b) part of the problem we utilize the diamond-shaped cursor that is located at the top of the line on this plot. Click-and-drag this cursor down the line to the point at which the entry under the “Temperature (T):” label reads “1123” (i.e., 850°C). The value of the diffusion coefficient at this temperature is given under the label “Diff Coeff (D):”. For our problem, this value is 1.2 \( \times \) 10\(^{-14} \) m\(^2\)/s.
5.22 (a) Using Equation 5.9a, we set up two simultaneous equations with $Q_d$ and $D_0$ as unknowns as follows:

$$\ln D_1 = \ln D_0 - \frac{Q_d}{R} \left( \frac{1}{T_1} \right)$$

$$\ln D_2 = \ln D_0 - \frac{Q_d}{R} \left( \frac{1}{T_2} \right)$$

Solving for $Q_d$ in terms of temperatures $T_1$ and $T_2$ (873 K [600°C] and 973 K [700°C]) and $D_1$ and $D_2$ (5.5 x $10^{-14}$ and 3.9 x $10^{-13}$ m$^2$/s), we get

$$Q_d = -R \frac{\ln D_1 - \ln D_2}{1/T_1 - 1/T_2}$$

$$= - \frac{(8.31 \text{ J/mol} \cdot \text{K}) \left[ \ln \left( 5.5 \times 10^{-14} \right) - \ln \left( 3.9 \times 10^{-13} \right) \right]}{\frac{1}{873 \text{ K}} - \frac{1}{973 \text{ K}}}$$

$$= 138,300 \text{ J/mol}$$

Now, solving for $D_0$ from Equation 5.8 (and using the 600°C value of $D$)

$$D_0 = D_1 \exp \left( \frac{Q_d}{RT_1} \right)$$

$$= (5.5 \times 10^{-14} \text{ m}^2/\text{s}) \exp \left[ \frac{138,300 \text{ J/mol}}{(8.31 \text{ J/mol} \cdot \text{K})(873 \text{ K})} \right]$$

$$= 1.05 \times 10^{-5} \text{ m}^2/\text{s}$$

(b) Using these values of $D_0$ and $Q_d$, $D$ at 1123 K (850°C) is just

$$D = (1.05 \times 10^{-5} \text{ m}^2/\text{s}) \exp \left[ -\frac{138,300 \text{ J/mol}}{(8.31 \text{ J/mol} \cdot \text{K})(1123 \text{ K})} \right]$$

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5.27 (a) We are asked to calculate the diffusion coefficient for Mg in Al at 450°C. Using the data in Table 5.2 and Equation 5.8

\[ D = D_0 \exp \left( -\frac{Q_d}{RT} \right) \]

\[ = \left( 1.2 \times 10^{-4} \text{ m}^2/\text{s} \right) \exp \left[ -\frac{131,000 \text{ J/mol}}{(8.31 \text{ J/mol - K})(450 + 273 \text{ K})} \right] \]

\[ = 4.08 \times 10^{-14} \text{ m}^2/\text{s} \]

(b) This portion of the problem calls for the time required at 550°C to produce the same diffusion result as for 15 h at 450°C. Equation 5.7 is employed as

\[ D_{450}^t = D_{550}^t \]

Now, from Equation 5.8 the value of the diffusion coefficient at 550°C is calculated as

\[ D_{550} = \left( 1.2 \times 10^{-4} \text{ m}^2/\text{s} \right) \exp \left[ -\frac{131,000 \text{ J/mol}}{(8.31 \text{ J/mol - K})(550 + 273 \text{ K})} \right] \]

\[ = 5.76 \times 10^{-13} \text{ m}^2/\text{s} \]

Thus,

\[ t_{550} = \frac{D_{450}^t}{D_{550}^t} \]

\[ = \frac{(4.08 \times 10^{-14} \text{ m}^2/\text{s})(15\text{ h})}{(5.76 \times 10^{-13} \text{ m}^2/\text{s})} = 1.06 \text{ h} \]