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Gas Flow

• Flow Regimes
  – Nature of the Gas (Knudsen’s Number)
  – Relative Quantity (Reynolds’ Number)
  – Turbulent
  – Laminar
  – Viscous
  – Molecular
  – Transition
Knudsen’s Number (Kn)

\[ Kn = \frac{\lambda}{d} \]

- \( Kn < 0.01 \) Viscous Flow - gas flow determined by gas-gas collisions
- \( 1 > Kn > 0.01 \) Transition Flow
- \( Kn > 1.0 \) Molecular Flow - gas flow determined by gas-wall collisions
Reynolds’ Number

\[ R = \frac{U \rho d}{\eta} \]

where:
- \( U \) = gas stream velocity
- \( \rho \) = mass density of the gas
- \( d \) = diameter of the tube
- \( \eta \) = viscosity

- \( R > 2200 \) Turbulent Flow
- \( R < 1200 \) Laminar Flow
- \( 1200 < R < 2200 \) Turbulent or Viscous Depending on the Geometry
Flow Regimes

- **Laminar Viscous Flow** - Ordered flow of gas in streamlines. Dominated by gas-gas collisions
  - $\text{Kn} < 0.01$, $R < 1200$

- **Molecular Flow** - Dominated by gas-wall collisions. Assume diffuse reflection at surfaces (molecules are re-emitted in a direction independent of the incident direction). Gas molecules do not collide with each other.
  - $\text{Kn} > 1$, $R < 1200$

- **Turbulent Flow** -
  - $\text{Kn} < 0.01$, $R > 2200$

- **Transition Flow** - Gas is neither viscous nor molecular. Flow is dominated by both gas-gas collisions and gas-wall collisions
  - $1 > \text{Kn} > 0.01$
Definitions

- **Throughput (Q)** - Quantity of gas (the volume of gas at a known pressure) that passes a plane in a known time. Units = Pa-m\(^3\)/s or Watts (energy flow -- energy it takes to transport the molecules across a plane)
  
  \[ Q = P \frac{dV}{dt} \]
  
  where: P=pressure and \( \frac{dV}{dt} \) = volumetric flow rate

- **Mass Flow** - The quantity of a substance (kg) that passes a plane in a known time. Units = kg/s

- **Molecular Flow** - The quantity of a substance (number of molecules N) that passes a plane in a known time. Units = N/s

- **Conductance** - The ability of an object to transport gas between two pressures regimes. Units = m\(^3\)/s
  
  \[ C = \frac{Q}{(P_2 - P_1)} \]
Viscous flow -- Orifice

- Orifice – tube of zero length
  - As P2 drops (P2/P1) decreases which induces a gas flow through orifice A. As P2/P1 decreases the gas flow through the orifice will increase until it reaches a maximum. At this maximum, this ratio of inlet pressure (P1) to outlet pressure (P2) has reached a “critical pressure ratio.” at this ratio the flow becomes “choked” at the speed of sound (343m/s)
Viscous Flow -- Orifice

\[ Q = A P_1 C' \left( \frac{2\gamma kT}{\gamma - 1} m \right)^{\frac{1}{2}} \left( \frac{P_2}{P_1} \right)^{\frac{1}{\gamma}} \left[ 1 - \left( \frac{P_2}{P_1} \right)^{\frac{\gamma - 1}{\gamma}} \right]^{\frac{1}{2}} \]

for:

\[ 1 > \frac{P_2}{P_1} \geq \left( \frac{2}{\gamma - 1} \right)^{\frac{\gamma}{\gamma - 1}} \]

where:

\[ \gamma = \frac{C_P}{C_V}, \gamma = 1.4 \text{ (diatomic species)}, 1.667 \text{ for (monatomic ), and 1.333 (triatomic)} \]

\( C' \) is a factor which reduces the cross sectional area because of the high speed of gas flow "vena contracta" \( \approx 0.85 \)
Viscous Flow -- Orifices

For \( \frac{P_2}{P_1} \geq \left( \frac{2}{\gamma - 1} \right)^{\frac{\gamma}{\gamma - 1}} = 0.525 \) for air (\( \gamma = 1.4 \))

\[ Q = A P_1 C' \left( \frac{2\gamma}{\gamma + 1} \frac{kT}{m} \right)^{\frac{1}{2}} \left( \frac{2}{\gamma + 1} \right)^{\frac{1}{\gamma - 1}} \]

Flow is independent of \( P_2 \)

Flow is said to be “critical” or “choked” – limited by gas stream traveling at the speed of sound
Viscous Flow -- Orifices

For Air at Room Temperature

\[ C \left( \frac{l}{s} \right) = \frac{7.66 \times 10^5 C' A (m^2)}{1 - \frac{P_2}{P_1}} \left( \frac{P_2}{P_1} \right)^{0.714} \left[ 1 - \left( \frac{P_2}{P_1} \right)^{0.286} \right]^{\frac{1}{2}} \]

for: \(1 > \frac{P_2}{P_1} \geq 0.52\)
Viscous Flow – Long Round Tubes

Hagan-Poiseuille Equation

\[ Q = \frac{\pi d^4 \left( P_1 + P_2 \right)}{128 \eta l} \frac{(P_1 - P_2)}{2} \]

For Air at Room Temperature

\[ C \left( \frac{l}{s} \right) = 1.38 \times 10^6 \frac{d \text{ (meters)}^4}{l} \frac{P_1 + P_2 \text{ (Pascals)}}{2} \]
Viscous Flow – Long Round Tubes

• Assumptions for Hagan-Poiseuille Equation
  – Fully developed flow – ie velocity profile is not position dependent
  – Laminar flow
  – Zero wall velocity
  – Incompressible gas – $U(\text{mach number}) = \frac{U}{U_{\text{sound}}} < \frac{1}{3}$ (ie non-choked flow)
Molecular Flow -- Orifice

\[ Q_1 = P_1 \frac{dv}{dt}, PV = NKT \]

\[ P_1 \frac{dv}{dt} = \frac{dN}{dt} kT, \]

\[ Q_1 = kT \frac{dN}{dt}, \frac{dN}{dt} \frac{1}{A} = \Gamma \]

\[ Q_1 = kT \Gamma A, \Gamma = \frac{1}{4} n v_{(avg)}, n = \frac{P}{kT} \]

\[ Q_1 = \frac{1}{4} P_1 A v_{(avg)}, Q_2 = \frac{1}{4} P_2 A v_{(avg)} \]

\[ Q = Q_1 - Q_2 : Q = \frac{1}{4} v_{(avg)} (P_1 - P_2) \]

For Air at Room Temperature

\[ C\left( \frac{m^3}{s} \right) = 116A(m^2) \]

\[ C\left( \frac{m^3}{s} \right) = 11.6A(cm^2) \]
Molecular Flow – Long Round Tube

\[ C = \frac{\pi}{12} v \frac{d^3}{l} \]

For Air at Room Temperature

\[ C \left( \frac{m^3}{s} \right) = 121 \frac{d^3(m)}{l(m)} \]