The BEC: Coherent Matter Waves

- PASI-2000 Lectures by William P. Reinhardt, University of Washington, Seattle, WA, 98195, USA. Rein@chem.washington.edu
- with help from Sam McKinney (thanks, Sam!)
- support from PASI, US-NSF, NIST and ONR.
PASI - Ushuaia - 2000

Will a 2nd Darwin Emerge?

- The Bose - Einstein Condensate
  - A coherent many body quantum system
  - Non-linear
    \[ \Rightarrow \text{Solitons} \]
    \[ \Rightarrow \text{Vortices} \]
    \[ \Rightarrow \text{CHAOS} \]

- Acknowledgements:
  - PASE!
  - NIST: Clark, Fedor, Burnett
  - Phillip's Group
  - U.O. Carr, Haung, McKinney, Brand, Thouless
Outline:

Lect 1: What is the BEC? How is it made? Mean-Field Theory | 1st Approx. Suddenly "Coherence" Analytic Mean-field Theory & non-linear collisions \[ \Rightarrow \text{solutions} \]

Lect 2: What "are" solutions? How might they be made? What did we simulate in Lect #2? Phase-number "uncertainty"

Lect 3: Atom Interferometry BEC Interferometry Solitons "Phase Engineered" \[ \bullet \text{Vortices} \bullet \text{CHAOS} \]
PERSON OF THE CENTURY

TIME

ALBERT EINSTEIN

DECEMBER 23, 1999 - JANUARY 3, 2000
www.time.com
2. Hr. Einstein legte eine Arbeit über die Quantentheorie des ein-atomigen idealen Gases (zweite Abhandlung) vor.


"... for each temperature, there exists a saturation density of the ideal gas, such that molecules in excess of this density do not participate in the thermal agitation."
DeBroglie wavelength \( \lambda_{dB} = \frac{\hbar}{\sqrt{2\pi m kT}} \)

At critical temperature and density, average distance between atoms becomes comparable to \( \lambda_{dB} \)

Result: Most atoms end up in same wavefunction

Fig. 3. BEC occurs when the deBroglie wavelengths of the atoms in the gas become comparable to the average distance between gas atoms.
Time Line of Highlights:

1924-25 Bose, Einstein

'50's Hecht, '70's Stwalley, Nosenov

\[ H^+ \quad H^+ \]

\textit{triplet-spin potential does not bind a state!}

Neither does

\[ \uparrow \text{e} \quad \uparrow \text{e} \]
$\text{H}_2$ triplet, singlet

Born-Oppenheimer

Potentials

Energy $V_{\text{eff}} \approx 53,000 \text{ K}$

$H: H = H + H$

$0.75 \text{ Å}$

Proton 1  Proton 2
$H_2$: triplet, singlet

Born-Oppenheimer potentials

$V_{\text{eff}} \sim 53,000 \, \text{K}

H: H = H + H

\text{proton 1} \rightarrow \text{proton 2}

0.75 \, \text{Å}

\text{R}
Alkali

$\text{Na} \uparrow \text{Na} \downarrow$

$v_{\text{eff}}$

$R$

$Rb_2$

	triplet ground

	many vibration states
1997 Nobel Prize in Physics
"Optical Cooling" W. Phillips
S. Chu

\[ \text{Na} \rightarrow 500 \text{m/sec} \quad \text{mm/sec} \quad \text{laser} \]

\[ \text{resonance} \]

\[ \text{Fresnel shifter (30 cm)} \]

\[ \text{Lewman, Cowell (JILA), Ketterle (MIT)} \]

\[ \text{Hanlet (Rice)} \]

\[ \text{spin polarized} \]

\[ \text{RF} \]

\[ \text{Na} \]

\[ \text{Ci} \]

\[ \text{Focussed Evap Cooling} \]

\[ \approx 20 \text{nK} \]

\[ \text{(H)} \]
The first condensate
JILA 1995
Many scientists from many countries contributed over a 25 year period!
Now BEC, all over the world: Ùshuaia?
False Color Velocity Distributions

High temp: Isotropic: Thermal

\[ e^{-\frac{1}{2}mv^2}{kT} \]

Low temp: Oscillator trap an-isotropic momentum distribution
Fig. 4. Horizontal sections taken through the velocity distribution at progressively lower values of \( \nu_{\text{evap}} \) show the appearance of the condensate fraction.
Site, Number scales

"typical" or "old days"

$N \approx 10^3 \rightarrow 10^4$ (Li, Na, He)

$N \approx 10^3 \rightarrow 10^4$ (set by visibility

(p $\approx 10^3 \rightarrow 10^5$ (set by visibility

(1/cm$^3$) 3-body

Recombination)

Atomic site $1,2\,\text{Å}$

Interaction length $2 \rightarrow 50\,\text{Å}$ (actual $\geq 50\,\text{Å}$)

Particle sep $\approx 200\,\text{Å}$

trap sizes $L \approx 100\,\mu\text{m}$, Don Low, Matt
• Mean Field Theory (...) 58

Ginzburg / Pitaevskii / Gross

Basic Idea:

Each Particle Occupies

a single "orbital" \( \Psi_i (x_i) \)

\[
\begin{align*}
\frac{-\nabla^2_i}{2m} \Psi_i & + V + g \Psi_i^2 \Psi_i \\
& + \sum_{j \neq i} \int \Psi_j^* (x) u(x - x_i) \Psi_j (x) dx = \mu \Psi_i
\end{align*}
\]
But as $T \to 0$

all particles end up in

a single "with"

and as $k \to 0$

$V(\vec{x}_i - \vec{x}_j) \to \frac{4\pi \hbar^2}{m} \delta(\vec{x}_i - \vec{x}_j)$

Fermi: reproduces $k \to 0$

$\text{s-wave scattering...}$

$-\frac{\hbar^2}{2m} \nabla^2 \Phi + V_{\text{tot}} \Phi + \frac{(N-1)4\pi \hbar^2}{m} |\Phi|^{2} \Phi$

$= \lambda \Phi$
Collective Excitations of a Bose-Einstein Condensate in a Dilute Gas

D. S. Jin,* J. R. Ensher, M. R. Matthews, C. E. Wieman, and E. A. Cornell*

Joint Institute for Laboratory Astrophysics, National Institute of Standards and Technology
and University of Colorado, Boulder, Colorado 80309-0440
and Physics Department, University of Colorado, Boulder, Colorado 80309-0440
(Received 23 May 1996)
Emergence of interaction effects in Bose-Einstein condensation

M. Holland, D. Jin*, M. Mathews, J. Ensher, M. Chiofalo†, J. Cooper, C. Wieman, and E. Cornell*
JILA, National Institute of Standards and Technology, and Department of Physics, University of Colorado,
Boulder, CO 80309-0440.
(October 14, 1996)

$10^4 N \nu^{1/2} (\text{Hz}^{1/2})$

FIG. 3. Release energy as a function of interaction strength. Comparison of mean-field theory (solid line) and experimental data points (o). Inset shows horizontal (x) and vertical (o) RMS widths against the mean-field predictions (dashed and solid lines).

$10^4 N \nu^{1/2} (\text{Hz}^{1/2})$

FIG. 4. Cloud shape parameter as a function of interaction strength. Shown are the mean-field theory (solid line) and experimental data points (o).
Is "theory" over?

Not at all ...

---

Popov (Theory) Dodd et al. (M87)

• Experiment Jin et al. (JICA)
So will the BEC
Be completely uninteresting

Resort to "collisions"

Narashimachar, Waldb, Schenzle, Civale, Zoller
PRA 1996

Predict \Rightarrow\text{Wave Interference}!
Observation of Interference Between Two Bose Condensates


Interference between two freely expanding Bose-Einstein condensates has been observed. Two condensates separated by ~40 micrometers were created by evaporatively cooling sodium atoms in a double-well potential formed by magnetic and optical forces. High-contrast matter-wave interference fringes with a period of ~15 micrometers were observed after switching off the potential and letting the condensates expand for 40 milliseconds and overlap. This demonstrates that Bose condensed atoms are "laser-like"; that is, they are coherent and show long-range correlations. These results have direct implications for the atom laser and the Josephson effect for atoms.
Fig. 2. Interference pattern of two expanding condensates observed after 40 ms time-of-flight, for two different powers of the argon ion laser-light sheet (raw-data images). The fringe periods were 20 and 15 μm, the powers were 3 and 5 mW, and the maximum absorptions were 90 and 50%, respectively, for the left and right images. The fields of view are 1.1 mm horizontally by 0.5 mm vertically. The horizontal widths are compressed fourfold, which enhances the effect of fringe curvature. For the determination of fringe spacing, the dark central fringe on the left was excluded.
This is exciting: \(10^6-10^{10}\) atoms are coherent matter wave cloud.

Let's look at collisions and eigenstates "inside" a trap at high density.

Step 1: Divide

\[
\left(-\frac{\hbar^2}{2m} \nabla^2 + V + V_{\text{trap}} + (n-1)\frac{4\pi\hbar^2}{m} \right) \psi = \mu \psi
\]

by "\((n-1)\frac{4\pi\hbar^2}{m}\)"

\[
(\frac{\hbar^2}{2m} \nabla^2 + V + \text{keV}) \xi = \mu \xi
\]

\[
\xi \sim \left(\frac{1}{\hbar^2}\right)^{1/2} \xi = \frac{\xi}{\text{cm}^3}
\]

\(\xi\) = healing length. Important scale size?
Step #2: derive analytically
Soluble model
(pseudo 1-dimensional)

Summer 1996: motivation
Ketterle had trap

• and we wanted a soluble problem (expected to learn something!)

Summer 2000: The Hannover Group
in Germany has made
a pseudo 1-dim condensate

Idea:

\[ \text{Many } \Psi \]

Then \[ \Psi(x,y,t) \approx \Psi(x, y) + \Psi(x) \]
\[ y(x) = f(x) \text{ which satisfies } \]

\[-f''(x) + f(x) = \alpha f(x) \]

with boundary conditions \( f(0) = 0 \) \( f(L) = 0 \)

\( f'(x) \) is an integrating factor:

\[ (f'(x))^2 = f(x) - f(x) \text{ } + \text{ const} \]

\[ dt = \frac{df}{\sqrt{1 - \alpha^2 f^2}} \]

\[ t = \int \frac{dx}{\sqrt{1 - \alpha^2 f^2 (1 - mx^2)}} \]

\( f = \text{Jacobi Elliptic Function} \)
\[ \int_{0}^{\infty} \frac{dx}{(1-x^2)(1-mx^2)^{1/2}} \]

is actually familiar at 2 limits:

\[ m \to 0 \]

\[ z = \int_{0}^{\infty} \frac{dx}{\sqrt{1-x^2}} = \arcsin(y) \]

\[ \to y = \sin(z), \quad m \to 0 \]

\[ \text{limit} \]

\[ m \to 1 \]

\[ t = \int_{0}^{\infty} \frac{dx}{(1-x^2)} = \tanh^{-1}(y) \]

\[ \to y = \tanh(t), \quad m \to 1 \]

\[ \text{limit} \]
Thus $\varphi(x) = f(x)$ is known and the eigenvalues found.

$\text{SN} \left( \frac{x}{L}, 2 \right)$

$\left( \frac{x'}{L} \right) \rightarrow 0$

$\tanh \left( \frac{\frac{3}{2} x'}{L} \right)$

Jacobi Elliptic "sine" function

$\text{"Particle in Box"}$

$1 \omega^2 \rightarrow n_0$

Ginzburg / Pitaevskii / Grosse

We need to satisfy BC at $0, L$

$\text{SN} \left( \frac{2}{L}, 2, n \right)$

$m \approx 1 - 10^{-9}$
Collisions of Bose-Einstein condensates

Solutions of the nonlinear Schrödinger equation for successive collective excited states exhibit nodal structure resembling that of a spectral sequence...

$n = 1$

$n = 2$

$n = 3$

$n = 4$

$n = 5$

$n = 6$

$n = 7$

$n = 8$

$n = 9$

$n = 10$

$n = 20$

tending eventually to free -particle-in-a-box solutions:

1997 March Meeting of the American Physical Society Kansas City, MO
Where do such "ode" eigenfunctions come from?

\[ \text{const} \times 15 \text{ digits!} \]

change notation! 

\[(f')^2 = f^4 + f^2 < \]

try \( f \to q \) (cosh) \( j \) \( f' \to \) momentum

\[ \text{Ans:} \quad q^2 - q^4 + q^2 = \text{const} \]

\[ V(q) \]

\[ q \]
zero-energy

Intra-trap (high density)

Collision:

\[ \begin{align*}
\text{Barrier} & \\
\text{Removal Barrier at } t \to \infty & \text{ follow evolution of "notch"}
\end{align*} \]
These are the dark and gray solutions of Zakharov & Shabat. Solutions are usually parameterized as \( f(x - ct, \eta, \mu) \).

But \( \Delta x \) is a constant out of motion if \( \Delta x = \pi + S \).

Then: \( c = c_{Bogoliubov} \sin(\frac{S}{2}) \).

Josephson Effect Analog \( \Delta x \) drives the current.
Periodic waves in shallow water. The original caption read: "As they near shallow water close to the coast of Panama, huge deep-sea waves, relics of a recent storm, are transformed into waves that have crests, but little or no troughs. A light breeze is blowing diagonally across the larger waves to produce a cross-chop. Three army bombers, escorted by a training ship, are proceeding from Albrook Field, Canal Zone, to David, Panama."

(Taken from National Geographic 63 (1933).)
What has actually been done?

\[ \phi_L^{N/2} \]

\[ \phi_R^{N/2} \]

Sudden approximation

\[ \rightarrow (\phi_L + e^{i\pi \phi_R})^N \]

But:

\[ \langle (\phi_L + e^{i\pi \phi_R})^N | \phi_L^{b/2} \phi_R^{w_2} \rangle \]

\[ = \text{const}! \Rightarrow \text{all } \pi \text{ equal prob!} \]
X completely ungradable

As in \( \Delta N \Delta \phi \sim 1 \) uncertainty!

But if \( \Phi = (\Phi_2 + e^{i\pi N})^N \)

we have no idea where the particles are, thus a phase off set may be maintained exactly!
Last chance to observe
Reinhardt "twin dilatation"
(en el fin del mundo.....)

- Will the solitons be real?
- Matter-wave interferometry
- glass engineer condensate
- Why are solitons so stable? (think...)
- Propaganda: Sam
  will discuss chaos
Generating Solitons by Phase Engineering of a Bose-Einstein Condensate


Quantum phase engineering is demonstrated with two techniques that allow the spatial phase distribution of a Bose-Einstein condensate (BEC) to be written and read out. A quantum state was designed and produced by optically imprinting a phase pattern onto a BEC of sodium atoms, and matter-wave interferometry with spatially resolved imaging was used to analyze the resultant phase distribution. An appropriate phase imprint created solitons, the first experimental realization of this nonlinear phenomenon in a BEC. The subsequent evolution of these excitations was investigated both experimentally and theoretically.
Matter wave Interferometry

Pritchard et. al. PRL 56, 827, 86
60, 515, '89
Sin An Lee et. al. PRL 25, 2637 '85

\[ I(x) = \int \psi^* \psi \, dx \]

\[ \Delta p = 2 \hbar k \]
Why 2\,\hbar\,c ?

Classical EM

\[\hbar/2\]
plane separated by \(\hbar/2\)
(matter wave – light wave)

Quantum Field (Roman)

\[\hbar\rightarrow\hbar\text{ recoil}\]

\[\hbar\text{ Stimulated Emission} \rightarrow \hbar\]
(gaistic matter – particulate light)
Kotzura et al., Physics Group
PRC 84, 021 (1999)

$\omega \rightarrow \omega + \delta$

$\theta \approx \pi$

Let $\text{BEC}_{2k \hbar} = |e\rangle$

$\text{BEC} = |g\rangle$
Multiple order Bragg diffraction
The Bragg interferometer allows us to probe the phase of the BEC wavefunction.

\[ \Delta \tau = \frac{1}{2} \]

The Bragg interferometer is depicted with three outputs. The phase shift between the input and the output is indicated as \( \Delta \tau \).

\[ \Delta \tau = \frac{1}{2} \]
The Bragg interferometer allows us to probe the phase of the BEC wavefunction.
By writing a phase of $\pi$ on the top half of the condensate, we demonstrate phase analysis with the interferometer.
Figure 3

Denschlag et al.
Why so stable
(Contrary to expectations ...)

K. Huang & C.N. Yang
Phys Rev 105, 767 (1957)

\[ E^{\pi^0} = \frac{2}{d} \frac{n d (k t)^2}{2m} + \frac{4\pi^2 t^2}{m \nu} \]

\[ n_0 = n_1, n_2, \ldots = 0 \]

\[ \left[ n^2 \right] = \left[ n \right] \]

\[ = \frac{4\pi^2 t^2}{m \nu} \frac{N(N-1)}{2} \equiv \text{pair energy} \]
Why does this stabilize an excited state?

Excitations from 

\( (0, 0, 0, 0, \ldots) \)

to 

\( (1, b-1, 0, 0, \ldots) \)

are also very expensive.

- Energy \( E(N_0) \)
  
\[ N_2 = N - N_0 \]

\[ 1 - \frac{n^2}{N^2} \]
Cost to move from

\((\mu, 0, 0, 0, 0, \ldots)\)

\(\rightarrow\) \((\mu - 1, 1, 0, 0, 0, \ldots)\)

is \(\sim \frac{N\mu^2 e^2}{mV}\)

\(\Rightarrow\) Bose condensation in

"State space" \(d > 0\)

---

NB for \(d < 0\), want \(\sum \frac{\mu^2}{2}\) small

\(\Rightarrow\) all \(\mu_i = 0\)

\(\Rightarrow\) condensation is in

"coordinate space"
Propaganda Section

Sam McKinnay

will discuss chaos in the BEC!
Dynamical and Wave Chaos in the Bose-Einstein Condensate

Sam McKinney and W.P. Reinhardt

- Symmetry Breaking and Phase Rigidity
- Phase Rigidity in 2D: Bose Einstein Speckle
- Laser Speckle and Scars
- Statistical Properties
Symmetry Breaking

⇒ Generalized Rigidity (P.W. Anderson, 1984)

- Symmetry is broken in the BEC because an arbitrary phase is chosen.
- Broken symmetry results in rigidity and topological defects (solitons, domain walls)
- So, just as when you drop a piece of chalk it shatters into domains, each of with its own lattice orientation, a shocked BEC should shatter into domains of constant phase separated by solitons.
Phase Rigidity in Quasi-1D
Phase Rigidity in 2D

- Produce 2D ground state of repulsive condensate in square box with rigid walls via complex time evolution.
- Imprint phase ramp at oblique angle to give condensate an initial velocity.
- Vary the strength of the shock by varying initial velocity.
Time Evolution of Shocked BEC

box length = 10\xi; v_i = 15.7 \xi/t_n \approx 11.1c_s

Phase Evolution

Density Evolution
Shocked BEC vs. Laser Speckle
Laser Speckle

- Intensity pattern produced by coherent (usually monochromatic) light reflecting from a rough surface or propagating through a medium with random refractive index fluctuations.
- Can be simulated as superposition of monochromatic plane waves in 3D with random directions and phases, projected onto 2D screen.
Heller’s “Scars”

• Ridge-like structures seen in quantum wavefunctions for chaotic enclosures.
• Originally produced as superposition in 2D of monochromatic plane waves with random directions and phases.
k-space Intensity

(a) Scars

(b) Speckle

(c) BEC with weak shock $v_i = 4.44c_s$ $(t=60)$

(d) BEC with strong shock $v_i = 11.11c_s$ $(t=60)$
Time Evolution of Shocked BEC

box length = 10\(\xi\); \(v_i = 6.28\ \xi/t_n \approx 4.44c_s\)
Relaxation

\[ v_i = 4.44c_s \]

\[ v_i = 8.88c_s \]

\[ v_i = 9.77c_s \]

\[ v_i = 15.6c_s \]
Intensity Distribution Functions

(a) Scars

(b) Speckle

(c) BEC with \( v_i = 4.44c_s \)

(d) BEC with \( v_i = 11.11c_s \)
Auto-correlation Functions

(a) Scars

(b) Speckle

(c) BEC with $v_i = 4.44c_s$

(d) BEC with $v_i = 11.11c_s$
Summary and Conclusions

- Phase rigidity is observed in 1D and 2D.
- A shocked BEC in 2D exhibits a speckle-like pattern b/c non-linearity in the NLSE produces dispersion in k-space, the effect of which is similar to projecting a 3D superposition of plane waves onto a 2D screen.
- The size scale is determined by the initial speed, not by the healing length \( \xi \).
- For strong shocks structure forms on a scale smaller than \( \xi \), implying that the NLSE should break down.
- For weak shocks there is a pseudo-relaxation in k-space as kinetic energy is converted to potential energy.
Reinhardt’s PASI BEC Lecture Summary

- The gaseous BEC is a strongly coherent and non-linear quantum system.
- H, Na, Rb, Li condensates (and spin mixtures) have been made in many labs, and many different confinement geometries.
- Mean-field theory works very well indeed!
- Solitons predicted by mean field theory have now been seen experimentally: important verification of phase control of quantum dynamics (vortices also described by mean field theory have now been observed!)
- Mach-Zehnder matter wave interferometry confirms phase imprinting, and subsequent soliton generation
- Phase-number uncertainty plays an important role in allowing a phase relationship to be fixed between two condensates
- Solitons are far more stable than expected in traditional condensed matter theories
- When shocked, the phase rigidity of the BEC creates broken symmetry phase domains & can generate dynamical wave chaos.