Quantum Teleportation

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\[ |\psi_{in}\rangle \rightarrow \text{Classical channel} \rightarrow \rho_{out} \]

\[ \text{Unknown input beam} \rightarrow \text{Alice} \rightarrow \text{Bob} \rightarrow \text{Teleported output beam} \]

Entanglement

Support:
QUIC funded by DARPA via ARO, NSF, ONR

http://www.eco.caltech.edu/~qoptics
**Quantum Teleportation**

**Input - unknown quantum state**

**Output - quantum state “identical” to input**

- Classical communication channel
- Shared quantum entanglement via quantum channel

**Protocol**
- Alice makes an “appropriate” (projective) measurement of the unknown state together with her component of the entangled state shared with Bob.
- Alice then sends this (random) classical information from the measurement to Bob.
- Bob uses the received information to perform a unitary transformation on his component of the entangled state to transform it into an output state “identical” to the input state.

**Note**
- Input state is unknown (and “unknowable”)
- Neither classical nor quantum channels individually carry any information about the unknown input.

**Quantum Teleportation**

\[ |\psi\rangle_{in,1,2} = |\psi\rangle_{in} \otimes |EPR\rangle_{1,2} \]

**Overall system state** -

\[ |\psi\rangle_{in,1,2} = \frac{1}{2} \sum_{i=1}^{4} |EPR_i\rangle_{in,1} U_i |\psi\rangle_2 \]

**Rewrite identically as** -

where the four "Bell states" are

\[ |EPR_i\rangle_{1,2} = \frac{1}{\sqrt{2}} \left( |\uparrow\downarrow\rangle_{1,2} + |\downarrow\uparrow\rangle_{1,2} \right) \]

\[ \quad + \left( |\uparrow\downarrow\rangle_{1,2} - |\downarrow\uparrow\rangle_{1,2} \right) \]

\[ \quad + \left( |\uparrow\uparrow\rangle_{1,2} + |\downarrow\downarrow\rangle_{1,2} \right) \]

\[ \quad + \left( |\uparrow\uparrow\rangle_{1,2} - |\downarrow\downarrow\rangle_{1,2} \right) \]

Quantum Teleportation, cont.

- Projective measurement in Bell basis by Alice finds $i_0$, hence

$$|\psi\rangle_{in,1,2} \rightarrow U_{i_0} |\psi\rangle_2$$

- Classical information of outcome $i_0$ (2 bits) sent to Bob

- Bob uses these 2 bits to select the inverse operation $U_{i_0}^{-1}$ to perform on EPR beam 2, and thus generates the output

$$U_{i_0}^\dagger U_{i_0} |\psi\rangle_2 \rightarrow |\psi\rangle_{out}$$

- Net effect - Output state identical to input state emerges from Bob's station
Quantum versus Classical Teleportation

- Classical channel shared by Alice and Bob -

- Quantum and classical channels shared by Alice and Bob -

Overlap of input and output states? Quantify via fidelity of entanglement $F$, where

$$F = \langle \psi_{in} | \rho_{out} | \psi_{in} \rangle$$

- Bounds on $F$ for classical teleportation?

1. For distribution of polarization states,
   $$F_{\text{classical}} < 0.67.$$  
2. For distribution of coherent states,
   $$F_{\text{classical}} < 0.50.$$
Essence of Quantum Teleportation

Disembodied Transport of Quantum State

\[ \text{"The Whole Thing"} \]

\[ \text{Input} \rightarrow \text{Output} \]

- How to quantify similarity of input and output?
- Choose observable $\hat{E}_d$ with measurement outcomes $\lambda$

\[ \Rightarrow \text{Measurement statistics for input} \quad \text{for output} \]

\[ P_{\text{in}}(\lambda) = \langle \text{in} | \hat{E}_d | \text{in} \rangle \quad P_{\text{out}}(\lambda) = \text{Tr} \left( \rho_{\text{in}} \hat{E}_d \right) \]

- Quantify similarity of $P_{\text{in}}$ and $P_{\text{out}}$ by statistical overlap $S_E$

\[ S_E = \int \sqrt{P_{\text{in}}(\lambda) P_{\text{out}}(\lambda)} \, d\lambda \]

\[ 0 \leq F \leq 1 \]

* For any observable $\hat{E}_d$

\[ S_E^2 \geq \text{Fidelity } F \]

Fidelity captures "quality" of teleportation protocol for any possible measurement

\[ \Rightarrow \text{Contrast} = \{ (q, p), (pq, p) \} \]
Criteria for *Genuine* Quantum Teleportation

\[ F = \langle \psi_{\text{in}} | \rho_{\text{out}} | \psi_{\text{in}} \rangle \]

1. An unknown quantum state propagates to Alice's station. Note that this state may be a component of and entangled with a larger system.

2. A "recreation" of this quantum state emerges from Bob's receiving terminal. Note that this teleported state should maintain any entanglement that may have existed for the original input state to Alice's station, and should be available for inspection via arbitrary quantum measurements by any subsequent "user."

3. The fidelity of input and output states is higher than that which could have been achieved if Alice and Bob shared only a classical communication channel. \[ = \text{Braunstein, Fuchs, HK}, \text{J. Mod. Opt. 47, 367 (2000)} \]

Photon Polarization -


(1) No input


(2) No output

(3) Low fidelity for both, \( F \ll 2/3 \)


Simulation of teleportation
Continuous Quantum Variables - Infinite Dimensional Hilbert Space

- Let's focus on the specific example of a harmonic oscillator \((x, p)\).
  - For example, trapped atom or the electromagnetic field.

- State space of an oscillator:
  \[
  a_0 \left| 0 \right> + a_1 \left| 1 \right> + a_2 \left| 2 \right> + \ldots
  \]
  
  (compare to a qubit: \(\alpha \left| 0 \right> + \beta \left| 1 \right>\))

- Wigner-function representation:

- Postpone for now the real (quantum information) action -
  - tensor product of several oscillators.
Squeezed Light

- Travelling wave of the electromagnetic field -

- Field quadratures $E_1$ and $E_2$ analogous to $x$ and $p$

Coherent state:

Squeezed state:
**EPR Entanglement**

- Making entanglement:
  - Large squeezing → near perfect entanglement
  - Zero squeezing → zero entanglement

- 1992 Demonstration of EPR Paradox*
  "Entanglement on Demand"

- Detecting entanglement:

  - Entanglement completely parameterized by \( x \) and \( p \)

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Teleportation of Broad Bandwidth Quantum Information

Wigner Distribution for EPR State

\[ W(x_1, p_1, x_2, p_2) \]

\[ \text{to Alice} \quad \text{to Bob} \]

\[ \sim \exp \left\{ - \left[ (x_1 - x_2)^2 + (p_1 + p_2)^2 \right] e^{+2\alpha} \right. \]

\[ - \left[ (x_1 + x_2)^2 + (p_1 - p_2)^2 \right] e^{-2\alpha} \}

\[ \xrightarrow{\alpha \to 0} \]

\[ f(x_1 - x_2) f(p_1 + p_2) \]

"Quantum twins"

Entanglement for continuous quantum variables for \( \alpha > 0 \!\)!

- Nonseparability criteria - Duan et al., PRL 88, 2722 (2002)

- Ou, Pereira, HNC, Beng, Appl. Phys. Lett. 68, 3663 (92); Simon, PRL 64, 2726 (2000).
Quantum Teleportation of Broad Bandwidth via EPR State (circa 1935)*

$x_{1,2}$ quadrature amplitudes for EPR beams* -

\[ x_1, x_2 \]

\[ \text{time} \]

$x_{in}$ quadrature of unknown input state -

\[ x_{in} \]

\[ \text{time} \]

Alice "sums" and detects $x_{in} + x_1$ -

$\rightarrow$

Alice sends classical information $l_x \sim x_{in} + x_1$ to Bob

*Wigner distribution for EPR state -

\[ W(x_1; p_1; x_2; p_2) = \exp \left( i \left( (x_1;i)x_2 \right)^2 + (p_1 + p_2)^2 \right) e^{+2\imath t} \]

\[ = \frac{1}{4} \left( 1 \pm (x_1;i)x_2 \pm (p_1 + p_2) \right) \]
Quantum Teleportation via EPR State, cont.

From $i_x$, Bob generates field $x_{Bob} \sim x_{in} + x_1$ and (-)sums with his copy of EPR beam $x_2$

and then “creates” output “identical” to unknown input -

Analogous protocol for $p_{in} \rightarrow p_{out}$
Unconditional Quantum Teleportation

A beam of light was sent from Alice to Bob without physically propagating through the intervening space.

Generating EPR Beams \& Entanglement

\begin{align*}
\hat{\chi}_i &= e^{r} \hat{\chi}_i^{(0)} \\
\hat{\rho}_i &= e^{-r} \hat{\rho}_i
\end{align*}

\begin{align*}
\hat{\chi}_{ii} &= e^{-r} \hat{\chi}_{ii}^{(0)} \\
\hat{\rho}_{ii} &= e^{+r} \hat{\rho}_{ii}^{(0)}
\end{align*}

\text{(0) = vacuum state}

\text{Squeezing}

\text{EPR - beam splitting relations}

\begin{align*}
\hat{\chi}_1 &= \frac{1}{\sqrt{2}} [e^{r} \hat{\chi}_i^{(0)} + e^{-r} \hat{\chi}_{ii}^{(0)}] \\
\hat{\chi}_2 &= \frac{1}{\sqrt{2}} [e^{r} \hat{\chi}_i^{(0)} - e^{-r} \hat{\chi}_{ii}^{(0)}] \\
\hat{\rho}_1 &= \frac{1}{\sqrt{2}} [e^{-r} \hat{\rho}_i^{(0)} + e^{r} \hat{\rho}_{ii}^{(0)}] \\
\hat{\rho}_2 &= \frac{1}{\sqrt{2}} [e^{-r} \hat{\rho}_i^{(0)} - e^{r} \hat{\rho}_{ii}^{(0)}]
\end{align*}

\text{Note -}

\begin{align*}
(\hat{\chi}_1 - \hat{\chi}_2) &= \sqrt{2} e^{-r} \hat{\chi}_{ii}^{(0)} \to 0 \\
(\hat{\rho}_1 + \hat{\rho}_2) &= \sqrt{2} e^{+r} \hat{\rho}_{ii}^{(0)} \to 0
\end{align*}

\text{quantum twins}

\text{entangled}
Alice's measurement - EPR$_1$ and 14$\hbar$m$>

- State to be teleported

**Homodyne Detection of $\hat{X}$ quadrature**

**Homodyne Detection of $\hat{P}$ quadrature**

**Unknown state**

$(\hat{X}_{in}, \hat{P}_{in})$

**More transformation at MAlice**

\[
\hat{X}_x = \frac{1}{\sqrt{2}} (\hat{X}_{in} - \hat{X}_{1}) \quad ; \quad \hat{X}_p = \frac{1}{\sqrt{2}} (\hat{X}_{in} + \hat{X}_{1})
\]

\[
\hat{P}_x = \frac{1}{\sqrt{2}} (\hat{P}_{in} - \hat{P}_{1}) \quad ; \quad \hat{P}_p = \frac{1}{\sqrt{2}} (\hat{P}_{in} + \hat{P}_{1})
\]

**Alice measures**

\[
\begin{cases}
\hat{X}_x \rightarrow X_x \\
\hat{P}_p \rightarrow P_p
\end{cases}
\]

*Note: limit $r \rightarrow \infty$, Alice learns nothing about $(\hat{X}_{in}, \hat{P}_{in})$*
Bob's Reconstruction of $|\Psi_{in}\rangle$ to obtain Teleported State $|\Psi_{out}\rangle$

- **Rewrite EPR2 as (simple identity)**

  $\hat{x}_2 = \hat{x}_{in} - (\hat{x}_1 - \hat{x}_2) - \sqrt{2} \hat{x}_x$

  $\hat{p}_2 = \hat{p}_{in} + (\hat{p}_1 + \hat{p}_2) - \sqrt{2} \hat{p}_p$

- **Phase-space displacement of $(\hat{x}_2, \hat{p}_2)$ by $(x_x, p_p)\sqrt{2}$ at $m_{Bob}$**

  $\hat{x}_{out} = \hat{x}_{in} - \sqrt{2} e^{-r} \hat{x}_x^{(o)} \Rightarrow \hat{x}_{in}$

  $\hat{p}_{out} = \hat{p}_{in} + \sqrt{2} e^{-r} \hat{p}_p^{(o)} \Rightarrow \hat{p}_{in}$

- **Squeezed conditional upon Alice measurement**
Quantum Tariffs and the 3 Vacuums
(or what is that 4.77 dB in the data to follow????)

Unknown State

"p"

"x"

1 vacuum unit

"x"

"p"

Alice

photocurrent $I^x$

photocurrent $I^p$

Optimal estimate of both "x" and "p"...
Quantum Tariff of 1 vacuum unit

Bob

photocurrent $I^x$

photocurrent $I^p$

field $\mathcal{E} \sim \frac{|I^x| + i |I^p|}{t}$

2 vacuum units

Translate classical currents back to quantum field ... Quantum Tariff of 1 vacuum unit

Vacuum Arithmetic:
$1_{(in)} + 1_{(detect \ "x","p")} + 1_{(broadcast \ \mathcal{E})} = 3$
($= + 4.77 \text{ dB}$)
The feed-forward gain is the only parameter Bob must fix (once).

The maximum overlap between the teleported and incident states occurs for an overall gain of 1.

This optimizes Victor's Fidelity.
Fix gain $g$ for 4.77 dB increase in $\sigma_W$ above vacuum-state limit. Subsequent comparison of theory and experiment without adjustable parameters.
Variance for Teleported Field as a Function of Bob’s Gain $g$

\[ \sigma_w \]

$\sigma_w^{x,p}$

- [dB]

Gain $g^2$ [dB]

- $g = 1$

---

- **without EPR beams**
- **with EPR beams**
Fidelity of Teleportation as Inferred by Victor

Without correction for losses, efficiency!

\[ F = 0.58 \pm 0.02 > F_0 = 0.50 \]

Legend:
- with EPR beams - quantum teleportation
- without EPR beams - classical teleportation

Graph showing fidelity F vs. gain \( g^2 \) [dB] with quantum and classical teleportation.
So what?

This is the first and only demonstration of full\* quantum teleportation:

*full:
  • Quantum state enters Alice’s station
  • Quantum state leaves Bob’s station
  • High fidelity of output to input.

unconditional fidelity \( = 0.58 > F_{\text{classical}} = 0.50 \)

- Hierarchy of fidelity thresholds

Is there anything else we can do with this?
Quantum Information Processing with Continuous Quantum Variables

- quantum logic with continuous variables
- universal computation
- quantum error correction
- quantum cryptography
- quantum dense coding
- quantum networks

Implementations:
- Nonlinear optics - X(2)
- Cavity QED
- Spin squeezing in atomic vapors (Polzik)
Dense Coding*

Teleportation:

2 classical bits

EPR state

Dense Coding:

2 classical bits

1 qubit

EPR state

- Send 2 classical bits via one qubit, plus entanglement.

S. L. Braunstein & H. J. Kimble, in preparation
PRA 61, 042302 (2000)
Cavity QED with Cold Atoms
H. Jeff Kimble, PASI, 18 October, 2000

Caltech Quantum Optics Group

FUNDING:
NSF, DARPA via ARO, ONR
HP Research Labs

http://www.cco.caltech.edu/~qoptics
• Cavity QED with strong coupling
  • Internal degrees of freedom for atomic dipole + cavity field
  • External degrees of freedom - atomic center-of-mass motion

• Quantum information science
  • Distributed quantum networks

• Trapping and tracking single atoms in cavity QED
  • 1 atom bound in orbit with single photons
  • 1 atom trapped in a FORT in a regime of strong coupling

(http://www.cco.caltech.edu/~qoptics)
Cavity QED Rates and Ratios

**Strong Coupling**
\[
g / (\gamma, \kappa, 1/T) \gg 1
\]

---

**Critical photon number**

\[
m_0 \approx \frac{\gamma^2}{2g^2} < 1
\]

Nonlinear optics with one photon per mode

\[
m_0 \sim 10^{-4} \text{ photons}
\]

**Optical Information**

\[
I_0 \sim g^2 \frac{T}{\kappa} > 1
\]

Real time dynamics with one and the same atom

\[
I_0 \sim 10^6 - 10^7
\]

**Critical atom number**

\[
N_0 \approx \frac{2\gamma\kappa}{g^2} < 1
\]

Single-atom switching of optical cavity response

\[
N_0 \sim 10^2 \text{ atoms}
\]
Nonlinear Optics with Single Atoms and Photons
H. J. Kimble et al., Caltech

$g_0 = 20$ MHz
Critical photon number $m_0 = 0.02$ photons

Christina Hood
Quentin Turchette
Quantum Phase Gate


![Graph showing probe phase shift vs. number of intracavity pump photons]

**Conditional dynamics** ↔ **Quantum logic**

\[ e_+ |+\rangle + e_- |\rangle \]

\[ U_{op} \]

\[ U_{op} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{i\phi_a} & 0 & 0 \\ 0 & 0 & e^{i\phi_b} & 0 \\ 0 & 0 & 0 & e^{i\phi_a + \phi_b + \Delta} \end{bmatrix} \]

**Single-qubit phase shifts**

\[ \phi_a \approx (17.5 \pm 1)^\circ \quad \phi_b \approx (12.5 \pm 1)^\circ \]

**Conditional phase shift**

\[ \Delta \approx (16 \pm 3)^\circ \]
Strong Coupling in Cavity QED

Stochastic Nature of Atomic Beams and Resulting Indeterminism

• Atomic beams (optical and microwave regimes) -
  • Critical photon number $m_0 \sim 0.1$ photon
  • Optical information $I_0 \sim 1$ bit
  • Atomic arrival time is unknown a priori leading to intrinsic indeterminism
  • Knowledge of atom-field interaction
    • Post-diction based upon atomic detection

• In regime of strong coupling $m_0 \ll 1$ photon, $I_0 \gg 1$ bit, controlled quantum dynamics require $N = 1, 2, 3, \ldots$ atoms “on demand”

• Cavity QED with localized atoms -

Quantum Information Science
• Quantum measurement
• Quantum logic and computation
• Quantum communication
• Quantum-classical interface
Cavity QED by the Numbers: N=1, 2, ... Atoms

A Localized Atom
Optical dipole-force trap, ion trap, magnetic trap

Ion traps: Wineland, Blatt, Hughes, Maleki, Walther, ...

Lattices: Haensch, Phillips, Salomon, Jessen, ...

OPTICAL LATTICE

Crystal of lattice parameter $\lambda/2$

lattice provides cooling + trapping
Spherical Mirror, Fabry-Perot Cavity

- High Reflectivity Surfaces - Finesse ~ 470,000
- Length actively stabilized to ~ $10^{-15}$ m

Record finesse - $F = 1.9 \times 10^8$, $R = 0.9999984$

BK7 Substrate

1mm

Length $l \sim 10 - 50$ μm; Mode waist $w_0 \sim 15$ μm
Whispering Gallery Modes of Quartz Microspheres


\[ f = 2.2 \times 10^6 \quad Q = 8 \times 10^9 \]

Dragiyi, Ilchanko
Hansre et al., ENS

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Single atoms
Few photons
Photonic Bandgap Cavity and Planar Magnetic Microtrap

$m_0 \sim 10^{-8}$ photons!

Single defect photonic bandgap nano-cavity

Current-carrying wires for magnetic micro-trap

Hideo Mabuchi
Michael Roukes

Axel Scherer
The Mechanical Consequences of Strong Coupling

- Strong coupling for *internal* degrees of freedom:
  \[ \hbar g_0 > \hbar \gamma \]

- Strong coupling for *external* degrees of freedom:
  \[ 5 \text{ mK} \quad \leftrightarrow \quad \hbar g_0 > \hbar \gamma \approx k_B T_{\text{doppler}} \quad \leftrightarrow \quad 120 \mu\text{K} \]

- Spatial dependence of atom-photon interaction \( g(r) \) provides trapping potential

Theory: Haroche; Walther (1991), ... Scully et al. (1996) (μwave), ... Parkins, Tan, Doherty, Walls (1996); Ritsch (1997) (optical)
Some Formalism - CQED + CM motion

\[ E \rightarrow (\text{decay}) \rightarrow (\text{probe}) \rightarrow \text{atom} \]

Reversible interactions:

\[ H = \frac{p^2}{2m} + \hbar (\omega_{\text{atom}} - \omega_{\text{probe}}) \sigma^+ \sigma^-
+ \hbar (\omega_{\text{cavity}} - \omega_{\text{probe}}) a^+ a^-
+ \hbar g(\hat{a}) (a^+ \sigma + \sigma^ a^+ )
+ \hbar (E_\text{a}^+ + E^* a^+) \]

+ Irreversible processes -

- atomic spontaneous decay
- cavity decay

\[ \frac{\partial \hat{\rho}}{\partial t} = \cdots \]

Separation of Time Scales

- Assume $T_{\text{internal}} \ll T_{\text{external}}$
  - atomic dipole + cavity field
  - $\delta^{-1}, \kappa^{-1} $
  - $\sim 10^{-8} \text{ sec}$
  - recoil frequency
  - $\frac{\hbar}{2m} \sim 10^{-9} \text{ sec}$

- Quasi-classical motion of center of mass
  $\Rightarrow$ Askin; Gordon, Cohen-Tannoudji ...

Fokker-Planck eqn. for atomic CM motion + master eqn. for atomic dipole + cavity field

$\Rightarrow$ external

Effective potential

$$ U(\vec{r}) = -\int \langle F(\vec{r}) \rangle \cdot d\vec{r} $$

Conservative

$$ F(\vec{r}) = -\hbar \omega_0 \nabla q(\vec{r}) \left[ \hat{a}^\dagger \hat{a} + \hat{a} \hat{a}^\dagger \right] $$
The Marriage of Laser Cooling and Trapping with Cavity QED

• Goal: localized atoms in a regime of strong coupling

• Avenue: integration of laser cooling and trapping with cavity QED
Single Atoms Bound in Orbit with Single Photons

Kevin Birnbaum
Theresa Lynn  Christina Hood

Joe Buck
EXPERIMENT - Single-atom trapping with $\bar{n} \sim 1$ photon
C. J. Hood and T. L. Lynn

\[ H = \begin{cases} \frac{\alpha}{\hbar} = 110 \text{ MHz} \\ \frac{\Delta}{\hbar} = 220 \text{ MHz} \\ \frac{\omega_{\text{atom}}}{\hbar} = 26 \text{ MHz} \\ \frac{\omega_{\text{probe}}}{\hbar} = 37 \text{ MHz} \end{cases} \]

(a)

(b)

\[ \bar{m} \]

\[ \text{time (ms)} \]
Orbital Period as a Function of Amplitude of Oscillation

Cavity Transmission

Time, msec

Transmission with no atom

$\bar{n} - 1$ photon

$T_1, T_2$

$A_1, A_2$

Trigger

$\omega_{probe}, \omega_{cavity}, \omega_{atom}$

40MHz, 100MHz
Anharmonic Trapping Potential for $\bar{n} \sim 1$ photon and $N = 1$ atom

$T(\mu s)$

Experiment

Pseudo-potential $U(\rho)$ from quantum master equation for experimental $\bar{n}$, $\Delta_{AC}$, $\Delta_{p}$

$T(\mu s)$

Simulation

Radius $\rho$

2.6 mK
9.3 $\mu$m
- Atoms localized at peak of a single standing-wave antinode.
- Heating in this dimension leads to escape.
- Orbital periods separate by angular momentum. **Conservative** radial motion dominates diffusion and standing-wave motion.
Cavity QED versus "Traditional" Free Space Trapping

Doherty, Lynn, Hood, HSK,
EBA (2009)
quant-ph/0006015

Cavity QED -
Pseudo-potential $U$
Diffusion $D$

Free Space -
Pseudo-potential $V$
Diffusion $D$

"Standard" Semiclassical Theory
Optical Information - Characterize ability to sense atomic motion within the cavity

\[ I = \frac{\text{Optical Information}}{\text{atom}} = R \Delta t \]
\[ I \sim \frac{g^2 \Delta t}{\kappa} \]

\[ \approx 1 \text{ } \mu \text{wave cavity QED} \]
\[ \approx 10^6 - 10^7 \text{ current work} \]

More generally, compare to fluorescence from single atom in free space

\[ \gamma \sim 10^7/s \longleftrightarrow R \sim 10^9/s \]

In principle enhancement in ability to extract information via cavity QED
Reconstruction of Single-Atom Trajectories

- Quantum master equation:
  - Photocurrent $i(t)$ to Intracavity photon number $n(t)$ to Coupling coefficient $g(\rho(t))$ to Radial atomic position $\rho(t)$

- Position $\rho(t)$ and potential $U(\rho(t))$ allow reconstruction of atomic trajectory $\{\rho(t), \theta(t)\}$

$$y = \rho \cos(\theta)$$
$$z = \rho \sin(\theta)$$
Laboratory Reconstruction of Single-Atom Trajectory

One atom bound in orbit with $\bar{n} = 1$ photon

http://www.its.caltech.edu/~qoptics/atomorbits
Laboratory Reconstruction of Single-Atom Trajectory

One atom bound in orbit with $\bar{n} = 1$ photon

http://www.its.caltech.edu/~qoptics/atomorbits/
Validation of Reconstruction Algorithm

- Numerical Simulation via quantum master equation (A. Doherty, A. S. Parkins)
  - Fully quantum for internal degrees of freedom (atom dipole + cavity field)
  - Wave-packet dynamics of atom in quais-classical approximation
- "Actual" atomic trajectory
- Trajectory reconstructed from photocurrent (including quantum noise)
The Atom-Cavity Microscope (ACM) - A New Time-Resolved Microscopy near the Standard Quantum Limit

C. J. Hood, T. W. Lynn, A. Doherty, A. S. Parkins, and H. J. Kimble,
Science 287, 1447 (2000)

$\tau \approx 10^{-5} \text{ sec for } \text{SNR} \approx 1$

- Detection capability enhanced by $1/N_0 \gg 1$
- Measured sensitivity $\sim 20\text{nm/Hz}^{1/2}$ radial
- Inferred sensitivity $\sim 0.2\text{nm/Hz}^{1/2}$ axial

- Determine full quantum susceptibility near (and perhaps beyond) SQL
- Measure amplitude and phase of field

Quantum feedback control in cavity QED* - One and the same quantum system in real time

- Kalman Filter (observer) tracks in real time the radius, radial momentum, angular momentum, and angle.
- Make angle observable by adding non-symmetric laser mode.
- Feedback to strength of trapping potential and non-symmetric laser mode to control/cool atom.

Kalman State estimator: Implemented on a XILINX Virtex FPGA, for fast reaction time.

*R. Legere and H. Mabuchi
Quantum Feedback for Real-Time Control of Atomic Motion

Modulate

Cs

detector

i(t)
Photocurrent

DSP

Angular Momentum

time (μs)

Radial Coordinate

time (μs)

Using small 1,0 mode for feedback to:
• Track absolute angle from noisy measurements of I(ρ)
• Damp angular momentum

Kalman Track

Simulated Trajectory
Trapping Single Atoms in Cavity QED with a FORT

Dan Stamper-Kurn

Jason McKeever

Christoph Naegerl

Jun Ye (JILA)

David Vernooy
upstairs chamber: collection MOT, 10^{-8} torr

UHV downstairs chamber: 10^{-10} torr
(order of minutes for collision lifetime)

- Cesium atoms (∼10^4) in a magneto-optical trap MOT
- Polarization-gradient cooling to 2µK
- Drop
Real-Time Tracking and Trapping of Single Atoms in Cavity QED

CQED probe ➔ FORT beam

Probe transmission

13 msec

FORT remains on

Turn off FORT to probe

FORT TRIGGERed on by individual atomic transit

Turn off FORT to probe
Lifetime for Single Trapped Atoms in Cavity QED

DATA
FIT - Trap lifetime (1/e) 28.5 ms
THEORY - C. Gardiner

Limits?
- Current - fluctuations in FORT beam
  C. Gardiner (1999)
- Ultimate - $10^2$ sec set by background gas
What are the eigenstates?

Carmichael et. al.: delocalized states over many standing waves
here: localized

Well-dressed states for wave-packet dynamics in cavity QED
- See also Scully, W. and 
  et al. ...
Cavity QED in a FORT - Real-Time Tracking and Cooling

CQED probe

FORT beam

\[
\begin{align*}
l_e, 0 > & \\
l_g, 1 > & \\
l_g, 0 > & \\
\end{align*}
\]

\[
\begin{align*}
l_e, n > & \\
l_g, n + 1 > & \\
l_e, n - 1 > & \\
l_g, n > & \\
\end{align*}
\]

C. Naegerl, D. Vernooy, J. Ye

S. van Enk

• Select an axial well via probe excitation

FORT ON

Transmission

1 atom → 17 ms

Time after FORT turned on [ms]
New Avenues for Quantum-State Synthesis and Measurement -
Quantum State Exchange between Motion and Light*

- Spectacular advances in the manipulation of the quantum states of motion for a single bound atom (ion)
  - Monroe, Wineland et al., NIST; Blatt et al., Innsbruck; …

- Utilize with quantum-state exchange
  - New sources for manifestly quantum light
  - New possibilities for quantum measurement

\[
\text{Atomic motion} \quad (q, p)
\]

\[
\text{Light} \quad (Q, P)
\]

- Manipulate atomic wave packets with light
  - Cooling of atomic motion (\textit{the vacuum-state of light is easy to make!})
  - EPR \((q, p)\) at a distance
  - Teleportation of atomic wave packets
  - Cavity QED + trapped ions - Blatt et al.; H. Walther et al. - ion-trap laser

Teleporting the Wave Function of a Massive Particle

*EPR Light Source* → *Bob* → *Bell-state analysis*

*Entangled beams* → *Alice* → *Victor*

*Classical Information $\alpha$* → *D($\alpha$) → Victor*

*A. S. Parkins and HJK, quant-ph/-9909021.*
QUANTUM STATE TRANSFER*

Combine standing and flying qubits to realize quantum networks for distributed quantum computation and communication.