Lecture Notes before Annotations

Lecture Notes after Annotations

\[ \sigma^2 \leftarrow s^2 \]

\[ \times \quad \text{s.d.}(x) = \frac{s}{n} \]

Estimation of \( \sigma^2 \)

\[ s^2 = \frac{\sum_{i=1}^{n} e_i^2}{n-2} = \frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{n-2} = \frac{\sum_{i=1}^{n} (y_i - [\hat{\beta}_0 + \hat{\beta}_1 x_i])^2}{n-2} \]

This estimate has \( n-2 \) degrees of freedom because two unknown parameters are estimated.

\[ y = b_0 + b_1 x \]

\( \mu_x, \sigma \)

\( \sigma \) does not depend on \( x \)

\[ y = \beta_0 + \beta_1 x \]

\[ H_0: \text{a curve is linear} \]

\[ \beta_1 = 0 \]
If \( P \)-value < 0.05 then your chances of rejecting a true null hypothesis is less than 0.05

A Type I error occurs when one rejects a true null hypothesis.
A Type II error occurs when one does not reject a false null hypothesis.

The \( p \)-value is a measure of confidence in our rejection of the null hypothesis.

\[
H_0: \beta_1 = 0 \\
H_0: \beta_1 = 0 
\]

Test of Hypothesis for \( \beta_0 \) and \( \beta_1 \)
\[
H_0: \beta_1 = 0 \text{ vs. } H_1: \beta_1 \neq 0
\]
Reject \( H_0 \) at \( \alpha \) level if \( |t| = \left| \frac{\hat{\beta}_1 - 0}{SE(\hat{\beta}_1)} \right| > t_{\alpha/2}
\]

Tire example:
\[
\left( \frac{\hat{\beta}_1}{SE(\hat{\beta}_1)} \right) = \left( \frac{-7.28}{0.614} \right) = -11.86, \quad t_{0.025} = 2.365
\]

Strong evidence that mileage affects tread depth.
(Two-sided p-value)

\[
\alpha = 0.05, 0.01, 0.10
\]

Reject if \( P \)-value \( \leq \alpha \)

\[
M_y = \beta_0 = \text{constant}
\]

If \( P \)-value < 0.05 then your chances of rejecting a true null hypothesis is less than 0.05

\[
\begin{array}{c}
\frac{t}{\overline{50}} \quad \frac{t_0}{\overline{50}}
\end{array}
\]

Analysis of Variance
\[
H_0: \beta_1 = 0 \text{ vs. } H_0: \beta_1 \neq 0
\]
\[
\nu = \frac{\text{MSR}}{	ext{MSE}}
\]

The Mean Square (MS) is the Sums of Square divided by its degrees of freedom, e.g.
\[
\text{MSE} = \text{SSE/(df = 2531.529/7 = 361.6}
\]
\[ H_0 : \beta_1 = 0 \text{ vs. } H_0 : \beta_1 \neq 0 \]

\[ F = \frac{\text{MSR}}{\text{MSE}} \]

\[ \frac{5087.2}{361.6} = 140.71 \]

\[ F = 140.70 \]

\[ (-11.8621) \]

\[ r^2 = 1 \]

\[ m_y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 \]

\[ H_0 : \beta_1 = 0 \text{ vs. } H_a : \beta_1 \neq 0 \]

ANOVA Test

\[ H_0 : \beta_1 = \beta_2 = 0 \]

**Prediction of a Future y or Its Mean**

For a fixed value of \( x \), are we trying to predict
- the average value of \( y \)?
- the value of a future observation of \( y \)?

**Example:**

- Do I want to predict the average selling price of all 4,000 square feet houses in my neighborhood.

- Or do I want to predict the particular future selling price of my 4,000 square feet house?

**Which prediction is subject to the most error?**
Prediction of a Future $y$ or Its Mean: Prediction Interval or Confidence Interval

For a fixed value of $x^*$, are we trying to predict
-the average value of $y$?
-the value of a future observation of $y$?

\[
\hat{y} = 4 + 5 \cdot x
\]
\[
x = 400 \Rightarrow \\
\mu_y = 4 + 5(400) = 2004 \\
\mu_\hat{y} = 4 + 5(400) = 2004
\]
JMP: Prediction of the Mean of $y$

\[ y = b_0 + b_1 x \]

JMP: Prediction of a Future Value of $y$
Formulas for Confidence and Prediction Intervals

A 100(1 - α)% CI for $\mu^*$ is given by

$$\hat{\mu}^* - t_{n-1,\alpha/2} \sqrt{\frac{1}{n} + \frac{(x^* - \bar{x})^2}{S_{xx}}} \leq \mu^* \leq \hat{\mu}^* + t_{n-1,\alpha/2} \sqrt{\frac{1}{n} + \frac{(x^* - \bar{x})^2}{S_{xx}}}$$  \hspace{1cm} (10.18)

where $\hat{\mu}^* = \hat{\beta}_0 + \hat{\beta}_1 x^*$ and $\bar{x} = \sqrt{\text{MSE}}$ is the estimate of $\sigma$.

A 100(1 - α)% PI for $Y^*$ is given by

$$\hat{Y}^* - t_{n-1,\alpha/2} \sqrt{\frac{1}{n} + \frac{(x^* - \bar{x})^2}{S_{xx}}} \leq Y^* \leq \hat{Y}^* + t_{n-1,\alpha/2} \sqrt{\frac{1}{n} + \frac{(x^* - \bar{x})^2}{S_{xx}}}$$  \hspace{1cm} (10.19)

where $\hat{Y}^* = \hat{\beta}_0 + \hat{\beta}_1 x^*$.

Prediction Interval of Chapter 7:

$100(1 - \alpha)%$ PI for a future observation $X \sim N(\mu, \sigma^2)$ is given by

$$\bar{x} - t_{n-1,\alpha/2} \sqrt{\frac{1}{n} + \frac{1}{n}} \leq X \leq \bar{x} + t_{n-1,\alpha/2} \sqrt{\frac{1}{n} + \frac{1}{n}}$$
Confidence and Prediction Intervals with JMP
### CI for Mean for \( \mu^* \)

<table>
<thead>
<tr>
<th>Mileage (in 1000 Miles)</th>
<th>Grove Depth (in mile)</th>
<th>Predicted Grove Depth (in mile)</th>
<th>Lower 95% Mean Grove Depth (in mile)</th>
<th>Upper 95% Mean Grove Depth (in mile)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>394.33</td>
<td>332.997636</td>
<td>398.296983</td>
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<tr>
<td>2</td>
<td>4</td>
<td>329.5</td>
<td>331.514187</td>
<td>339.535999</td>
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<td>267.194859</td>
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<td>28</td>
<td>183.81</td>
<td>158.779167</td>
<td>133.800899</td>
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<td>32</td>
<td>150.33</td>
<td>127.569687</td>
<td>108.917836</td>
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<tr>
<td>10</td>
<td>25</td>
<td>*</td>
<td>*(179.621042)</td>
<td>*(198.738823)</td>
</tr>
</tbody>
</table>

Date: 6/20/2012

Unit 10 - Stat 571 - Ranaei V. Leva
### Prediction Interval for $Y^*$

<table>
<thead>
<tr>
<th>Miles (in 1000)</th>
<th>Grove Depth (in mts)</th>
<th>Predicted Grove Depth (in mts)</th>
<th>Lower 95% Indiv Grove Depth (in mts)</th>
<th>Upper 95% Indiv Grove Depth (in mts)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>394.33</td>
<td>360.639667</td>
<td>413.419682</td>
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<td>4</td>
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<td>204.83</td>
<td>215.024167</td>
<td>167.269439</td>
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<tr>
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<td>179</td>
<td>185.901667</td>
<td>137.099817</td>
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<td>8</td>
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<td>163.83</td>
<td>156.779167</td>
<td>106.28034</td>
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<td>127.656667</td>
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<td>10</td>
<td>25</td>
<td>*</td>
<td>(175.621042)</td>
<td>(&lt;227.7884)</td>
</tr>
</tbody>
</table>
Prediction for the Mean of Y or a Future Observation of Y

- Point estimate prediction is the same in both cases: 178.62
- But the error bands are different
  - Narrower for the mean of Y: [158.73, 198.51]
  - Wider for a future value of Y: [129.44, 227.80]