Unit 7: Inference for Single Samples

Statistics 571: Statistical Methods
Ramón V. León
Inference About the Mean and Variance of a Normal Population

Applications:

• Monitor the mean of a manufacturing process to determine if the process is under control
• Evaluate the precision of a laboratory instrument measured by the variance of its readings
• Prediction intervals and tolerance intervals which are methods for estimating future observations from a population.

By using the central limit theorem (CLT), inference procedures for the mean of a normal population can be extended to the mean of a non-normal population when a large sample is available.
Inferences on Mean (Large Samples)

- Inferences on \( \mu \) will be based on the sample mean \( \bar{X} \), which is an unbiased estimator of \( \mu \) with variance \( \frac{\sigma^2}{n} \).
- For large sample size \( n \), the CLT tells us that \( \bar{X} \) is approximately \( N(\mu, \sigma^2/n) \) distributed, even if the population is not normal.
- Also for large \( n \), the sample variance \( s^2 \) may be taken as an accurate estimator of \( \sigma^2 \) with negligible sampling error. If \( n \geq 30 \), we may assume that \( \sigma = s \) in the formulas.
Confidence Intervals on the Mean:
Large Samples

$$P \left[ -z_{\alpha/2} \leq Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \leq z_{\alpha/2} \right] = 1 - \alpha$$
Confidence Intervals on the Mean

\[
\bar{x} - z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + z_{\alpha/2} \frac{\sigma}{\sqrt{n}}
\]

\[
\bar{x} - z_{\alpha} \frac{\sigma}{\sqrt{n}} \leq \mu \quad \text{(Lower One-Sided CI)}
\]

\[
\mu \leq \bar{x} + z_{\alpha} \frac{\sigma}{\sqrt{n}} \quad \text{(Upper One-Sided CI)}
\]

\[
\frac{\sigma}{\sqrt{n}} \quad \text{is the standard error of the mean}
\]
Sample Size Determination for a $z$-interval

• Suppose that we require a $(1-\alpha)$-level two-sided CI for $\mu$ of the form $[\bar{x} - E, \bar{x} + E]$ with a margin of error $E$.

• Set $E = z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$ and solve for $n$, obtaining $n = \left[ \frac{z_{\alpha/2} \sigma}{E} \right]^2$.

• Calculation is done at the design stage so a sample estimate of $\sigma$ is not available.
• An estimate for $\sigma$ can be obtained by anticipating the range of the observations and dividing by 4.

  Based on assuming normality since then 95% of the observation are expected to fall in $[\mu - 2\sigma, \mu + 2\sigma]$
Example 7.1 (Airline Revenue)

- Airline wants to estimate mean share of revenue per ticket within $5 using a 99% confidence interval.
- Anticipated range of the data is $200.

Anticipated range of $200 divided by 4

$$n = \left[ \frac{z_{\alpha/2} \sigma}{E} \right]^2 = \left[ \frac{z_{0.01/2} \sigma}{E} \right]^2 = \left[ \frac{2.576 \times 50}{5} \right]^2 = 664$$
Example 7.2

Test $H_0 : \mu = 42,000$ vs. $H_1 : \mu \neq 42,000$ at 10% level of significance

Can test three ways:

1. Calculate z-statistics:

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{42196 - 42000}{500/\sqrt{32}} = 2.217$$

Reject if $|z| > z_{.05} = 1.645$: Reject
Example 7.2 (Continued)

2. Calculate the P-value:

\[
P\left(\left|z\right| \geq \frac{42196 - 42000}{500/\sqrt{32}}\right) = P\left(\left|z\right| \geq 2.217\right) = 2 \left[1 - \Phi(2.217)\right] = 2 \left[1 - 0.987\right] = 0.026
\]

Reject if P-value is less than 0.1: Reject
Example 7.2 (Continued)

3. Calculate a 90% CI for $\mu$

$$
\left[ \bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \right] = \left[ 42196 \pm 1.645 \frac{500}{\sqrt{32}} \right] = [42051, 42341]
$$

Reject if $\mu = 42,000$ does not fall in the CI: Reject

Which way do you prefer?
Power Calculation for One-sided Z-tests

\[ \pi(\mu) = P[\text{Test rejects } H_0 \mid \mu] \]

Consider the problem of testing \( H_0: \mu \leq \mu_0 \) vs. \( H_1: \mu > \mu_0 \). The power function of the \( \alpha \)-level upper one-sided \( z \)-test for this problem can be derived as follows:

\[
\pi(\mu) = P \left( \bar{X} > \mu_0 + z_\alpha \frac{\sigma}{\sqrt{n}} \mid \mu \right)
\]

\[
= P \left( Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} > z_\alpha + \frac{\mu_0 - \mu}{\sigma/\sqrt{n}} \mid \mu \right)
\]

\[
= 1 - \Phi \left[ z_\alpha + \frac{(\mu_0 - \mu)\sqrt{n}}{\sigma} \right]
\]

\[
= \Phi \left[ -z_\alpha + \frac{(\mu - \mu_0)\sqrt{n}}{\sigma} \right].
\] (7.7)
Power Calculation for One-sided Z-tests

\[ \mu_0 + z_\alpha \sigma \sqrt{n} \]

Accept \( H_0 \)

Reject \( H_0 \)

\[ \alpha = \pi(\mu_0) \]

Power \( \pi(\mu_1) \)

\[ \bar{X} \sim N \left( \mu, \frac{\sigma^2}{n} \right) \]
Power Functions Curves

Probability of Rejection is even smaller for $\mu < \mu_0$ so simple null hypothesis $H_0: \mu = \mu_0$ can be replaced by composite null hypothesis $H_0: \mu \leq \mu_0$

It is easier to detect a big difference from $\mu_0$.

**Figure 7.2** Power Function Curves for the Upper One-Sided $\alpha$-Level $z$-Test with Two Different Sample Sizes, $n$ and $n' > n$
Example 7.3 (SAT Couching: Power Calculation)

\[ H_0 : \mu \leq 15 \text{ vs. } H_1 : \mu \geq 15 \quad (0.05 - \text{level test}) \]

\[
\pi(\mu) = \Phi \left[ -z_\alpha + \frac{(\mu - \mu_0) \sqrt{n}}{\sigma} \right]
\]

\[
\pi(30) = \Phi \left[ -1.645 + \frac{(30 - 15) \sqrt{20}}{40} \right] = \Phi(0.032) = 0.513
\]

Probability of detecting a change of 30 points after couching

6/24/2003   Unit 7 - Stat 571 - Ramón V. León
Power Calculation Two-Sided Test

Read on your own the derivation on pages 244-245 and Example 7.4
Power Curve for Two-sided Test

It is easier to detect large differences from the null hypothesis.

Larger samples lead to more powerful tests.

Figure 7.4  Power Function Curves for the $\alpha$-Level Two-Sided $z$-Test with Two Different Sample Sizes, $n$ and $n' > n$
Sample Size Determination for a One-Sided $z$-Test

• Determine the sample size so that a study will have sufficient power to detect an effect of practically important magnitude

• If the goal of the study is to show that the mean response $\mu$ under a treatment is higher than the mean response $\mu_0$ without the treatment, then $\mu - \mu_0$ is called the treatment effect

• Let $\delta > 0$ denote a practically important treatment effect and let $1 - \beta$ denote the minimum power required to detect it. The goal is to find the minimum sample size $n$ which would guarantee that an $\alpha$-level test of $H_0$ has at least $1 - \beta$ power to reject $H_0$ when the treatment effect is at least $\delta$. 
Sample Size Determination for a One-sided Z-test

Because Power is an increasing function of \( \mu - \mu_0 \), it is only necessary to find \( n \) that makes the power \( 1 - \beta \) at \( \mu = \mu_0 + \delta \).

\[
\pi(\mu_0 + \delta) = \Phi\left(-z_{\alpha} + \frac{\delta \sqrt{n}}{\sigma}\right) = 1 - \beta \quad \text{[See Equation (7.7), Slide 11]}
\]

Since \( \Phi(z_{\beta}) = 1 - \beta \) we have \( -z_{\alpha} + \frac{\delta \sqrt{n}}{\sigma} = z_{\beta} \).

Solving for \( n \), we obtain

\[
n = \left[\frac{(z_{\alpha} + z_{\beta}) \sigma}{\delta}\right]^2
\]
Example 7.5 (SAT Coaching: Sample Size Determination)

Refer to Example 7.3. Calculate the number of students that must be tested in order to have at least 90% power for detecting an increase of 30 points or more in the mean SAT-V scores due to the coaching program.

We have $\alpha = 0.05$, $z_\alpha = 1.645$, $\beta = 0.10$, $z_\beta = 1.282$, $\sigma = 40$, and $\delta = \mu - \mu_0 = 30 - 15 = 15$. Substituting these values in (7.10), we obtain

$$n = \left[ \frac{(1.645 + 1.282)40}{15} \right]^2 = 60.92 \text{ or } 61.$$ 

Therefore at least 61 students must be tested. 

◆
Sample Size Determination for a Two-Sided z-Test

\[ n = \left[ \frac{\left( z_{\alpha/2} + z_\beta \right) \sigma}{\delta} \right]^2 \]

Read on you own the derivation on pages 248-249

**Example 7.6 (Control Chart: Sample Size Determination)**

In Example 7.4 we found that using a sample size of 5 to monitor the cereal filling process gave only 22% power to detect a shift of one standard deviation (0.1 oz.) from the target mean weight of 16 oz. What is the minimum sample size required to guarantee at least 75% power?

We have \( \alpha = 0.0027, \ z_{\alpha/2} = z_{0.0135} = 3.0, \ 1 - \beta = 0.75, \ z_\beta = z_{0.25} = 0.675, \) and \( \delta/\sigma = 1.0. \) Substituting these values in (7.11), we obtain

\[ n = \left[ \frac{(3.0 + 0.675)}{1} \right]^2 = 13.51 \ 	ext{or} \ 14. \]

Read on your own Example 7.4
Inference on Mean (Small Samples)

The sampling variability of \( s^2 \) may be sizable if the sample is small (less than 30). Inference methods must take this variability into account when \( \sigma^2 \) is unknown.

Assume that \( X_1, \ldots, X_n \) is a random sample from an \( N(\mu, \sigma^2) \) distribution. Then \( T = \frac{\bar{X} - \mu}{S/\sqrt{n}} \) has a \( t \)-distribution with \( n - 1 \) degrees of freedom (d.f.)
Confidence Intervals on Mean

\[
1 - \alpha = P \left[ -t_{n-1,\alpha/2} \leq T = \frac{\bar{X} - \mu}{S/\sqrt{n}} \leq t_{n-1,\alpha/2} \right] \\
= P \left[ \bar{X} - t_{n-1,\alpha/2} \frac{S}{\sqrt{n}} \leq \mu \leq \bar{X} + t_{n-1,\alpha/2} \frac{S}{\sqrt{n}} \right]
\]

\[
\bar{X} - t_{n-1,\alpha/2} \frac{S}{\sqrt{n}} \leq \mu \leq \bar{X} + t_{n-1,\alpha/2} \frac{S}{\sqrt{n}} \] [Two-Sided 100(1-\alpha)% CI]

\[
t_{n-1,\alpha/2} > z_{\alpha/2} \Rightarrow t - \text{interval is wider on the average than } z\text{-interval}
\]
Example 7.7

Table 7.4 gives 29 measurements of the density of earth (expressed as multiples of the density of water, i.e., in grams/cc) made in 1798 by the British scientist Henry Cavendish. Estimate the density of earth from these measurements using a 95% CI.

<p>| | | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>5.50</td>
<td>5.61</td>
<td>4.88</td>
<td>5.07</td>
<td>5.26</td>
<td>5.55</td>
<td>5.36</td>
<td>5.29</td>
<td>5.58</td>
</tr>
<tr>
<td>5.57</td>
<td>5.53</td>
<td>5.62</td>
<td>5.29</td>
<td>5.44</td>
<td>5.34</td>
<td>5.79</td>
<td>5.10</td>
<td>5.27</td>
</tr>
<tr>
<td>5.42</td>
<td>5.47</td>
<td>5.63</td>
<td>5.34</td>
<td>5.46</td>
<td>5.30</td>
<td>5.75</td>
<td>5.86</td>
<td>5.85</td>
</tr>
</tbody>
</table>
Example 7.7 (Continue)

The box plot and normal plot for these data are shown in Figure 7.5. Both plots are satisfactory and no outliers are indicated. Therefore we proceed with formal calculations. For these data, the sample mean $\bar{x} = 5.448$, the sample standard deviation $s = 0.221$, and hence the SEM, $s/\sqrt{n} = 0.221/\sqrt{29} = 0.041$. The d.f. are $29 - 1 = 28$, and the required critical point is $t_{28,.025} = 2.048$ from Table A.4. The 95% CI is calculated to be

$$[5.448 - 2.048 \times 0.041, 5.420 + 2.048 \times 0.041] = [5.363, 5.532].$$
Example 7.7 in JMP

Note: JMP file available for download at the course home page.

JMP Distribution Platform

Confidence Intervals

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Lower CI</th>
<th>Upper CI</th>
<th>1-Alpha</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>5.454138</td>
<td>5.366636</td>
<td>5.54164</td>
<td>0.950</td>
</tr>
<tr>
<td>Std Dev</td>
<td>0.230039</td>
<td>0.182554</td>
<td>0.311117</td>
<td></td>
</tr>
</tbody>
</table>
Testing for Normality in JMP

JMP Distribution Platform

- Density Options:
  - Display Options
  - Histogram Options
  - Normal Quantile Plot
  - Outlier Box Plot
  - Quantile Box Plot
  - Stem and Leaf
  - CDF Plot
  - Test Mean
  - Test Std Dev
  - Confidence Interval

- Fit Distribution:
  - Normal
  - LogNormal

- Parameter Estimates:
  - Type: Location Mu
  - Estimate: 5.454138
  - Location Sigma
  - Estimate: 0.230039
  - Dispersion Sigma
  - Estimate: 0.182554

- Goodness-of-Fit Test:
  - Shapiro-Wilk Test
  - VV: 0.977696
  - Prob=VV: 0.793
Example 7.8 (Tear Strength of Rubber)

The following are the measurements made on the tear strengths of 16 sample sheets of a silicone rubber used in a high voltage transformer. Calculate a 95% lower confidence limit on the mean tear strength $\mu$.

Tear Strength in PSI

<p>| | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>33.74</td>
<td>34.40</td>
<td>32.62</td>
<td>32.57</td>
<td>34.69</td>
<td>33.78</td>
<td>36.76</td>
<td>34.31</td>
</tr>
<tr>
<td>37.61</td>
<td>33.78</td>
<td>35.43</td>
<td>33.22</td>
<td>33.53</td>
<td>33.68</td>
<td>33.24</td>
<td>32.98</td>
</tr>
</tbody>
</table>

(a)

(b)
Example 7.8  (Omitting Outliers)

Figure 7.7  Box Plot and Normal Plot for Tear Strength Data after Omitting Outliers: (a) Box plot omitting outliers \( n = 14 \); (b) Normal plot omitting outliers \( n = 14 \);

The summary statistics for the data after omitting the two outliers are \( \bar{x} = 33.712 \), \( s = 0.798 \), and \( n = 14 \). Therefore a lower 95\% confidence limit calculated using \( t_{13,0.05} = 1.771 \) is

\[
\mu \geq 33.712 - 1.771 \frac{0.798}{\sqrt{14}} = 33.334.
\]
Example 7.8 in JMP

To get a one-sided confidence bound get a confidence interval with twice the $\alpha$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Lower CI</th>
<th>Upper CI</th>
<th>1-Alpha</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>33.7121</td>
<td>33.3344</td>
<td>34.0898</td>
<td>0.90</td>
</tr>
<tr>
<td>Std Dev</td>
<td>0.7980</td>
<td>0.6085</td>
<td>1.1855</td>
<td></td>
</tr>
</tbody>
</table>

Outliers excluded using the option in the **Rows** menu.
### T-test for the Mean

**Table 7.5** Level $\alpha$ Tests on $\mu$ When $\sigma^2$ Is Unknown (Test Statistic: $t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$)

<table>
<thead>
<tr>
<th>Testing Problem</th>
<th>Hypotheses</th>
<th>Reject $H_0$ if</th>
<th>$P$-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper One-Sided</td>
<td>$H_0: \mu \leq \mu_0$</td>
<td>$t &gt; t_{n-1,\alpha}$ (or $\bar{x} &gt; \mu_0 + t_{n-1,\alpha} \frac{s}{\sqrt{n}}$)</td>
<td>$P(T_{n-1} \geq t)$</td>
</tr>
<tr>
<td>vs.</td>
<td>$H_1: \mu &gt; \mu_0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lower One-Sided</td>
<td>$H_0: \mu \geq \mu_0$</td>
<td>$t &lt; -t_{n-1,\alpha}$ (or $\bar{x} &lt; \mu_0 - t_{n-1,\alpha} \frac{s}{\sqrt{n}}$)</td>
<td>$P(T_{n-1} \leq t)$</td>
</tr>
<tr>
<td>vs.</td>
<td>$H_1: \mu &lt; \mu_0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Two-Sided</td>
<td>$H_0: \mu = \mu_0$</td>
<td>$</td>
<td>t</td>
</tr>
<tr>
<td>vs.</td>
<td>$H_1: \mu \neq \mu_0$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Example 7.9 (Tear Strength of Rubber)

After omitting the outliers, test whether the mean tear strength exceeds a lower specification limit of 33 psi.

\[ H_0 : \mu \leq 33 \text{ vs. } H_1 : \mu > 33 \]

95\% lower confidence bound \( \mu \geq 33.334 \) so \( H_0 \) is rejected at the \( \alpha = 0.05 \) level

Another way:

\[ t = \frac{33.712 - 33}{0.798/\sqrt{14}} = 3.343 > 1.771 = t_{13,.05} \text{ so we reject } H_0 \]

Using t table can only say P-value is between .0025 and .005
Example 7.9 in JMP

Exact P-value
JMP Power Animation (Calculation)

Calculation based on the Noncentral t-Distribution as discussed on Pages 253 and 254 of your textbook

Alternative value can be changed
Inference on Variance

Assume that $X_1, \ldots, X_n$ is a random sample from an $N(\mu, \sigma^2)$ distribution

$$\chi^2 = \frac{(n-1)S^2}{\sigma^2}$$ has a Chi-square distribution with $n - 1$ d.f.

![Graph showing lower and upper critical points of a chi-square distribution](image)

**Figure 7.8** Lower and Upper $\alpha/2$ Critical Points of a $\chi^2_{n-1}$ Distribution

$$1 - \alpha = P \left[ \chi^2_{n-1, 1-\alpha/2} \leq \frac{(n-1)S^2}{\sigma^2} \leq \chi^2_{n-1, \alpha/2} \right]$$
CI for $\sigma^2$ and $\sigma$

$$1 - \alpha = P \left[ \chi^2_{n-1, 1-\frac{\alpha}{2}} \leq \frac{(n-1)S^2}{\sigma^2} \leq \chi^2_{n-1, \frac{\alpha}{2}} \right]$$

$$= P \left[ \frac{(n-1)S^2}{\chi^2_{n-1, \frac{\alpha}{2}}} \leq \sigma^2 \leq \frac{(n-1)S^2}{\chi^2_{n-1, 1-\frac{\alpha}{2}}} \right].$$

It follows that the $100(1 - \alpha)\%$ CI for $\sigma^2$ has the form:

$$\frac{(n-1)s^2}{\chi^2_{n-1, \frac{\alpha}{2}}} \leq \sigma^2 \leq \frac{(n-1)s^2}{\chi^2_{n-1, 1-\frac{\alpha}{2}}} \quad \text{(Two-Sided CI),} \quad \text{(7.17)}$$

where $s^2$ is the observed value of the sample variance. A $100(1 - \alpha)\%$ CI for $\sigma$ is

$$s \sqrt{\frac{n-1}{\chi^2_{n-1, \frac{\alpha}{2}}}} \leq \sigma \leq s \sqrt{\frac{n-1}{\chi^2_{n-1, 1-\frac{\alpha}{2}}}} \quad \text{(Two-Sided CI).} \quad \text{(7.18)}$$
Confidence Intervals for \( \sigma \)

**Example 7.10** *(Thermometer Precision: Confidence Interval)*

Refer to Example 5.5 concerning verification of the advertised precision of \( \sigma = 0.01^\circ \text{F} \) for a thermometer. Verification is done by taking 10 repeat measurements of a controlled temperature bath. Suppose that the sample variance of these 10 measurements is \( s^2 = 2.0 \times 10^{-4} \). Then a 90% CI for \( \sigma \) is

\[
\left[ \sqrt{\frac{9s^2}{X^2_{9,.05}}} , \sqrt{\frac{9s^2}{X^2_{9,.95}}} \right] = \left[ \sqrt{\frac{9 \times 2 \times 10^{-4}}{16.92}} , \sqrt{\frac{9 \times 2 \times 10^{-4}}{3.325}} \right] = [0.0103, 0.0233].
\]

The claimed value, \( \sigma = 0.01 \), does not lie in this interval. Hence the data do not support the claim at the 10% level of significance. Since the lower confidence limit is greater than 0.01, we conclude that \( \sigma > 0.01 \).

See Slide 29

90% CI for \( \sigma \) using the tear strength data of Example 7.8 with outliers removed
Hypothesis Test on Variance

In the previous example, a CI was used to test hypotheses of the form
\[ H_0: \sigma^2 = \sigma_0^2 \text{ vs. } H_1: \sigma^2 \neq \sigma_0^2. \]
The same test can be performed directly using the **chi-square statistic**:
\[ \chi^2 = \frac{(n - 1)s^2}{\sigma_0^2}. \]  \hspace{1cm} (7.21)

When \( H_0 \) is true, this statistic has a \( \chi^2_{n-1} \) distribution. This leads to the following \( \alpha \)-level two-sided test:

Reject \( H_0 \) if \( \chi^2 > \chi^2_{n-1, \alpha/2} \) or if \( \chi^2 < \chi^2_{n-1, 1-\alpha/2} \).

**One side tests are similar**
JMP Test for $\sigma$

Tear Strength data of Example 7.8 with outliers omitted
Prediction Intervals

• Many practical applications call for an interval estimate of
  – an individual (future) observation sampled from a population
  – rather than of the mean of the population.

• An interval estimate for an individual observation is called a prediction interval

Prediction Interval Formula:

\[
\bar{x} - t_{n-1,\alpha/2} s \sqrt{1 + \frac{1}{n}} \leq X \leq \bar{x} + t_{n-1,\alpha/2} s \sqrt{1 + \frac{1}{n}}
\]
Confidence vs. Prediction Interval

Prediction interval of a single future observation:

\[
\bar{x} - t_{n-1,\alpha/2}s\sqrt{1 + \frac{1}{n}} \leq X \leq \bar{x} + t_{n-1,\alpha/2}s\sqrt{1 + \frac{1}{n}}
\]

As \( n \to \infty \) interval converges to \([\mu - z_{\alpha/2}\sigma, \mu + z_{\alpha/2}\sigma]\)

Confidence interval for \(\mu\):

\[
\bar{x} - t_{n-1,\alpha/2}s\sqrt{\frac{1}{n}} \leq \mu \leq \bar{x} + t_{n-1,\alpha/2}s\sqrt{\frac{1}{n}}
\]

As \( n \to \infty \) interval converges to single point \(\mu\)
Example 7.12: Tear Strength of Rubber

Suppose that the 14 measurements in Example 7.8 of the tear strength of rubber (after omitting the two outliers) are regarded as measurements of 14 successive batches.

Run chart shows process is predictable.
(Should really use a moving range and individual X control chart)
Example 7.12: Tear Strength of Rubber

95% Prediction interval for future batch:

\[ \bar{x} - t_{14-1,.025}s\sqrt{1 + \frac{1}{n}} \leq X \leq \bar{x} + t_{14-1,.025}s\sqrt{1 + \frac{1}{n}} \]

\[ 33.712 \pm 2.160 \times 0.798 \sqrt{1 + \frac{1}{14}} = [31.928, 35.496] \]

95% Confidence interval for the mean \( \mu \):

\[ \bar{x} - t_{14-1,.025}s\sqrt{\frac{1}{n}} \leq \mu \leq \bar{x} + t_{14-1,.025}s\sqrt{\frac{1}{n}} \]

\[ 33.712 \pm 2.160 \times 0.798 \sqrt{\frac{1}{14}} = [33.252, 34.172] \]
Prediction Intervals in JMP

Pop up menu of JMP’s **Fit Model** platform in the Analyze menu
Prediction Intervals in JMP
Tolerance Intervals

Suppose we want an interval which will contain at least .90 = 1 - \( \gamma \) of the strengths of the future batches (observations) with 95% = 1 - \( \alpha \) confidence.

Note that this statistical interval is even wider than the prediction interval.

\[
[\bar{x} - Ks, \bar{x} + Ks] = 33.712 \pm 2.529 \times 0.798 = [31.694, 35.730]
\]

JMP 5 has tolerance intervals per my suggestion.
Tolerance Intervals in JMP 5