Unit 13: Analysis of Multifactor Experiments

Analysis of Multifactor Experiments

<table>
<thead>
<tr>
<th>Table 13.1</th>
<th>Data from a Balanced Two-Way Layout</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factor A</td>
<td>Factor B Levels</td>
</tr>
<tr>
<td>Levels</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>$Y_{i11}, \ldots, Y_{i1s}$</td>
</tr>
<tr>
<td>2</td>
<td>$Y_{i1t1}, \ldots, Y_{i1ts}$</td>
</tr>
<tr>
<td>$\vdots$</td>
<td>$\vdots$</td>
</tr>
<tr>
<td>$i$</td>
<td>$Y_{11i}, \ldots, Y_{1is}$</td>
</tr>
<tr>
<td>$\vdots$</td>
<td>$\vdots$</td>
</tr>
<tr>
<td>$a$</td>
<td>$Y_{11a}, \ldots, Y_{1as}$</td>
</tr>
</tbody>
</table>

Balanced two-way layout:
- $n$ = replicates
- Two factors: So there are $ab$ treatment combinations
- Completely randomized design
Example

**Example 13.1 (Bonding Strength of Capacitors: Parameter Estimates)**

Capacitors are bonded to a circuit board used in high voltage electronic equipment. Engineers designed and carried out an experiment to study how the mechanical bonding strength of capacitors depends on the the type of substrate (factor A) and the bonding material (factor B). There were three types of substrates: aluminium oxide (Al₂O₃) with bracket, Al₂O₃ without bracket, and beryllium oxide (BeO) without bracket. Four types of bonding material were used: Epoxy I, Epoxy II, Solder I, and Solder II. Four capacitors were tested at each factor level combination. Simulated bonding strength data are given in Table 13.2. Calculate the estimates of the parameters of model (13.3) for these data.

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Model for Balanced Two-Way Layout

\[ Y_{ijk} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij} + \varepsilon_{ijk} = \mu_{ij} + \varepsilon_{ijk} \]

\((i = 1, \ldots, a; \ j = 1, \ldots, b; \ k = 1, \ldots, n)\)

\[ \sum_{i=1}^{a} \tau_i = 0, \sum_{j=1}^{b} \beta_j = 0, \]

\[ \sum_{i=1}^{a} (\tau\beta)_{ij} = 0 \quad \text{for all} \ j \]

\[ \sum_{j=1}^{b} (\tau\beta)_{ij} = 0 \quad \text{for all} \ i \]

\((\tau\beta)_{ij}\) is called the \((i, j)\)th row-column interaction

The \(\varepsilon_{ijk}\)'s are independent and identically distributed (i.i.d.) 
\(N(0, \sigma^2)\) random errors.
Interactions

\[ \mu_{ij} - \mu_{i\cdot} = \left( \mu + \tau_i + \beta_j + (\tau\beta)_{ij} \right) - \left( \mu + \tau_r + \beta_j + (\tau\beta)_{rj} \right) = (\tau_i - \tau_r) + ((\tau\beta)_{ij} - (\tau\beta)_{rj}) \] which depends on \( j \)

Similarly,

\[ \mu_{ij} - \mu_{\cdot j} = (\beta_j - \beta_j') + ((\tau\beta)_{ij} - (\tau\beta)_{ij'}) \] which depends on \( i \)

With no interaction one can understand the variation in \( y \) in terms of the A and B effects. With an interaction knowing the A and B effect is not enough.
The model (i.e. substrate, material, and their interaction) explains 93% of the variation in bonding strength.

Notice that the interaction is significant.
JMP 4 Analysis of Two-Way Layout

Here we can see graphically the nature of the interaction between substrate and material.

Residual Diagnostic Plots

The usual residual diagnostic plots can (should) be used with the two-way layout
Do You Need to Know More?

573 Design of Experiments (3) One-way ANOVA, multiple range tests, equal and unequal variances, transformations; factorial experiments, completely randomized designs, analysis of covariance, split-plot and nested designs, fractional factorials, sequential designs. Prereq: 571. Sp