1. Suppose that a population of units has a life distribution with a constant hazard rate equal to 5 (hr.)\(^{-1}\).
   
a. What is the population’s survivor function?

b. What is the population’s cumulative distribution function?

c. What is the population’s density function?

d. What is the population’s integrated hazard function?

e. What is the units’ mean time to failure?
2. Recall that the mean and variance of a Gumbel distribution with location parameter $\mu$ and scale parameter $\sigma$ are given by:

\[
\text{Mean} = \mu - \gamma \sigma \\
\text{Variance} = \left(\frac{\pi^2}{6}\right)\sigma^2
\]

where $\gamma = 0.5772$ is Euler’s constant and $\pi = 3.14$.

Suppose that you have an uncensored sample of Weibull data. Further suppose that the sample mean and variance of the log of these data are 10 and 4 respectively.

a. What are a method-of-moments estimates for the parameters, $\mu$ and $\sigma$, of the Gumbel distribution of the log data?

b. What are a method-of-moments estimates for the parameters, $\alpha$ and $\eta$, of the Weibull distribution of the data?

c. Do you think that the hazard rate of the distribution of the data is increasing, decreasing, or constant? Why?

d. Why did we assume above that the data was uncensored?
3. Suppose that you have \( n = 12 \) light bulbs with independent exponential distributions with hazard rates equal to 2. What is the distribution of the illumination time? What are the parameters of this distribution?

4. Suppose that you have the following failure times in hours:

\[ 1, 2, 4, 5, 6, 8, 12, 15, 18 \]

a. Sketch the empirical survivor function for these data:

b. Calculate a nonparametric 95% confidence interval for the probability of surviving ten hours or more of a randomly selected unit from the population from which these data was drawn.

c. Calculate a nonparametric 95% confidence interval for the integrated hazard at ten hours of the population from which these data was drawn.
5. Suppose that you have the following data in hours:

   Failure times: 1, 2, 4, 5, 6, 8, 12, 15, 18
   Removal times: 3, 7, 13, 20

   a. Sketch the product-limit estimator of the survivor function of the population from which these data was drawn.

   b. Calculate a nonparametric 95% confidence interval for the probability of surviving ten hours or more of a randomly selected unit from the population from which these data was drawn.

   c. Calculate a nonparametric 95% confidence interval for the integrated hazard at ten hours of the population from which these data was drawn.
6. Suppose that a Weibull plot has a slope of 2 and a y-intercept of -10. What are graphical estimates for the Weibull parameters $\alpha$ and $\eta$?

7. Suppose that a lognormal plot has a slope of 2 and a y-intercept of -10. What are graphical estimates for the lognormal parameters $\mu$ and $\sigma$?

8. Suppose that some mechanical parts have the failure time distribution from which the following plot came from:

![Plot of failure time distribution](image)

What can you tell an engineer about these mechanical parts? Why?