Variability Basics

God does not play dice with the universe.
– Albert Einstein

Stop telling God what to do.
– Niels Bohr

Variability Makes a Difference!

Little’s Law: TH = WIP/CT, so same throughput can be obtained with large WIP, long CT or small WIP, short CT. The difference? Variability!

Penny Fab One: achieves full TH (0.5 j/hr) at WIP=W0=4 jobs if it behaves like Best Case, but requires WIP=27 jobs to achieve 95% of capacity if it behaves like the Practical Worst Case. Why? Variability!
Tortise and Hare Example

Two machines:
- subject to same workload: 69 jobs/day (2.875 jobs/hr)
- subject to unpredictable outages (availability = 75%)

Hare X19:
- long, but infrequent outages

Tortoise 2000:
- short, but more frequent outages

Performance: Hare X19 is substantially worse on all measures than Tortoise 2000. Why? Variability!

Variability Views

Variability:
- Any departure from uniformity
- Random versus controllable variation

Randomness:
- Essential reality?
- Artifact of incomplete knowledge?
- Management implications: robustness is key
Probabilistic Intuition

Uses of Intuition:
- driving a car
- throwing a ball
- mastering the stock market

First Moment Effects:
- Throughput increases with machine speed
- Throughput increases with availability
- Inventory increases with lot size
- Our intuition is good for first moments

Probabilistic Intuition (cont.)

Second Moment Effects:
- Which is more variable – processing times of parts or batches?
- Which are more disruptive – long, infrequent failures or short frequent ones?
- Our intuition is less secure for second moments
- Misinterpretation – e.g., regression to the mean
Variability

**Definition:** Variability is anything that causes the system to depart from regular, predictable behavior.

**Sources of Variability:**
- setups
- machine failures
- materials shortages
- yield loss
- rework
- operator unavailability
- workspace variation
- differential skill levels
- engineering change orders
- customer orders
- product differentiation
- material handling

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Measuring Process Variability

\[ t_c = \text{mean process time of a job} \]

\[ \dot{t}_c = \text{standard deviation of process time} \]

\[ c_c = \frac{\sigma_c}{t_c} = \text{coefficient of variation, CV} \]

*Note: we often use the “squared coefficient of variation” (SCV), \( c_c^2 \)*
Variability Classes in Factory Physics

Effective Process Times:
- *actual* process times are generally LV
- *effective* process times include setups, failure outages, etc.
- HV, LV, and MV are all possible in effective process times

Relation to Performance Cases: For balanced systems
- MV – Practical Worst Case
- LV – between Best Case and Practical Worst Case
- HV – between Practical Worst Case and Worst Case

Measuring Process Variability – Example

<table>
<thead>
<tr>
<th>Trial</th>
<th>Machine 1</th>
<th>Machine 2</th>
<th>Machine 3</th>
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<td>15</td>
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<td>(c_e)</td>
<td>25.1</td>
<td>13.2</td>
<td>43.2</td>
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<tr>
<td>(s_e)</td>
<td>2.5</td>
<td>15.9</td>
<td>127.0</td>
</tr>
<tr>
<td>(c_v)</td>
<td>0.1</td>
<td>1.2</td>
<td>2.9</td>
</tr>
</tbody>
</table>

Class: LV MV HV
Natural Variability

**Definition:** variability without explicitly analyzed cause

**Sources:**
- operator pace
- material fluctuations
- product type (if not explicitly considered)
- product quality

**Observation:** natural process variability is usually in the LV category.

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Down Time – Mean Effects

**Definitions:**

\[
\begin{align*}
l_0 & = \text{base process time} \\
c_0 & = \text{base process time coefficient of variability} \\
r_0 = \frac{1}{l_0} & = \text{base capacity (rate, e.g., parts/hr)} \\
m_f & = \text{mean time to failure} \\
m_r & = \text{mean time to repair} \\
c_r & = \text{coefficient of variability of repair times (}\sigma_r/m_r)\end{align*}
\]
Down Time – Mean Effects (cont.)

**Availability:** Fraction of time machine is up

\[ A = \frac{m_f}{m_f + m_r} \]

**Effective Processing Time and Rate:**

\[ r_e = Ar_0 \]

\[ t_e = t_0 / A \]

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Totoise and Hare - Availability

**Hare X19:**

\[ t_0 = 15 \text{ min} \]

\[ \sigma_0 = 3.35 \text{ min} \]

\[ c_0 = \sigma_0 / t_0 = 3.35 / 15 = 0.05 \]

\[ m_f = 12.4 \text{ hrs (744 min)} \]

\[ m_r = 4.133 \text{ hrs (248 min)} \]

\[ c_e = 1.0 \]

**Tortoise:**

\[ t_0 = 15 \text{ min} \]

\[ \sigma_0 = 3.35 \text{ min} \]

\[ c_0 = \sigma_0 / t_0 = 3.35 / 15 = 0.05 \]

\[ m_f = 1.9 \text{ hrs (114 min)} \]

\[ m_r = 0.633 \text{ hrs (38 min)} \]

\[ c_e = 1.0 \]

**Availability:**

\[ A = \frac{m_f}{m_f + m_r} = \frac{744}{744 + 248} = 0.75 \]

\[ A = \frac{m_f}{m_f + m_r} = \frac{114}{114 + 38} = 0.75 \]

No difference between machines in terms of availability.
Down Time – Variability Effects

**Effective Variability:**

\[ t_e = t_0 / A \]
\[ \sigma_e^2 = \left( \frac{\sigma_0}{A} \right)^2 + \frac{(m^2 + \sigma_r^2)(1 - A)t_0}{Am_r} \]
\[ c_e^2 = \frac{\sigma_e^2}{t_e} = c_0^2 + (1 + c_r^2)A(1 - A) \frac{m_r}{t_0} \]

**Variability depends on repair times in addition to availability**

**Conclusions:**

- Failures inflate mean, variance, and CV of effective process time
- \( \text{Mean} (t_e) \) increases proportionally with \( 1/A \)
- \( \text{SCV} (c_e^2) \) increases proportionally with \( m_r \)
- \( \text{SCV} (c_e^2) \) increases proportionally in \( c_r^2 \)
- For constant availability \( (A) \), long infrequent outages increase SCV more than short frequent ones

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Tortoise and Hare - Variability

**Hare X19:**

\[ t_e = \frac{t_A}{A} = \frac{15}{0.75} = 20 \text{ min} \]
\[ c_e^2 = c_0^2 + (1 + c_r^2)A(1 - A) \frac{m_r}{t_0} = \]
\[ 0.05+(1+1)0.75(1-0.75) \frac{248}{15} = \]
\[ 6.25 \; \text{high variability} \]

**Tortoise 2000**

\[ t_e = \frac{t_A}{A} = \frac{15}{0.75} = 20 \text{ min} \]
\[ c_e^2 = c_0^2 + (1 + c_r^2)A(1 - A) \frac{m_r}{t_0} = \]
\[ 0.05+(1+1)0.75(1-0.75) \frac{38}{15} = \]
\[ 1.0 \; \text{moderate variability} \]

*Hare X19 is much more variabile than Tortoise 2000!*
Setups – Mean and Variability Effects

Analysis:

\[ N_s = \text{average no. jobs between setups} \]
\[ t_s = \text{average setup duration} \]
\[ \sigma_s = \text{std. dev. of setup time} \]
\[ c_s = \frac{\sigma_s}{t_s} \]
\[ t_c = t_0 + \frac{t_s}{N_s} \]
\[ \sigma_c^2 = \sigma_s^2 + \frac{N_s - 1}{N_s} t_s^2 \]
\[ c_c^2 = \frac{\sigma_c^2}{t_c^2} \]

Setups – Mean and Variability Effects (cont.)

Observations:

- Setups increase mean and variance of processing times.
- Variability reduction is one benefit of flexible machines.
- However, the interaction is complex.
Setup – Example

Data:

- Fast, inflexible machine – 2 hr setup every 10 jobs
  \( t_0 = 1 \text{ hr} \)
  \( N_s = 10 \text{ jobs/setup} \)
  \( t_c = 2 \text{ hrs} \)
  \( t_e = t_0 + t_c / N_s = 1 + 2 / 10 = 1.2 \text{ hrs} \)
  \( r_e = 1/t_e = 1 / (1 + 2 / 10) = 0.8333 \text{ jobs/hr} \)

- Slower, flexible machine – no setups
  \( t_0 = 1.2 \text{ hrs} \)
  \( r_e = 1/t_0 = 1/1.2 = 0.833 \text{ jobs/hr} \)

Traditional Analysis: No difference!

Setup – Example (cont.)

Factory Physics Approach: Compare mean and variance

- Fast, inflexible machine – 2 hr setup every 10 jobs

  \( t_0 = 1 \text{ hr} \)
  \( c_0^2 = 0.0625 \)
  \( N_s = 10 \text{ jobs/setup} \)
  \( t_c = 2 \text{ hrs} \)
  \( c_c^2 = 0.0625 \)
  \( t_e = t_0 + t_c / N_s = 1 + 2 / 10 = 1.2 \text{ hrs} \)
  \( r_e = 1/t_e = 1 / (1 + 2 / 10) = 0.8333 \text{ jobs/hr} \)

\[
\sigma_e^2 = \sigma_0^2 + \frac{\sigma_c^2}{N_s} \times \frac{N_s - 1}{N_s^2} = 0.4475
\]

\( c_e^2 = 0.31 \)
Setup – Example (cont.)

• Slower, flexible machine – no setups
  \[ t_0 = 1.2 \text{ hrs} \]
  \[ c_0^2 = 0.25 \]
  \[ r_s = 1/ t_0 = 1/1.2 = 0.833 \text{ jobs/hr} \]
  \[ c_r^2 = c_0^2 = 0.25 \]

Conclusion: *Flexibility can reduce variability.*

Setup – Example (cont.)

**New Machine:** Consider a third machine same as previous machine with setups, but with shorter, more frequent setups

\[ N_s = 5 \text{ jobs/setup} \]
\[ t_s = 1 \text{ hr} \]

**Analysis:**

\[ r_s = 1/ t_s = 1 /(1 + 1/5) = 0.833 \text{ jobs/hr} \]
\[ \sigma_s^2 = \sigma_0^2 + \frac{\sigma_r^2}{N_s} + \frac{N_s - 1}{N_s^2} r_s^2 = 0.2350 \]
\[ c_r^2 = 0.16 \]

**Conclusion:** *Shorter, more frequent setups induce less variability.*
Other Process Variability Inflators

Sources:
- operator unavailability
- recycle
- batching
- material unavailability
- et cetera, et cetera, et cetera

Effects:
- inflate $t_c$
- inflate $c_v$

Consequences: Effective process variability can be LV, MV, or HV.

Illustrating Flow Variability

Low variability arrivals

\[ \begin{array}{c}
\textbf{smooth!} \\
\end{array} \]

High variability arrivals

\[ \begin{array}{c}
\textbf{bursty!} \\
\end{array} \]
**Measuring Flow Variability**

\[ t_a = \text{mean time between arrivals} \]

\[ r_a = \frac{1}{t_a} = \text{arrival rate} \]

\[ \sigma_a = \text{standard deviation of time between arrivals} \]

\[ c_a = \frac{\sigma_a}{t_a} = \text{coefficient of variation of interarrival times} \]

---

**Propagation of Variability**

\[ c_d(i) = c_a(i+1) \]

**Single Machine Station:**

\[ c_d(i) = u^2 c_a + (1-u^2) c_a^2 \]

where \( u \) is the station utilization given by \( u = r_d f_e \)

**Multi-Machine Station:**

\[ c_d = 1 + (1-u^2)(c_a^2 - 1) + \frac{u^2}{m} (c_a^2 - 1) \]

where \( m \) is the number of (identical) machines and \( u = \frac{r_d f_e}{m} \)
Propagation of Variability

High Utilization Station
High Process Var

Low Flow Var  High Flow Var

Low Utilization Station
High Process Var

Low Flow Var  Low Flow Var

Propagation of Variability

High Utilization Station
Low Process Var

High Flow Var  Low Flow Var

Low Utilization Station
Low Process Var

High Flow Var  High Flow Var
Variability Interactions

Importance of Queueing:
- manufacturing plants are queueing networks
- queueing and waiting time comprise majority of cycle time

System Characteristics:
- Arrival process
- Service process
- Number of servers
- Maximum queue size (blocking)
- Service discipline (FCFS, LCFS, EDD, SPT, etc.)
- Balking
- Routing
- Many more

Kendall's Classification

A/B/C

A: arrival process
B: service process
C: number of machines

M: exponential (Markovian) distribution
G: completely general distribution
D: constant (deterministic) distribution.
Queueing Parameters

- \( r_a \) = the rate of arrivals in customers (jobs) per unit time (\( t_a = 1/r_a \) = the average time between arrivals).
- \( c_a \) = the CV of inter-arrival times.
- \( m \) = the number of machines.
- \( r_e \) = the rate of the station in jobs per unit time = \( m/t_e \).
- \( c_e \) = the CV of effective process times.

\( u = \text{utilization of station} = r_a/r_e. \)

Note: A station can be described with 5 parameters.

Queueing Measures

Measures:
- \( C_T_q \) = the expected waiting time spent in queue.
- \( C_T \) = the expected time spent at the process center, i.e., queue time plus process time.
- \( \text{WIP} \) = the average WIP level (in jobs) at the station.
- \( \text{WIP}_q \) = the expected WIP (in jobs) in queue.

Relationships:
- \( C_T = C_T_q + t_e \)
- \( \text{WIP} = r_a \times C_T \)
- \( \text{WIP}_q = r_a \times C_T_q \)

Result: If we know \( C_T_q \), we can compute \( \text{WIP}, \text{WIP}_q, C_T. \)
The G/G/1 Queue

Formula:

\[ CT_q = V \times U \times t \]
\[ = \left( \frac{c_u^2 + c_e^2}{2} \right) \left( \frac{u}{1-u} \right) t_e \]

Observations:

- Useful model of single machine workstations
- Separate terms for variability, utilization, process time.
- \( CT_q \) (and other measures) increase with \( c_u^2 \) and \( c_e^2 \)
- Flow variability, process variability, or both can combine to inflate queue time.
- Variability causes congestion!

The G/G/m Queue

Formula:

\[ CT_q = V \times U \times t \]
\[ = \left( \frac{c_u^2 + c_e^2}{2} \right) \left( \frac{u^{\left[2(m+1)\right]-1}}{m(1-u)} \right) t_e \]

Observations:

- Useful model of multi-machine workstations
- Extremely general.
- Fast and accurate.
- Easily implemented in a spreadsheet (or packages like MPX).
### VUT Spreadsheet

<table>
<thead>
<tr>
<th>MEASURE</th>
<th>STATION</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arrival Rate (parts/hr)</td>
<td>( r_a )</td>
<td>10.000</td>
<td>9.800</td>
<td>9.310</td>
<td>8.845</td>
<td>7.960</td>
</tr>
<tr>
<td>Arrival CV</td>
<td>( r_a^2 )</td>
<td>1.000</td>
<td>0.947</td>
<td>1.331</td>
<td>6.212</td>
<td>3.573</td>
</tr>
<tr>
<td>Natural Process Time (hr)</td>
<td>( t_n )</td>
<td>0.090</td>
<td>0.090</td>
<td>0.095</td>
<td>0.090</td>
<td>0.090</td>
</tr>
<tr>
<td>Natural Process CV</td>
<td>( t_n^2 )</td>
<td>0.500</td>
<td>0.500</td>
<td>0.500</td>
<td>0.500</td>
<td>0.500</td>
</tr>
<tr>
<td>Number of Machines</td>
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<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
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<tr>
<td>MTTF (hr)</td>
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<td>MTTR (hr)</td>
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<td>Availability</td>
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<td>0.962</td>
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<tr>
<td>Effective Process Time (failures only)</td>
<td>( t_e^f )</td>
<td>0.091</td>
<td>0.091</td>
<td>0.099</td>
<td>0.092</td>
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<tr>
<td>Effective Process Time (failures only)</td>
<td>( t_e^f )</td>
<td>0.956</td>
<td>0.956</td>
<td>6.729</td>
<td>2.289</td>
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<td>Jobs Between Setups</td>
<td>( N_s )</td>
<td>100.000</td>
<td>100.000</td>
<td>100.000</td>
<td>100.000</td>
<td>100.000</td>
</tr>
<tr>
<td>Setup Time (hr)</td>
<td>( t_s )</td>
<td>0.000</td>
<td>0.500</td>
<td>0.500</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Setup Time CV</td>
<td>( t_s^2 )</td>
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<td>1.000</td>
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</tr>
<tr>
<td>Eff Process Time (failures/setsups)</td>
<td>( t_e^f )</td>
<td>0.091</td>
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<tr>
<td>Eff Process Time Var (failures/setsups)</td>
<td>( t_e^f )</td>
<td>0.888</td>
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<td>Eff Process CV (failures/setsups)</td>
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<td>Departure CV</td>
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<td>0.947</td>
<td>1.331</td>
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<td>3.573</td>
<td>2.845</td>
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<td>Yield</td>
<td>( y )</td>
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<td>0.956</td>
<td>0.950</td>
<td>0.980</td>
<td>0.980</td>
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<tr>
<td>Final Departure Rate</td>
<td>( r_a^*y )</td>
<td>9.800</td>
<td>9.130</td>
<td>8.845</td>
<td>7.960</td>
<td>7.562</td>
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<tr>
<td>Final Departure CV</td>
<td>( y(1-y) )</td>
<td>0.948</td>
<td>1.314</td>
<td>5.952</td>
<td>3.316</td>
<td>2.752</td>
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<td>Utilization</td>
<td>( u )</td>
<td>0.990</td>
<td>0.940</td>
<td>0.966</td>
<td>0.812</td>
<td>0.731</td>
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<td>Throughput</td>
<td>( TH )</td>
<td>9.800</td>
<td>9.310</td>
<td>8.845</td>
<td>7.960</td>
<td>7.562</td>
</tr>
<tr>
<td>Average Time (hr)</td>
<td>( CT )</td>
<td>0.879</td>
<td>1.744</td>
<td>11.768</td>
<td>1.669</td>
<td>0.720</td>
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<tr>
<td>Cycle Time (hr)</td>
<td>( CT_r )</td>
<td>0.970</td>
<td>1.804</td>
<td>11.871</td>
<td>1.760</td>
<td>0.812</td>
</tr>
<tr>
<td>Cumulative Cycle Time (hr)</td>
<td>( ZeCT_d(t=0) )</td>
<td>0.970</td>
<td>2.089</td>
<td>14.481</td>
<td>16.441</td>
<td>17.253</td>
</tr>
<tr>
<td>WIP in Queue (jobs)</td>
<td>( r_aCT )</td>
<td>8.708</td>
<td>17.436</td>
<td>117.676</td>
<td>16.488</td>
<td>7.202</td>
</tr>
<tr>
<td>WIP (jobs)</td>
<td>( r_aCT )</td>
<td>9.697</td>
<td>18.395</td>
<td>118.714</td>
<td>17.604</td>
<td>8.120</td>
</tr>
<tr>
<td>Cumulative WIP (jobs)</td>
<td>( Z_eCT(t) )</td>
<td>9.697</td>
<td>28.092</td>
<td>146.407</td>
<td>164.411</td>
<td>172.531</td>
</tr>
</tbody>
</table>

---

### Seeking Out Variability

#### General Strategies:
- look for long queues (Little's law)
- focus on high utilization resources
- consider both flow and process variability
- ask “why” five times

#### Specific Targets:
- equipment failures
- setups
- rework
- operator pacing
- anything that prevents regular arrivals and process times
Variability Pooling

**Basic Idea:** the CV of a sum of independent random variables decreases with the number of random variables.

**Example (Time to process a batch of parts):**

\[ t_0 = \text{time to process single part} \]
\[ \sigma_0 = \text{standard deviation of time to process single part} \]
\[ c_0 = \frac{\sigma_0}{t_0} = \text{CV of time to process single part} \]

\[ t_0(\text{batch}) = nt_0 \]
\[ \sigma_0^2(\text{batch}) = n\sigma_0^2 \]
\[ \frac{c_0^2(\text{batch})}{t_0(\text{batch})} = \frac{n\sigma_0^2}{nt_0} = \frac{\sigma_0^2}{n} = \frac{c_0^2}{n} \]

\[ c_0(\text{batch}) = \frac{c_0}{\sqrt{n}} \]

Safety Stock Pooling Example

- PC’s consist of 6 components (CPU, HD, CD ROM, RAM, removable storage device, keyboard)
- 3 choices of each component: \(3^6 = 729\) different PC’s
- Each component costs $150 ($900 material cost per PC)
- Demand for all models is normally distributed with mean 100 per year, standard deviation 10 per year
- Replenishment lead time is 3 months, so average demand during LT is \(\theta = 25\) for computers and \(\theta = 25(729/3) = 6075\) for components
- Use base stock policy with fill rate of 99%
Pooling Example - Stock PC’s

Base Stock Level for Each PC:

\[ R = \theta + z \sigma \]

\[ = 25 + 2.33(\sqrt{25}) \]

\[ = 25 + 11.55 \]

\[ = 36.55 \]

On-Hand Inventory for Each PC:

\[ I(R) = R - \theta + B(R) \approx R - \theta = z \sigma \]

\[ = 36.55 - 25 \]

\[ = 11.55 \text{ units} \]

Total On-Hand Inventory:

\[ 11.55 \times 729 \times \$900 = \$7,873,200 \]

Pooling Example - Stock Components

Necessary Service for Each Component:

\[ S = (0.99)^{1/6} = 0.9983 \Rightarrow z_s = 2.93 \]

Base Stock Level for Each Component:

\[ R = \theta + z_s \sigma \]

\[ = 6075 + 2.93(\sqrt{6075}) \]

\[ = 6075 + 630 \]

\[ = 6705 \]

On-Hand Inventory Level for Each Component:

\[ I(R) = R - \theta + B(R) \approx R - \theta = z_s \sigma \]

\[ = 6705 - 6075 \]

\[ = 630 \text{ units} \]

Total Safety Stock:

\[ 630 \times 18 \times \$150 = \$615,600 \]

A 92% reduction!
### Basic Variability Takeaways

**Variability Measures:**
- CV of effective process times
- CV of interarrival times

**Components of Process Variability**
- failures
- setups
- many others - deflate capacity and inflate variability
- long infrequent disruptions worse than short frequent ones

**Consequences of Variability:**
- variability causes congestion (i.e., WIP/CT inflation)
- variability propagates
- variability and utilization interact
- pooled variability less destructive than individual variability