NUMERICAL SIMULATION OF ELECTROMECHANICAL DYNAMICS IN PACED CARDIAC TISSUE

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ABSTRACT
We study electromechanical dynamics in paced cardiac tissue using numerical simulations of a mathematical model that accounts for excitation-contraction coupling as well as mechanoelectrical feedback. A previously developed finite element based parallel platform is adopted. Extensive numerical simulations are carried out on a 2d tissue and a 3d tissue to investigate the influences of various parameters on the stability of propagating cardiac waves, including conduction velocity, pathological scars, contraction, and stretch activated channels.

INTRODUCTION
Sudden cardiac arrest (SCA), a leading cause of death in the industrialized world, kills over 350,000 Americans each year. SCA can often result from fatal arrhythmias such as ventricular fibrillation. Much research has demonstrated the connections between the fatal arrhythmias and the instabilities of the cardiac electrophysiology. Alternans of action potential duration (APD) is a well-known indicator of the instabilities. Discordant alternans, a phenomenon of two spatially distinct regions exhibiting APD alternans of opposite phases, amplifies dispersion of repolarization and can lead to conduction block and reentry [1][1]. However, the mechanisms of discordant alternans are not fully understood.

METHODS
Models from multiple physics fields are integrated, including electrophysiology, electromechanics, and mechanoelectrical feedback. A computational algorithm is developed based on finite element methods.

Cardiac Electrophysiology
Cardiac dynamics of electrophysiology in extended tissue are modeled using a reaction-diffusion equation:
\[
\frac{\partial v}{\partial t} = \text{div}(D \nabla v) - \frac{1}{C_m} (I_{\text{ion}} + I_{\text{ext}} + I_s)
\]  
where \( v \) represents the transmembrane voltage, \( D \) represents the diffusion matrix, operator \( \text{div}[\cdot] \) denotes the divergence with respect to spatial coordinates \( x \), \( C_m \) represents the transmembrane capacitance, \( I_{\text{ion}} \) is the total ionic current, \( I_{\text{ext}} \) is the excitation current and \( I_s \) is the stretch activated channel (SAC). The equation is supplemented by a number of ordinary differential equations, describing the variation of membrane conductance and ionic concentrations. In this study, the FMG model [3] is adopted to describe the ionic currents.

Cardiac Mechanics and Mechano-Electric Feedback
The quasi-static stress equilibrium of the heart is described by the following equation of balance of linear momentum:
\[
\text{div}[\tau] + B = 0
\]  
where \( B \) represents the given body force per unit reference volume, \( \tau \) represents the Eulerian Kirchhoff stress tensor, which is composed of the passive stress \( \tau_{\text{pas}} \) and the excitation induced contraction \( \tau_{\text{act}} \) [4]: \( \tau = \tau_{\text{pas}} + \tau_{\text{act}} \).

The resulting partial differential equations are solved using finite element method (FEM). The time integration of the reaction term is solved using forward Euler method with a time step of 0.005ms. The time integration of the diffusion term is solved using backward Euler method with a time step of 0.1ms.
Simulation Details

A square tissue (5cmx5cm) is stimulated at the left-bottom corner with a basic cycle length of 200ms. The tissue is represented by a finite element hexahedral mesh with 20402 nodes in total and size of each element is about 0.05cm. A square scar with a size of 0.05cm is presented at the center of the tissue for some simulations. The electrical propagation is isotropic along both x and y directions when not considering contraction.

RESULTS AND DISCUSSION

Influence of conduction velocity on formation of discordant alternans in a 2d tissue

Previous studies have shown that discordant alternans can be caused by interaction of conduction velocity and APD restitution [5]. In this study, we found that, when the conduction velocity was reduced by half in both directions of the tissue, the concordant distribution of APD alternans would change to discordant alternans.

Figure 1 shows the APD alternans distribution when conduction is normal. The APD in the entire tissue alternates concordantly between about 118ms and 162ms. The APD alternans at the 29th beat in Figures 1-3 are equal to $d_{29} - d_{28}$, where $d_{28}$ and $d_{29}$ are the APDs at the 28th and 29th beats, respectively. Figure 2 shows the APD alternans distribution in the tissue when the conduction is isotropic but slow in both directions. The nodal line remains still as the simulations lasts. This study gives another mechanism of formation of the discordant alternans: it can arise from the coupling of the slow conduction and the high pacing rate.

Influence of scar on cardiac electrophysiology in a 2d tissue

Myocardial scarring has been considered an important anatomic component of the substrate for arrhythmias like ventricular tachycardia and fibrillation [6]. In this study, we investigated the influence of the presence of a scar at the center of the tissue on the APD alternans distribution. The scar is formulated in the model by making the cells in the scar area always at resting potential, and making the electrical propagation impossible in such area.

When the conduction is normal, we found that, the part of the tissue near to the scar and far away from the stimulation site was stabilized. Fig. 3 shows that, in the area near the stimulation site, the APD alternates between about 116ms and...
165ms, while the green area that is far away from the stimulation site remains an APD of about 145ms. Although the discordant alternans is not formed, this heterogeneous distribution of APD over the tissue may play a role as a substrate to arrhythmias too.

When the conduction is slow, a sustained spiral wave breakup is observed, as shown in Fig. 4. The breakup of spiral waves may induce fatal cardiac arrhythmias such as ventricular fibrillation. Thus the scar obviously disturbs the stability of the cardiac electrophysiology.

**Influence of excitation induced contraction and mechano-electric feedback in a 3d tissue**

The tissue is seen as a bunch of fibers, with fiber orientation along vertical direction. The stress at the left-top and right-bottom corner is fixed. A scar is presented at the center of the tissue. Fig. 5 shows that the wave break happens in both top and middle panels, but is absent in bottom panels. Because of the occurrence of SAC at the left-top corner and the right-bottom corner, two electrical waves sweep horizontally in opposite directions and collide with each other, thus prohibiting the happening of spiral wave breakup. The SAC thus shows a capability of stabilizing the cardiac electrophysiology.

**CONCLUSION**

We have studied dynamics of cardiac waves in two tissues using numerical simulations through a parallel finite element platform. Special focus is on spatial patterns of alternans and on the development of spiral waves. A slow conduction speed and the presence of a scar can both disturb the stability of the cardiac electrophysiology. Moreover, the stretch activated channel is shown to be able to prevent the spiral wave breakup.

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**REFERENCES**


