Endogenous private safety investment and the willingness to pay for mortality risk reductions

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Abstract

When individuals cannot undertake safety-improving expenditures, the effect of an increase in the initial risk on the willingness to pay (WTP) for mortality risk reduction is positive because of the dead-anyway effect. When they can undertake safety-improving expenditures, the effect of an increase in the initial risk is governed by two effects: The dead-anyway effect which is positive and the high-payment effect which is negative. We treat the two types of risk-reducing expenditures, endogenous and exogenous, as inputs in a safety-improving technology function and find conditions that guarantee that the high-payment effect dominates.

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1. Introduction

The willingness to pay (WTP) for fatality risk reduction is both the main component of the value of a statistical life and an important input for the cost-benefit analysis of public projects or regulations that aim at reducing risks to human lives.\textsuperscript{1} Given this fact, it

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\textsuperscript{1}The value of a statistical life is equal to 100v/m if the WTP is v for a program reducing fatality risk by m percentage points (i.e., saving m out of 100 lives). The WTP is also used to measure benefits from various non-fatality risk reductions, such as a reduction in the risk of getting a target disease (e.g., Bleichrodt et al., 2003).
becomes important to determine how the WTP for fatality risk reduction changes when either the affected individuals become more wealthy or the initial fatality risk rises. Indeed, these questions have received prior attention in the literature. Jones-Lee (1974) and Weinstein et al. (1980) both find that the marginal willingness to pay (MWTP) for fatality risk reduction increases with both wealth and the initial fatality probability. As pointed out by Pratt and Zeckhauser (1996), though, the latter comparative static result, that an increase in the initial fatality risk leads to an increase in the MWTP for risk reduction, is highly dependent on the way in which Jones-Lee and Weinstein et al. construct their models: In both models, the affected individuals are powerless to take any actions on their own to reduce their fatality risks. Despite the fact they questioned this modeling assumption, Pratt and Zeckhauser did not analyze the implications of allowing individuals to decrease their own fatality risks. That is the purpose of this paper.²

Pratt and Zeckhauser (1996) argue that the comparative static results of Jones-Lee (1974) and Weinstein et al. (1980) arise from what they call the “dead-anyway effect.” Suppose that there are two states of the world corresponding to life and death. If the marginal utility of wealth is higher in the life state than in the death state, an increase in the probability of dying makes wealth less valuable in expectation, implying a higher MWTP for risk reduction. However, they also point out that if the individual can take actions on his own to reduce the risk, there is a “high-payment effect” that may work in the opposite direction of the dead-anyway effect. If the higher mortality risk induces higher private expenditure on risk reduction, final wealth is reduced and its marginal utility increases, which in turn reduces the MWTP for risk reduction.³

Empirical evidence suggests that the high-payment effect dominates the dead-anyway effect. Based on a survey of households in suburban Boston, Smith and Desvousges (1987) estimated individuals’ WTP for reductions in the risks of death from hazardous wastes and found that the MWTP decreases with the level of risk. Liu et al. (1997) performed a meta-analysis using data from different compensating wage differential studies, regressing the value of a statistical life on average income and risk levels. They found a statistically significant negative coefficient on the average risk level, which is consistent with the conclusion that the MWTP for risk reduction falls with the level of risk. Viscusi and Aldy (2003) perform a similar meta-analysis using more studies and alternative statistical techniques and reach the same conclusion. All of this evidence seems to indicate not only the existence of the high-payment effect, but its dominance over the dead-anyway effect.⁴

²There is an extensive literature on self-protection. The difference between self-insurance (the private effort to reduce the severity of a stochastic loss) and self-protection (the private effort to reduce the probability of a stochastic loss) was first emphasized by Ehrlich and Becker (1972). Dionne and Eeckhoudt (1985), Briys and Schlesinger (1990), and McGuire et al. (1991) study the relationship between the degree of risk aversion and the size of self-protection. Other previous studies that endogenize self-protection but have a different emphasis than that of this paper include Shogren and Crocker (1991) and Quiggin (1992, 2002).

³As the baseline mortality risk increases, quoting Pratt and Zeckhauser (1996, p. 752), “the ‘dead-anyway’ effect, ..., arises because the dollars the individual expends have a larger chance of coming from the low-valued state, namely ‘dead.’” On the other hand, “Risk aversion creates the ‘high-payment’ effect: The more risk is concentrated, the more those bearing the risk will be paying, thereby increasing their marginal utility of income.” In other words, the dead-anyway effect decreases the marginal utility of wealth while the high-payment effect increases it.

⁴On the other hand, the prediction of Jones-Lee and Weinstein et al. regarding wealth’s effect on the valuation of risk reductions has been widely supported by empirical evidence; see Liu et al. (1997) and Viscusi and Aldy (2003).
This paper looks at the dead-anyway effect and the high-payment in a theoretical setting and raises two significant questions. First, under what circumstances does the high-payment effect exist? In particular, under what circumstances does the individual increase his private risk-reduction expenditure when the probability of dying increases? Second, under what circumstances does the high-payment effect outweigh the dead-anyway effect, so that expanding the setting reverses the comparative static results of Jones-Lee (1974) and Weinstein et al. (1980) but is consistent with the empirical results of Smith and Desvousges (1987), Liu et al. (1997) and Viscusi and Aldy (2003)?

To address these questions, we assume that there is a production function for risk reduction with two inputs, the individual's private expenditure and an external expenditure. We then determine conditions on this production function that lead to the existence of a high-payment effect and to the high-payment effect outweighing the dead-anyway effect. The key condition for the existence of a high-payment effect is that an increase in the external expenditure does not make the private expenditure more productive. The high-payment effect outweighs the dead-anyway effect if a change in the private expenditure has a sufficiently large proportional impact on the marginal product of the external input.

We also reexamine, in our more general framework, the effect of initial wealth on the MWTP for risk reduction. The fact that individuals with higher initial wealth tend to spend more on safety improvement (and hence have a lower probability of death) generates a “reverse dead-anyway” effect. Since they spend more on self-protection, wealthier individuals have a lower probability of dying, which makes wealth more valuable in expectation, implying a lower MWTP for risk reduction. This reverse dead-anyway effect makes the impact of initial wealth on the MWTP for risk reduction less obvious than in Jones-Lee (1974) and Weinstein et al. (1980). Nonetheless, we find that this reverse dead-anyway effect is not strong enough to overcome the positive effect of wealth on the MWTP for risk reduction, upholding the qualitative finding of a positive correlation between MWTP for risk reduction and initial wealth.

The paper is organized as follows. In the next section, we study the properties of safety-producing technology. These properties prove to be critical in deriving the results in the following sections. In Section 3, we add to the standard setup a private safety-improving investment, and study how the individual’s optimal levels of investment and risk change with initial risk and wealth. Then, in Section 4, we investigate how the equilibrium MWTP for risk reduction is affected by initial risk and wealth. We conclude in Section 5.

2. Expenditures to reduce fatality risk

There are two sources of expenditures on risk reduction. One source, \( I \), is the investment made by the decision-maker to improve his own safety. The other source, \( G \), is an external source that is exogenous to the decision-maker. This external expenditure on risk reduction can have many sources. It may come from private donations by others, as is the case when a group raises money to pay for life-saving medical treatment for an individual. It may come from a public expenditure, such as when the highway department provides guardrails or other road safety measures. It may even come from an expenditure by the individual facing the risk but that is exogenous to the individual, such as the purchase of mandated automobile safety restraints. There is another interpretation of \( G \), as well. If one individual is endowed with a greater ability to reduce risk than another, as long as that endowment is
exogenous to the individual it could be captured by a different level of $G$. Throughout this paper we interpret $G$ as an external expenditure, in keeping with the desire to make our results applicable to cost-benefit analysis for public policy. The only key characteristic of $G$, though, is that its level is exogenous to the individual facing the risk.

The probability of survival is given by $q(I, G)$ when the decision-maker invests $I$ and the external source spends $G$ on fatality-risk reduction. For notational convenience, we use $q_1$ to stand for the first order partial derivative of $q$ with respect to the first argument (which is $I$), and $q_{12}$ to stand for the second order cross-partial derivative, etc. Since both expenditures improve the chances of survival, $q_1 > 0$ and $q_{2} > 0$. Diminishing returns implies that $q_{11} < 0$ and $q_{22} < 0$. The function $q(I, G)$ is, essentially, a production function for fatality risk reduction, with inputs $I$ and $G$. All of the preceding assumptions are straightforward and hold for any production function. What distinguishes risk-reduction technology from ordinary production, though, is the sign of the cross-partial $q_{12}$. In a standard production setting where the inputs are, say, capital and labor, additional capital makes labor more productive, so the cross-partial is positive. The synergies that occur in production might be absent from risk reduction. We provide two examples for $q_{12} \leq 0$ in the following. As a result, we shall consider both $q_{12} > 0$ and $q_{12} \leq 0$ in this paper.

First, suppose that the survival probability is increased through medical expenditures. Expenditures by the individual are captured by $I$, while expenditures by the external source are captured by $G$. In this setting both expenditures purchase the same good, and the survival probability depends only on the total expenditure, $I + G$, so that $q(I, G) = f(I + G)$. Consequently, the cross-partial $q_{12} = f'' = q_{11} = q_{22} < 0$, and this feature occurs whenever the two expenditures purchase the same good.

For the second example, consider the probability of surviving a driving experience. The private expenditure captures the costs of driving more slowly, and the external expenditure captures the costs of more or better guardrails. The effectiveness of driving more slowly decreases when the guardrails are better, and the benefits of the guardrail improvements diminish when the cars travel more slowly. The implications for the cross-partial are that $q_{12} \leq 0$.

To an individual, $q(0, G)$ is the initial probability of survival (hence, $1 - q(0, G)$ is the initial mortality risk), which varies with $G$, either due to individual differences in endowed ability or due to changes in the external expenditure. Since $q_{2}(0, G) > 0$, the effects of an increase in the initial survival probability are qualitatively equivalent to the effects of an increase in $G$, a fact that is used throughout the paper.

3. Equilibrium private expenditures on safety improvement

Following the simple setup in Jones-Lee (1974), we assume that an individual faces two existence states, “life” and “death”, with survival occurring with probability $q$. The individual’s utility derived from wealth, $W$, is state-dependent. Denoting the utility function under the state of “life” as $L(W)$ and the utility function under the state of “death” as $D(W)$, non-satiation and risk aversion in each state imply $L', D' > 0$ and $L'', D'' < 0$. Also following Jones-Lee, we assume $L(W) > D(W)$ and $L'(W) > D'(W)$ for all $W$. There are several reasons to believe that both of these assumptions are true. First, Pratt and Zeckhauser (1996) argue that the assumptions are valid as long as one is not too altruistic towards one’s heirs. Second, Viscusi and Evans (1990) support both assumptions empirically when the states correspond to remaining healthy or being injured instead of life.
and death. Third, the basic question being addressed is the value of saving a life, which implicitly assumes that the life is worth saving, or, mathematically, \( L(W) > D(W) \). Finally, there is an option value to being alive, in that someone who is alive could always choose to allocate an additional dollar of wealth in the same way it would be allocated if that person died, which implies that the utility generated by that extra dollar of wealth must be at least as high when the individual is alive as when he is dead. Consequently, \( L'(W) > D'(W) \).5

As discussed in the preceding section, the probability of survival is determined by an individual expenditure on safety improvement, \( I \), and an external expenditure, \( G \), and can be written as \( q(I, G) \). The external input \( G \) is exogenous to the individual, but may or may not be exogenous to society. The government, for example, may be able to undertake a program that changes \( G \), and hence the initial survival probability \( q(0, G) \). The final survival probability, on the other hand, is endogenous to the individual. This formulation allows for offsetting behavior. If, for example, the government undertakes a program that increases the initial survival probability, the individual might respond by investing less in safety improvements. In such cases, the impact of the government’s program is lessened by the offsetting behavior of the individual, and this in turn affects how much the individual is willing to pay for the reduction in the risk probability. This raises two important questions. First, how does the individual’s choice of the safety-improving investment \( I \) change when the initial survival probability changes? Second, how does the final survival probability change when the initial survival probability changes?

The individual chooses the safety-improving expenditure \( I \) to maximize expected utility

\[
EU = q(I, G)L(\frac{W_0}{C_0} - I) + [1 - q(I, G)]D(\frac{W_0}{C_0} - I),
\]

(1)

where \( W_0 \) is the individual’s initial wealth. The first order condition giving the optimal safety investment is

\[
q_1(I, G)[L(\frac{W_0}{C_0} - I) - D(\frac{W_0}{C_0} - I)] - q(I, G)L'(\frac{W_0}{C_0} - I)
- [1 - q(I, G)]D'(\frac{W_0}{C_0} - I) = 0
\]

(2)

or \( MWTP \)

\[
MWTP = \frac{L(\frac{W_0}{C_0} - I) - D(\frac{W_0}{C_0} - I)}{q(I, G)L'(\frac{W_0}{C_0} - I) + [1 - q(I, G)]D'(\frac{W_0}{C_0} - I)} = \frac{1}{q_1(I, G)}.
\]

(3)

The left side of (3) is the MWTP for risk reduction at the optimum and the right side is the marginal cost of risk reduction. Therefore, the first order condition simply says that the private safety-improving expenditure will be carried out to a point where the MWTP for risk reduction equals its marginal cost.

To see how the optimal safety-improving expenditure changes with the initial safety level \( q(0, G) \) or simply \( G \) (since the former is a strictly increasing function of the latter due to

5The risk in this paper can also be interpreted as associated with other hazards for which the two inequalities hold, such as loss of a child, loss of health, or imprisonment. For an example of an application of state-dependent preferences to settings other than death, see Neilson (1998).
6The second order derivative of the expected utility in (1) with respect to \( I \) can be derived as \( q_{11}(L - D) - 2q_1L' - D' + qL'' + (1 - q)D'' \) which is negative given the assumed properties of functions \( L, D, \) and \( q \). Therefore, the second order sufficient condition of the individual’s safety investment problem is satisfied.
\( q_2(0, G) > 0 \), we differentiate (2) with respect to \( G \), taking \( I \) as a function of \( G \). This yields\[ \frac{dI}{dG} = \frac{-q_{12} \cdot (L - D) + q_2 \cdot (L' - D')}{q_{11} \cdot (L - D) - 2q_1 \cdot (L' - D') + qL'' + (1 - q)D''}. \] (4)

The denominator of (4) is negative, given the assumptions on \( L(\cdot), D(\cdot), \) and \( q(\cdot, \cdot) \) discussed above. Therefore, a sufficient condition for the existence of offsetting behavior, namely \( dI/dG < 0 \), is \( q_{12} \leq 0 \). In this case, when external expenditures are increased to improve initial safety, the individual responds by investing less in safety improvements. Since the initial risk is determined by the level of external expenditure on risk reduction, as the level of external expenditure falls, the initial fatality risk rises. This means that when the initial risk increases, the individual invests more in safety improvement. These results are summarized as the following proposition:

**Proposition 1.** Under the maintained assumptions on the state-dependent utility functions \( L(W) \) and \( D(W) \) and the risk-reducing technology \( q(I, G) \), if \( q_{12} \leq 0 \) then individual safety-improving investment decreases with the external safety-improving expenditure, and therefore increases with the initial risk level.

Peltzman (1975) found that the supposed effects of various automobile safety designs (safety belts, energy-absorbing steering column, etc.) were largely offset by drivers’ more risky driving behavior. Extensive further empirical evidence has generally confirmed the existence of offsetting behavior in the area of auto safety.\(^7\) In terms of the notation of this paper, the mandated auto safety designs represent increases in external safety measure \( G \), which induce drivers to make less safety investment \( I \) on their own.

The equilibrium probability of survival is determined by \( q(I, G) \) given the response of \( I \) to the externally determined \( G \). Consequently, the effect on the equilibrium probability of survival of an increase in \( G \) is given by\[ \frac{dq}{dG} = q_1 \cdot \frac{dI}{dG} + q_2 = \frac{-(q_1 q_{12} - q_2 q_{11}) \cdot (L - D) - q_1 q_2 \cdot (L' - D') + q_2 \cdot [qL'' + (1 - q)D'']}{q_{11} \cdot (L - D) - 2q_1 \cdot (L' - D') + qL'' + (1 - q)D''} \] (5)

Under the given assumptions on \( L(\cdot), D(\cdot), \) and \( q(\cdot, \cdot) \), the denominator of the above expression is negative, as are the terms \(-q_1 q_2 \cdot (L' - D') \) and \( q_2 \cdot [qL'' + (1 - q)D''] \) in the numerator. If \[ q_1 q_{12} - q_2 q_{11} \geq 0, \] (6)
then \( dq/dG > 0 \), implying that a higher external expenditure leads to a higher equilibrium probability of survival. So, if the external source spends more on risk reduction, even though the individual might respond by spending less on risk reduction, the overall survival probability increases in equilibrium. The individual’s response only partially offsets any change in the externally determined initial risk.

Expression (6) is not terribly restrictive. First, note that (6) holds if and only if \( d(q_2/q_1)/dI \geq 0 \), which means that the marginal rate of technical substitution between \( I \) and \( G \)—the additional units of \( I \) needed for a one-unit reduction of \( G \), to maintain the same safety level—increases as \( I \) increases, while holding \( G \) constant. This in turn occurs if

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\(^7\)For example, see Keeler (1994), Peterson et al. (1995) and Sen (2001).
production exhibits diminishing marginal rate of technical substitution and if \( G \) is a normal input, in the sense that if input prices are held constant and output increases, demand for \( G \) increases.

If (6) holds, so that \( dq/dG > 0 \), a higher initial risk (corresponding to a lower \( G \)) leads to a higher equilibrium risk (corresponding to a lower \( q \)) even though the individual spends more on risk reduction when the initial risk is higher. These results are summarized in the following proposition.

**Proposition 2.** Under the maintained assumptions on the state-dependent utility functions \( L(W) \) and \( D(W) \) and the risk-reducing technology \( q(I, G) \), if expression (6) holds then the equilibrium probability of survival increases when the external expenditure \( G \) increases, and decreases when the initial risk level increases.

The effects of changes in initial wealth are also of policy interest. To that end, we use (2) to conduct a comparative statistics analysis of \( I \) with respect to \( W_0 \) and find

\[
\frac{dI}{dW_0} = \frac{-q_1 \cdot (L' - D') + qL'' + (1 - q)D''}{q_1 \cdot (L - D) - 2q_1 \cdot (L' - D') + qL'' + (1 - q)D''} > 0,
\]

where the inequality follows from the maintained assumptions on \( L, D, \) and the safety-producing technology \( q \). Furthermore, the effect on the equilibrium risk level of initial wealth is given by \( dq/dW_0 = q_1 \cdot dI/dW_0 > 0 \). This leads to the following proposition.

**Proposition 3.** Under the maintained assumptions on the state-dependent utility functions \( L(W) \) and \( D(W) \) and the risk-reducing technology \( q(I, G) \), both the equilibrium individual safety-improving investment and the equilibrium probability of survival increase with initial wealth.

4. The MWTP for risk reduction

Ignoring the possibility that an individual can make safety-improving expenditures to reduce the risk of fatality, both Jones-Lee (1974) and Weinstein et al. (1980) evaluated the WTP for risk reduction at the initial position \( (q(0, G), W_0) \), namely

\[
MWTP|_{I=0} = \frac{L(W_0) - D(W_0)}{q(0, G)L'(W_0) + [1 - q(0, G)]D'(W_0)},
\]

instead of at the equilibrium position as shown by the left-hand side of (3). The numerator in the above expression is the marginal expected utility of a one unit reduction (say, one percentage point) in risk, which is independent of the initial risk level \( 1 - q(0, G) \). The denominator is the marginal expected utility of wealth, which is smaller the larger the initial risk level given that \( L' > D' \) for any wealth level. Therefore, both Jones-Lee and Weinstein et al. concluded that the WTP for risk reduction increases with the initial risk level.

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8The MWTP identified in the left-hand side of (3) coincides with the MWTP given in (8) when the level of safety-improvement investment is restricted to zero.
From (3), in the setting in which individuals can undertake expenditures that reduce their mortality probabilities, the equilibrium MWTP for risk reduction is given by

$$\text{MWTP} = \frac{L(W_0 - I) - D(W_0 - I)}{q(I, G)L'(W_0 - I) + [1 - q(I, G)]D'(W_0 - I)}.$$ 

The dead-anyway effect occurs because an increase in initial risk (or a decrease in $G$) causes an increase in the equilibrium mortality probability (by Proposition 2), and since $L'(W) > D'(W)$, the denominator falls, thereby raising the MWTP for risk reduction. The high-payment effect occurs for two reasons. First, the increase in the initial risk causes an increase in the level of investment, which, given that $L'(W) > D'(W)$, causes $L(W_0 - I) - D(W_0 - I)$ to fall. Second, the same increase in the level of investment reduces wealth for other consumption, which, given the diminishing marginal utility of wealth in both states, causes the denominator—the marginal expected utility of wealth—to rise. This puts downward pressure on the MWTP. The question, then, is which effect dominates? If the dead-anyway effect dominates, an increase in the initial risk leads to a higher MWTP, but if the high-payment effect dominates, an increase in the initial risk leads to a lower MWTP.

By (3), in equilibrium the MWTP for risk reduction is equal to its marginal cost, or $1/q(I, G)$. Accordingly, it is possible to determine the effect of a change in the initial risk on the equilibrium MWTP by determining its effect on the equilibrium marginal cost. We have

$$\frac{d(MWTP)}{dG} = \frac{-q_1(dI/dG) + q_{12}}{q_1^2} = \frac{(2q_1q_{12} - q_2q_{11})(L' - D') - q_{12}[qL'' + (1 - q)D']}{q_1^2[q_{11} \cdot (L - D) - 2q_1 \cdot (L' - D') + qL'' + (1 - q)D']].}$$

Under the given assumptions on $L(\cdot), D(\cdot)$, and $q(\cdot, \cdot)$, the denominator of expression (9) is negative. If $q_{12} \geq 0$ then the numerator is positive and $d(MWTP)/dG < 0$. If, on the other hand,

$$2q_1q_{12} - q_2q_{11} < 0,$$

then the numerator is negative (noting that condition (10) implies $q_{12} < 0$), and $d(MWTP)/dG > 0$. Summarizing these results establishes the following proposition.

**Proposition 4.** Under the maintained assumptions on the state-dependent utility functions $L(W)$ and $D(W)$ and the risk-reducing technology $q(I, G)$, (i) if $q_{12} \geq 0$, then the equilibrium MWTP for risk reduction decreases with the external expenditure, and consequently, increases with the initial risk; (ii) if condition (10) holds, then the equilibrium MWTP for risk reduction increases with the external expenditure, and consequently, decreases with the initial risk.

Therefore, when the private safety investment and the external safety expenditure are complements ($q_{12} \geq 0$), incorporating the high-payment effect would not qualitatively change the previous result that valuation of risk reductions increases with the baseline risk. On the other hand, if the two safety inputs are substitutes ($q_{12} < 0$) and if condition (10) is satisfied, the high-payment effect dominates the dead-anyway effect, and the conventional result regarding the relationship between the valuation of risk reductions and the level of risk is overturned. Further insight into expression (10) can be generated by considering it...
together with expression (6). Rearranging the two inequalities yields

\[- \frac{q_{12}}{q_2} \in \left[ -\frac{1}{2} \frac{q_{11}}{q_1}, -\frac{q_{11}}{q_1} \right]. \tag{11}\]

The term \(-q_{12}/q_2\) is the magnitude of the proportional change in the marginal product of input \(j\) caused by a marginal change in input \(I\). So, the left-hand side of (11) is the proportional change in the marginal product of the external input \(G\) caused by a change in the input \(I\). The upper bound in (11) has the implication that a change in \(I\) has a larger (in magnitude) proportional impact on the marginal product of \(I\) than it does on the marginal product of \(G\). The lower bound implies that the change in \(I\) does not have “too small” a proportional impact on the marginal product of \(G\), where “too small” means less than half of the proportional impact on the marginal product of \(I\).

Proposition 4 states that, given the assumptions of the model, expression (10) is a sufficient condition for the high-payment effect to dominate the dead-away effect. To establish the reasonableness of condition (10), suppose that the risk-reduction technology is described by the function

\[ q(I, G) = f(I + G), \tag{12} \]

where \(f' > 0\) and \(f'' < 0\). Then \(q_1 = q_2 = f'\) and \(q_{11} = q_{12} = f''\), implying that \(q_1 q_{12} - q_2 q_{11} = 0\). Since \(q_{12} < 0\), it follows that \(2q_1 q_{12} - q_2 q_{11} < 0\), and condition (10) holds. In (12) the two inputs are perfect substitutes, and the high-payment effect dominates the dead-away effect.

As another example, suppose that the probability of death as a function of private and public safety expenditures takes the following multiplicatively separable form:

\[ p(I, G) = \frac{A^a g(G)}{(I + A)^a}, \tag{13} \]

where \(A\) and \(a\) are positive constants, \(g > 0\), \(g' < 0\) and \(g'' > 0\). In this specification, the probability of death is simply \(g(G)\), which decreases in the public safety expenditure \(G\), when the private safety investment \(I = 0\). Private investment \(I > 0\) would scale that mortality risk downward to \(A^a g(G)/(I + A)^a\). This specification is quite flexible with parameters \(A\) and \(a\) assigned different values. Under this mortality risk specification, \(q(I, G) = 1 - p(I, G) = 1 - (A^a g(G)/(I + A)^a)\). Then, \(q_1 = aA^a(I + A)^{-(a+1)} g(G) > 0\), \(q_2 = -A^a(I + A)^{-a} g'(G) > 0\), \(q_{11} = -a(a+1)A^a(I + A)^{-(a+2)} g(G) < 0\), and \(q_{12} = aA^a(I + A)^{-(a+1)} g'(G) < 0\). So

\[ q_1 q_{12} - q_2 q_{11} = -aA^2a(I + A)^{-2(a+1)} g(G) g'(G), \]

and

\[ 2q_1 q_{12} - q_2 q_{11} = a(a-1)A^{2a}(I + A)^{-2(a+1)} g(G) g'(G). \]

Therefore, (6) is satisfied for all positive \(a\)’s but (10) is satisfied only for \(a > 1\).

By incorporating private safety-improving expenditures, we find that the equilibrium MWTP for risk reduction decreases with the initial risk level for an important class of safety-improving production functions. In other words, the high-payment effect from a higher initial risk level, which is not adequately incorporated into previous analyses, could be strong enough to more than fully offset the dead-away effect. This result is important since individuals facing most real life fatality risks (e.g., fatal crimes and traffic accidents,
life-threatening diseases like AIDS, and so on) can do something on their own to reduce the risk in the first place. Therefore, their WTP values for further risk reduction are evaluated at the equilibrium positions rather than the initial positions. Indeed, in cost benefit analysis, the WTP values for the outputs of a project are always evaluated at the realized equilibrium.9

Given that the presence of safety-improving investment could reverse the previous result on the relationship between the WTP for risk reduction and the initial risk, the previous result on the positive relationship between the WTP for risk reduction and initial wealth should also be reconsidered. From (8), the previous result concerning the effect of the initial wealth is easy to understand. When \( W_0 \) increases, the numerator of (8)—the marginal expected utility of risk reduction—is larger since \( L' > D' \). At the same time, the denominator of (8)—the marginal expected utility of wealth—is smaller due to risk aversion in both states. Therefore, Jones-Lee (1974) and others concluded that the WTP for risk reduction increases with wealth.

As we demonstrated in the last section, however, wealthier individuals spend more on safety improvement and end up having a lower equilibrium risk level. Using Pratt and Zeckhauser’s (1996) terminology, this generates a reverse dead-anyway effect in the sense that the lowered risk of death increases the marginal expected utility of wealth. Obviously, this reverse dead-anyway effect from larger initial wealth works to reduce the WTP for risk reduction. The question is whether the reverse dead-anyway effect is strong enough to more than fully offset the effect considered in Jones-Lee.

Using (3) again, the overall effect of an increase in \( W_0 \) on the equilibrium MWTP is given by

\[
\frac{d(MWTP)}{dW_0} = -\frac{q_{11}(dI/dW_0)}{q_1^2} > 0,
\]

establishing our final result.

**Proposition 5.** Under the maintained assumptions on the state-dependent utility functions \( L(W) \) and \( D(W) \) and the risk-reducing technology \( q(I, G) \), the equilibrium MWTP for risk reduction increases with initial wealth.

Therefore, accounting for the fact that higher initial wealth would induce an individual to spend more on safety improvement does not change the qualitative finding by Jones-Lee of the positive relationship between the WTP for risk reduction and the initial wealth.

5. Conclusion

Previous studies concerning how the willingness to pay (WTP) for risk reduction is affected by a person’s initial risk and wealth did not account for the possibility that one may be able to improve one’s safety by investing in risk reduction. This paper endogenizes

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9See, for example, Dreze and Stern (1987), Snow and Warren (1996), Dahlby (1998), Sandmo (1998), Allgood and Snow (1998) and Liu (2003). This fact concerning the custom benefit-side measurement has been used in Browning et al. (2000) to judge the merit of alternative measures of the marginal cost of funds. On the other hand, note that the traditional analysis of relationship between the WTP for risk reduction and the initial risk level exemplified in Jones-Lee (1974) and Weinstein et al. (1980) is appropriate when individuals cannot reduce a fatal risk on their own, such as the risk of nuclear war or structured gambles like Russian Roulette.
the safety-improving decision and conducts a more general analysis of the relationship between the WTP for risk reduction and the levels of initial risk and wealth. It finds that the effect of a higher initial risk on the WTP for risk reduction could be either positive or negative (depending on the specification of safety-improving technology), rather than definitely positive as found by previous studies. That is, the high-payment effect could outweigh the dead-anyway effect, and does so for an important class of safety-improving production functions. On the other hand, while there is a reverse dead-anyway effect from larger initial wealth, which works to reduce the WTP for risk reduction, it is not strong enough to overcome the positive effect of larger wealth on the WTP found in previous studies.

The empirical implications of the propositions in this paper are as follows. When we consider individuals who ultimately differ only in total wealth \( W_0 \) (they may also differ in the equilibrium risk of fatality they face but it is so only because they have different wealth), we should expect a higher marginal willingness to pay (MWTP) for risk reduction from wealthier individuals. On the other hand, across individuals with identical total wealth but different equilibrium risks (the difference in initial risk may not be directly observable), those facing higher risks may have either higher or lower WTP for the same percentage point marginal reduction in the risk.

Even though the mortality probabilities we deal with here are likely to be very small, the results are still economically significant for three reasons. First, and as already mentioned, our results allow for an explanation of why the MWTP for risk reduction decreases with the size of the risk, as found in several empirical studies (Smith and Desvousges, 1987; Liu et al., 1997; Viscusi and Aldy, 2003). Second, evidence suggests that individuals vastly overweight probabilities of unlikely, bad outcomes (see Machina, 1982, for an early discussion). For example, in the Allais paradox individuals choose $1 million for sure over a lottery with a 10% chance of $5 million, an 89% chance of $1 million, and a 1% chance of $0, even though the second lottery has an expected value of $1.39 million. Rank-dependent expected utility models can accommodate the overweighing of unlikely, bad outcomes, and our approach allows for rank-dependence if the probability \( q(I, G) \) is interpreted as \( \pi(p(I, G)) \) where \( p \) is the probability of survival when the decision-maker invests \( I \) on safety improvements and the external source spends \( G \), and \( \pi \) is the probability transformation function from the rank-dependent expected utility specification. Finally, even if the mortality probability change and the MWTP turn out to be small, the ratio is the key component of the value of a statistical life, an important figure for policy analysis.

Existing studies on how various factors affect the WTP for fatality risk reductions go beyond the effects of the initial risk or wealth investigated in this paper. For example, Eeckhoudt and Hammitt (2001) studied how certain background risks (physical or financial) affect the value of a statistical life, i.e., the WTP for the reduction of mortality risk at the margin. In light of the findings in this paper, it is interesting to look at whether endogenizing private safety-improving effort would qualitatively alter the results in their study.

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References


