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Interim bribery in auctions ☆

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Abstract

Bidders can bribe the auctioneer before they bid, with the auctioneer lowering the winner's bid if the winner paid the bribe. In equilibrium bidders employ a cutoff strategy and corruption affects neither efficiency nor the bidders' expected payoffs.

Keywords: Auctions; Bribery; Corruption; Revenue equivalence

JEL classification: D44; K42

1. Introduction

In many cases, but not all, a sealed-bid auction has an auctioneer. Sometimes the auctioneer is a third party in the transaction, and sometimes it is an individual who works for the firm awarding the prize and who is given the task of collecting the bids from the bidders. The existence of an agent coming between the seller and the bidders raises the possibility of corruption in two ways. First, the auctioneer could look at the submitted bids and then solicit a bribe from the winner *after* the bids are submitted in exchange for changing the bid in a way that is favorable to the winner. In a standard high-bid auction, this would entail soliciting a bribe in exchange for lowering the winner's bid down to the second-highest bid. Alternatively, the auctioneer could solicit bribes from the bidders *before* the bids are submitted, in exchange for a promise to reduce the bidder's bid should that bidder be the winner. Several existing papers address *ex post* bribery that occurs after all of the bids are submitted.[1] This paper analyzes interim bribery that occurs after bidders learn their valuations but before the bids are submitted.

This is not simply an academic exercise, because interim bribery has been documented in actual auctions. In their bids for corporate waste-disposal contracts in New York City, Mafia families would sometimes pay bribes for an "undertaker's look" at the bids of the other bidders before making their own bids.[2] In 1997 a Covington, Kentucky, developer was shown the bids of two competing developers for a $37million dollar courthouse construction project.[3] In Chelsea, Massachusetts, in the 1980s, the city's auctioneer was accused of accepting bribes to rig

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[1] Lengwiler and Wolfstetter (2000) analyze auctions in which the winning bidder can bribe the auctioneer to change the bid after the auction has ended. Their results are similar to ours, although the results depend on the possibility of the corruption being detected and punished. Menezes and Monteiro (2006) consider a scenario in which there are two bidders and the auctioneer approaches one of them to solicit a bribe in return for changing the bid. The auctioneer can approach either the winner or the loser. Burguet and Perry (2002) study an auction in which one bidder is honest but one is corrupt. Burguet and Che (2004) and Celentani and Ganuza (2002) study a procurement auction in which the awarding of the contract is based on both the price and the quality of the project, and a corrupt auctioneer can manipulate the quality component in exchange for a bribe.

[2] Cowan and Century (2002, pp. 223–231).

[3] Crowley, Patrick, "Bid Scandal Bill in Trouble," *Cincinnati Enquirer*, January 21, 2000.

auctions in favor of certain bidders, one time serving as a bidder's agent in an auction he was running.[4]

2. Auctions with interim bribery

The seller of a single good faces n risk neutral potential buyers. The seller has hired an auctioneer to run a sealed-bid first-price auction, and pays the auctioneer a fixed wage (as opposed to a commission) in exchange for her services. The auctioneer approaches every bidder before the auction is held and tells them that if the bidder agrees to pay a bribe of α and is the highest bidder, he pays the second-highest bid. If the highest bidder did not pay the bribe, he pays his bid. Bribes are collected from all bidders who agreed to pay, even from losing bidders. Consequently, the game is a three-stage game. In the first stage the auctioneer sets α, in the second stage the bidders decide whether to pay α independently and simultaneously, and in the third stage the bidders choose their bids.

Bidders draw valuations $v_1, \ldots, v_n$ independently from the distribution F with support [0,1]. The value of the object to the seller is zero and the reserve price is zero. There is no entry fee, making it optimal for all bidders to bid. The seller is passive in this game and we ignore issues related to the detection and punishment of corruption.

Consider the subgame that follows the auctioneer's choice of α. The first task is to find the bids of bidders who do and do not pay the bribe. If a bidder pays the bribe and is the highest bidder, he pays the second highest bid. Therefore, after paying the bribe the bidder essentially participates in a second price auction, and his dominant strategy is to bid his valuation.

Lemma 1. Any bidder who pays the bribe bids his valuation, v_i.

Our main result concerns when bidders pay the bribe and when they do not. The next lemma states that bidders use cutoff strategies, that is, for bidder i there is a valuation v_i^* such that he pays the bribe when $v_i \geq v_i^*$ and does not pay the bribe when $v_i < v_i^*$.

Lemma 2. In any equilibrium every bidder uses a cutoff strategy.

Proof. See the Appendix.

If a bidder does not pay the bribe, he must pay his own bid if he wins. Consequently, and for the standard reasons, he bids less than his valuation. How much less depends on the behavior of other bidders. In a symmetric equilibrium if a bidder with valuation v_i declines the bribe, all bidders with lower valuations also decline the bribe. Let $b_1(v)$ denote the equilibrium bid function in a standard first-price sealed bid auction without bribery, given by

$$b_1(v) = v - \frac{1}{F^{n-1}(v)} \int_0^v F^{n-1}(y)dy. \tag{1}$$

[4] Murphy, Sean P., "Chelsea Businessman is Said to Allege Attempted Bribery," *Boston Globe*, September 22, 1993.

Lemma 3. If bidder i with valuation v_i does not pay the bribe and all the bidders with valuations below v_i do not pay the bribe, bidder i bids according to the function $b_1(v_i)$.

Proof. Let $b(v)$ denote the equilibrium bid function for bidders who choose not to pay the bribe. For the standard reasons, b is assumed to be increasing. By Lemma 1, all bidders who do pay the bribe bid their valuations, and $v \geq b(v)$ for all v. If bidder i does not pay the bribe, and all bidders with valuations below v_i also do not pay the bribe, bidder i only wins when his is the highest valuation. The theory of first price auctions then implies that, conditional on his own valuation being the highest, bidder i's optimal bid is then $b_1(v_i)$.

The next theorem shows that a symmetric equilibrium exists, and that there is only one symmetric equilibrium of the subgame.

Theorem 1. Given the amount of the bribe α, there exists a unique symmetric Bayesian–Nash equilibrium in which bidders with values in $[0,v^*)$ do not pay the bribe and bidders with values in $[v^*, 1]$ do pay the bribe, where v^* solves

$$\int_0^{v^*} [b_1(v^*) - b_1(v)]dF^{n-1}(v) = \alpha \tag{2}$$

Proof. See the Appendix.

A bidder with valuation v^* who pays the bribe earns expected surplus of $\int_0^{v^*} [v^* - b_1(v)]dF^{n-1}(v) - \alpha$, while a bidder with valuation v^* who does not pay the bribe earns expected surplus of $\int_0^{v^*} [v^* - b_1(v^*)]dF^{n-1}(v)$. The fact that the bidder is indifferent reduces to $\int_0^{v^*} [b_1(v^*) - b_1(v)]dF^{n-1}(v) = \alpha$, which is simply expression (2).

Given α, the equilibrium bid function $b(v)$ is increasing (with a jump at v^*), and a bidder with a valuation of 0 does not pay the bribe and earns zero expected surplus. Consequently, the Revenue Equivalence Theorem (Myerson, 1981) implies the following corollary.

Corollary 1. The auction with bribery is efficient, and any bribes paid are a transfer from the seller to the auctioneer.

Efficiency follows from the fact that the equilibrium bid function is increasing, and the fact that the bidder with the highest valuation wins suggests that no losing bidder will complain on fairness grounds because that bidder would not have won the auction in the absence of bribery, either. The efficiency result relies heavily on the assumption that all bidders have the chance to pay the bribe, though, because if only one bidder can pay the bribe, there is a possibility that when that bidder pays and bids his valuation, he outbids someone with a higher valuation but a lower bid.

Now turn attention to the decision facing the auctioneer. In the first period the auctioneer chooses the size of the bribe α to maximize her expected revenue. By Theorem 1, though, for any given α there is a unique threshold valuation v^*, and expression (2) implicitly defines a function $\alpha(v^*)$. The auctioneer's expected revenue is given by

$$R(v^*) = n(1 - F(v^*))\alpha(v^*). \tag{3}$$

Because $R(0)=R(1)=0$, the optimal threshold valuation, and therefore the optimal bribe is strictly positive. For example, when valuations are distributed uniformly on [0,1], the optimal (for the auctioneer) cutoff point is $v^{**}=n/(n+1)$, which is the expected value of the highest of the n valuations. So, only bidders with very high valuations pay the bribe.

Differentiating (3) and rearranging yields

$$b_1(v^*) - \frac{1-F(v^*)}{f(v^*)} b_1'(v^*) = \frac{\int_0^{v^*} b_1(v) dF^{n-1}(v)}{F^{n-1}(v^*)}. \tag{4}$$

From (4), the auctioneer's optimal bribe is set as if she were running an auction with a reserve price, with the bidders' valuations replaced by their bids and treating her own value as the expected highest bid among $n-1$ bidders conditional on none of their valuations exceeding v^*. Noting that the value of the bribe to a bidder derives from the expected savings from being able to pay the second-highest bid instead of the highest one, the auctioneer chooses the cutoff point for the bribe in a way that maximizes her expected share of these savings.

Appendix A

Proof of Lemma 2. Fix any equilibrium and consider the (right-continuous) cdf, $G_i(b)$, of the highest bid of bidders $j \neq i$. Also let $x_i(b)$ denote the probability of i winning with bid b against the rival bidders employing their equilibrium strategies. (Note that $x_i(b)$ may not equal $G_i(b)$ since a tie may arise at a mass point b.) Let B_c be the set of b's for which G is continuous, and let B_m be the set of b's for which G jumps. Then

$$U_{ic}(v) = \int_{b \le v, b \in B_c} (v-b) dG_i(b) + \sum_{b \le v, b \in B_m} (v-b)[G_i(b_+) - G_i(b_-)] - \alpha.$$

$U_{ic}(\cdot)$ is absolutely continuous and can be rewritten as

$$U_{ic}(v) = \int_{v'}^{v} G_i(s) ds + U_{ic}(v'), \tag{A1}$$

for any v'.

Now consider

$$U_{in}(v) = \sup_b (v-b) x_i(b).$$

It follows that

$$U_{in}(v) = \max_b (v-b) G_i(b),$$

since $(v-b)G_i(b)$ is an upper envelope of $(v-b)x_i(b)$. One can check that $U_{in}(v)$ is absolutely continuous, that the maximum is well defined (since an upper envelope is upper semicontinuous and the choice can be bound to a compact set without loss of generality), and that $f(b,v) \equiv (v-b)G_i(b)$ is differentiable in v for every b in the equilibrium support. Hence, one can invoke Theorem 2 of Milgrom and Segal (2002) to show that

$$U_{in}(v) = \int_{v'}^{v} G_i(b^*(s)) ds + U_{in}(v'), \tag{A2}$$

for $b^*(s) \in \arg\max_b (v-b) x_i(b)$.

It follows from (A1) and (A2) that

$$U_{ic}(v) - U_{in}(v) = \int_{v'}^{v} [G_i(s) - G_i(b^*(s))] ds + [U_{ic}(v') - U_{in}(v')]. \tag{A3}$$

Since $b^*(s) < s$ for almost every s, it is clear from (A3) that, whenever $U_{ic}(v') - U_{in}(v') > 0$, it must be that $U_{ic}(v) - U_{in}(v) > 0$ for $v > v'$, which proves that the equilibrium strategy must involve a cutoff strategy with some threshold v_i^*.

Proof of Theorem 1. By Lemma 2 every bidder uses a cutoff strategy, so there exist values $v_1^*, \ldots, v_n^*$ such that bidder i pays the bribe if $v_i \ge v_i^*$ and does not pay the bribe if $v_i < v_i^*$. It remains to show that $v_1^* = \ldots = v_n^* = v^*$.

We first show that the lowest of the n cutoff points is v^*. Suppose that player i has the lowest threshold point, v_i^*, and draws the valuation v_i. If he does not pay the bribe he bids the standard first-price equilibrium bid, since everyone below him also bids according to the standard first-price equilibrium bid function. Then, letting $H_i(\cdot)$ denote the cdf of the highest valuation of bidders $j \neq i$,

$$U_{ic}(v_i^*) = \int_0^{v_i^*} (v_i^* - b_1(v)) dH_i(v) - \alpha$$

and

$$U_{in}(v_i^*) = (v_i^* - b_1(v_i^*)) H_i(v_i^*).$$

Therefore

$$U_{ic}(v_i^*) - U_{in}(v_i^*) = \int_0^{v_i^*} (b_1(v_i^*) - b_1(v)) dH_i(v) - \alpha. \tag{A4}$$

The right-hand side is obviously increasing in v_i^* (since the bid function increases) and it is equal to zero when $v_i^* = v^*$. Consequently, there is no equilibrium in which the lowest threshold point is below v^*.

We next show that the highest of the n cutoff points is also v^*. Suppose that i has the highest threshold value $v_i^* > v^*$, and choose $v_i \varepsilon (v^*, v_i^*)$. It follows from Eq. (A3) that

$$U_{ic}(v_i) - U_{in}(v_i) = \int_{v^*}^{v_i} [G_i(s) - G_i(b^*(s))] ds + [U_{ic}(v^*) - U_{in}(v^*)]. \tag{A5}$$

By construction $U_{ic}(v^*) - U_{in}(v^*) = 0$. The integral in (A5) is positive for $v_i > v^*$, since $b^*(s) < s$ for almost all s. Consequently, bidder i wants to pay the bribe when he draws the valuation $v_i < v_i^*$, which is a contradiction.

References

Burguet, R., Perry, M., 2002. Bribery and favoritism by auctioneers in sealed-bid auctions. Working paper, Rutgers University.

Burguet, R., Che, Y.K., 2004. Competitive procurement with corruption. Rand Journal of Economics 35, 50–68.

Celentani, M., Ganuza, J.J., 2002. Corruption and competition in procurement. European Economic Review 46, 1273–1303.

Cowan, R., Century, D., 2002. Takedown: The Fall of the Last Mafia Empire. G.P. Putnam's Sons, New York.

Lengwiler, Y., Wolfstetter, E., 2000. Auctions and corruption. Working paper, Humboldt-Universitat zu Berlin.

Menezes, F.M., Monteiro, P.K., 2006. Corruption and auctions. Journal of Mathematical Economics 42, 97–108.

Milgrom, P., Segal, I., 2002. Envelope theorems for arbitrary choice sets. Econometrica 70, 583–601.

Myerson, R., 1981. Optimal auction design. Mathematics of Operations Research 6, 58–73.