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Reference Wealth Effects in Sequential Choice

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Abstract

It is argued that in order to accommodate experimentally-observed choice patterns, it is not enough to model the utility function as being dependent on changes from a reference wealth point. Instead, individuals should be modeled as treating decisions as part of an identifiable sequence of decisions, and utility should be a function of reference wealth, income so far from the sequence, and payoffs from the current decision. The three-argument utility function allows for risk aversion over gains and risk seeking over losses for the first choice in the sequence, and for the house money and break-even effects in later decisions.

Key words: Risk aversion, expected utility, reference wealth, house money effect

JEL Classification: D81

The purpose of this paper is to characterize how utility functions change during the course of a sequence of risky decisions. Historically, most applications of expected utility theory have treated final wealth as the carrier of value of the utility function. However, several researchers, most notably Markowitz (1952) and Kahneman and Tversky (1979), have suggested that the proper carrier of value is the change in wealth. Most attention has focused on the premise that utility functions tend to be concave over gains, convex over losses, and steeper for losses than for gains. These hypotheses have received experimental support,[1] and have led to such issues as framing (Tversky and Kahneman (1981)) and the idea that a loss matters more than an equivalent foregone gain (Thaler (1980)). It would appear, then, that a proper model of behavior toward risk would employ a utility function with two arguments: reference wealth and the change in wealth.[2] This paper contends that a two-argument utility function cannot accommodate two other patterns specified in the experimental literature, the house money effect and the break-even effect (see Thaler and Johnson (1990)),[3] and that to allow for these two effects a more complicated utility function is needed. When an individual faces a sequence of decisions, the house money and break-even effects can be accounted for if the utility function has three arguments: reference wealth, the income from the sequence so far, and the payoff from the current decision. This paper identifies properties of the three-argument utility function that allow preferences to exhibit the house money and break-even effects while retaining the underlying pattern of risk aversion over gains and risk seeking over losses.

The house money effect takes the following form. If an individual has already won a positive amount of money from the sequence of decisions, he is willing to take fair bets as long as the payoffs are small relative to the amount already won. Gamblers refer to this behavior as “playing with the house money.” They are willing to risk some, but not all, of their previous winnings. Thaler and Johnson (1990) and Keasey and Moon (1996) find evidence of the house money effect in their experiments. Evidence can also be found in game shows such as “Card Sharks” (Gertner, 1993) and “Jeopardy!” (Metrick, 1995). On the latter show, situations often arise in which a contestant who is already assured of winning the game still bets thousands of dollars on his ability to answer a single question, something he would probably not do outside of the context of the show. The house money effect is inconsistent with a two-argument utility function because individuals who are risk averse over gains are unwilling to take fair bets in the gains space. It is also possible to describe a loss version of the house money effect. If an individual has already lost some amount of money from the sequence of decisions, he is unwilling to take fair bets whose payoffs are small relative to the amount already lost.

The break-even effect states that if an individual has already lost some amount of money from the sequence of decisions, he is willing to take fair bets that give him a chance of breaking even. Kahneman and Tversky (1979) note the increased propensity of gamblers to bet on longshots at the end of the racing day, presumably in an attempt to cover earlier losses. Note that the break-even effect does not contradict the loss version of the house money effect: the house money effect governs behavior when payoffs are small relative to the previous losses and the break-even effect governs behavior when payoffs are either comparable to or larger than the previous losses. Thaler and Johnson’s (1990) experiments contain evidence of the break-even effect in addition to the house money effect.

Interestingly, consideration of the house money and break-even effects generates a utility function that closely resembles the one postulated by Markowitz (1952). The utility function he proposed is concave then convex over losses (moving from small to large), and convex then concave over gains. The utility function derived here is convex then concave then convex then concave, retaining the same shape as the Markowitz utility function. The differences lie in the derivation, location, and interpretation of the inflection points.

The usefulness of the model can be demonstrated by applying it to the analysis of the disposition effect, which is the tendency of investors to hold losing assets too long and sell winning assets too soon. Standard analysis of the disposition effect only considers whether the prior performance of the asset was a loss or a gain. The analysis here suggests that another factor is important: the size of the prior loss or gain relative to the potential loss or gain from holding the asset. If the prior loss or gain is large relative to potential future payoffs, the house money effect comes into play. If the prior loss or gain is small relative to potential future payoffs, the break-even effect comes into play, and predicts the opposite behavior. This prediction is evaluated using evidence from an experiment run by Weber and Camerer (1998), and their evidence supports the hypothesis that after a sufficiently large initial gain, investors tend to hold the asset, consistent with the house money effect.

We begin the analysis by reviewing some relevant experimental evidence in Section 1, and then in Section 2 it is demonstrated that a two-argument utility function does not have sufficient flexibility to accommodate all of the evidence. Section 3 introduces a three-argument utility function and provides conditions under which it can accommodate the experimental evidence. Section 4 compares the resulting model to the utility model of Markowitz (1952) and the editing model proposed by Thaler and Johnson (1990). Issues of dynamic consistency are explored in Section 5, and Section 6 applies the model to make new predictions regarding the disposition effect. Section 7 summarizes the key findings and discloses their importance for experimental examinations of choice behavior. Proofs are collected in an Appendix.

1. Patterns Emerging from the Experimental Evidence

This section recounts some experimental evidence related to the shape of utility functions. In particular, evidence regarding three patterns is presented. They are: (i) subjects are risk averse over gains and risk seeking over losses, (ii) the house money effect, and (iii) the break-even effect. Pattern (i) implies that the utility function cannot have the individual's final wealth position as its only argument, but must instead have reference wealth and the change in wealth as arguments, and patterns (ii) and (iii) are the basis of this paper's argument that the utility function requires three arguments to fit observed behavior.

Table 1 shows illustrative results from experiments run by Battalio, Kagel, and Jiranyakul (1990).[4] Subjects face pairs of choices that are identical in terms of final wealth positions, but differ in terms of gains or losses from a starting balance. Subjects are given an initial balance, and then choose between a sure thing or a risky gamble. The notation ($\$a, p_1; \$b, p_2$) in the table denotes a gamble which pays $\$a$ with probability p_1 and $\$b$ with probability p_2. For example, in their Question 2 subjects are given a starting balance of \$30, and then choose between a sure loss of \$11 or a 70:30 chance of losing \$20 or \$0. In their Question 2' subjects are given a starting balance of \$5, and then choose between a sure gain of \$14 or a 70:30 chance of winning \$5 or \$25. The two problems yield identical choices in terms of final wealth positions: either a sure final wealth gain of \$19 or a 70:30 chance of gaining \$10 or \$30. The subjects facing real payoffs split their choices evenly in Question 2, but 77 percent of them chose the less risky alternative in Question 2'. This evidence suggests that subjects do not evaluate choices according to final wealth positions, or, in other words, preference do not exhibit *asset integration.*

The choices made in Questions 1', 2', and 3' in Table 1 display risk aversion over gains while the choices made in Questions 1 and 2 display risk seeking over losses. A related issue is *framing* (Tversky and Kahneman (1981)), which refers to the empirically-validated proposition that the manner in which a decision problem is posed has a pre-

Table 1. Asset integration evidence from Battalio, Kagel, and Jiranyakul (1990).

Question	Initial balance	Sure gain	Risky gamble	Percent risk seeking
1	\$30	-\$12	(-\$20, .6; \$0, .4)	65
1'	\$ 5	\$13	(\$5, .6, \$25, .4)	45
2	\$30	-\$11	(-\$20, .7, \$0, .3)	50
2'	\$ 5	\$14	(\$5, .7; \$25, .3)	23
3'	\$ 5	\$12	(\$20, .6; \$0, .4)	19

dictable effect on the individual's choice. Questions 1 and 1' in Table 1 lead to identical probability distributions over payoffs. In Question 1, though, the decision problem is phrased as losses from an initial large gain, and in Question 1' the decision problem is phrased as additional gains from an initial small gain. When the decision problem is framed in terms of losses (Question 1), more than half of the subjects exhibit risk seeking, but when the decision problem is framed in terms of gains (Question 1'), more than half of the subjects exhibit risk aversion.

Thaler and Johnson (1990) uncover two other significant patterns of preferences toward risk in the presence of initial gains and losses. In their Experiment 4, subjects face a variety of gambles presented in two ways. In one setting subjects are told that they have received an initial payoff of $\$y$, and they then choose between a sure addition of \$0 or a 50:50 chance of winning or losing an additional $\$x$. In the other setting subjects are offered a one-stage gamble with the same final payoff distribution, that is, they choose between a sure gain of \$y or a 50:50 chance of $\$y + x$ or $\$y - x$. These experiments differ from those of Battalio, Kagel, and Jiranyakul in that Thaler and Johnson give subjects an initial payoff and then offer them a symmetric fair gamble, while Battlaio, Kagel, and Jiranyakul give subjects an initial payoff and then give them further gains or further losses, but not both. The results of Thaler and Johnson's experiments are shown in Table 2. Several important patterns emerge. First, note from the one-stage column that subjects are risk averse over gains (Problem 1) and risk seeking over losses (Problems 3 and 4). Also, the differences in choices between the one-stage and two-stage versions of the same problem provide further evidence of the failure of asset integration.

The two-stage version of Problem 1 shows the house money effect. After an initial gain, individuals become risk seeking over gambles that are "small" relative to the initial gain. In Problem 1, subjects initially gain \$15, and then choose whether or not to take a 50:50 chance of winning or losing an additional \$4.50. Thaler and Johnson found that 77 percent of the subjects took the gamble. The one-stage version of Problem 1 shows that without the initial gain, most subjects would be risk averse over this range of final payoffs. Because of the initial gain, subjects switch from being risk averse to risk seeking. Problem 4 shows that a reversed version of the house money effect occurs following an initial loss—subjects switch from being risk seeking over losses to risk averse. In Problem 4, subjects initially lose \$7.50, and then have the opportunity to take a 50:50 chance of winning or losing an additional \$2.25. Thaler and Johnson found that 60 percent of the

Table 2. Experiment 4 from Thaler and Johnson (1990)

	Intitial outcome	Gain or loss	Percent risk seeking	
Problem	y	x	Two-stage	One-stage
1	\$15	\$4.50	77	44
2	\$ 0	\$2.25	41	50
3	-\$ 2.25	\$2.25	69	87
4	-\$ 7.50	\$2.25	40	77

subjects chose not to take the additional risk. Significantly, Keasey and Moon (1996) confirm the house money effect following an initial gain, but fail to confirm it following an initial loss. It is useful to define the house money effect formally.

Definition. Suppose that an individual has an initial gain or loss of y. The individual's preferences satisfy the *house money effect* if there exists a function $\alpha(y)$ such tat after a gain ($y > 0$) the individual is unwilling to accept gambles of the form $(x, 1/2; -x, 1/2)$ for every $x \in (0, \alpha(y))$, and after a loss ($y < 0$) the individual is willing to accept gambles of the form $(x, 1/2; -x, 1/2)$ for every $x \in (0, \alpha(y))$.

This definition of the house money effect states that following a loss of y, the individual is unwilling to accept 50:50 fair bets whose payoffs are small relative to y, where "small" means that the payoffs have magnitude less than $\alpha(y)$. Presumably, $a(y) < |y|$. Similarly, following a gain of y, the individual is willing to accept 50:50 fair bets whose payoffs are small relative to y.

The two-stage version of Problem 3 shows a different pattern. In Problem 3, subjects initially lose \$2.25, and then choose whether or not to take a 50:50 chance of winning or losing \$2.25. A majority of the subjects were willing to take the additional risk. Yet, subjects were unwilling to take the additional risk after either a \$7.50 initial loss or a \$0 initial loss. Thaler and Johnson surmise that this choice pattern is driven by a break-even effect. Subjects are willing to take risks that present the possibility of recovering their original loss. Kahneman and Tversky (1979) also discuss the break-even effect in the context of horse races. Bettors are more prone to bet on long shots later in the racing day, presumably in an attempt to recover losses suffered earlier in the day (see also McGlothlin (1956)). The break-even effect can also be defined formally.

Definition. Suppose that an individual has an initial gain or loss of y. The individual's preferences satisfy the *break-even effect* if there exist functions $\beta(y)$ and $\gamma(y)$ such that after a loss ($y < 0$), for every $x \in (\beta(y), 0)$ and every $z \in (\gamma(y),\infty)$ the individual is willing to accept the gamble $(x, p; z, 1-p)$ where $p = z/(z-x)$; and after a gain ($y > 0$), for every $x \in (0,\beta(y))$ and every $z \in (-\infty, \gamma(y))$ the individual is unwilling to accept the gamble $(x, p; z, 1-p)$ where $p = z/(z-x)$.

The formal definition of the break-even effect states that after suffering an initial loss, for any small additional loss x (where "small" is determined by the function β) and large gain z (where "large" is determined by the function γ) the individual is willing to accept a fair gamble with payoffs x and z. Put another way, after losing y the individual is willing to bet as much as $\beta(y)$ as part of an actuarially fair chance to win at least $\gamma(y)$. The formal definition also includes a reverse break-even effect corresponding to the case of an initial gain. After an initial gain, the individual is unwilling to take fair gambles that have only small potential gains but might cause the individual to lose his previous winnings.

2. Two-Argument Utility Functions

The evidence presented in the preceding section suggests that gains or losses from the current decision matter and that reference wealth matters because of framing effects. A utility function with two arguments, reference wealth and the change in wealth, can capture this behavior. In this section we characterize some properties of a two-argument utility function and show that these properties are inconsistent with the house money and break-even effects.

Let w denote reference wealth and let x be the change in wealth from the current decision. Assume that the individual is an expected utility maximizer with utility function $v(x,w)$.[5] The experimental evidence suggests two properties: individuals exhibit risk aversion over gains and risk seeking over losses, and individuals are averse to fair gambles. To allow for the possibility of nondifferentiable utility functions, we phrase our properties in terms of concavity and convexity rather than the conventional second-derivative conditions.

Property 1. S-shaped utility. For all w, $v(x,w)$ is strictly convex in x for $x < 0$ and strictly concave in x for $x > 0$.

Property 2. Loss aversion.[6] For all w and all nondegenerate distributions $F(x)$ such that $\int x dF(x) = 0$, $\int v(x,w)dF(x) < v(0,w)$.

A utility function satisfying Properties 1 and 2 is shown in Figure 1, which is identical to the utility function proposed by Kahneman and Tversky (1979).

We now show that it is impossible for utility functions that have only two arguments to satisfy Properties 1 and 2 *and* exhibit the house money and break-even effects. Suppose that the individual faces a sequence of decisions and let y denote the income from the sequence so far. This income, termed "initial income," can either be added to the payoffs from the current gamble, making the utility function $v(y + x,w)$, or it can be added to reference wealth, making the utility function $v(x,w + y)$. The following series of propositions shows that neither of these formulations can accommodate the evidence.

Suppose first that initial income is added to the payoffs from the current gamble. Proposition 1 shows that the house money effect is incompatible with Property 1.

Proposition 1. Suppose that an individual's preference satisfy the house money effect and that all prior income is added to the payoffs of current gambles. Then the individual's utility function cannot be S-shaped.

The intuition is straightforward. Suppose that the individual experiences a gain in the first period, and that the gain is added to the payoffs of the gamble in the second period. The house money effect says that the individual should be willing to take a small fair bet. If the new gamble is small compared to the previous winnings, adding the initial gain to the

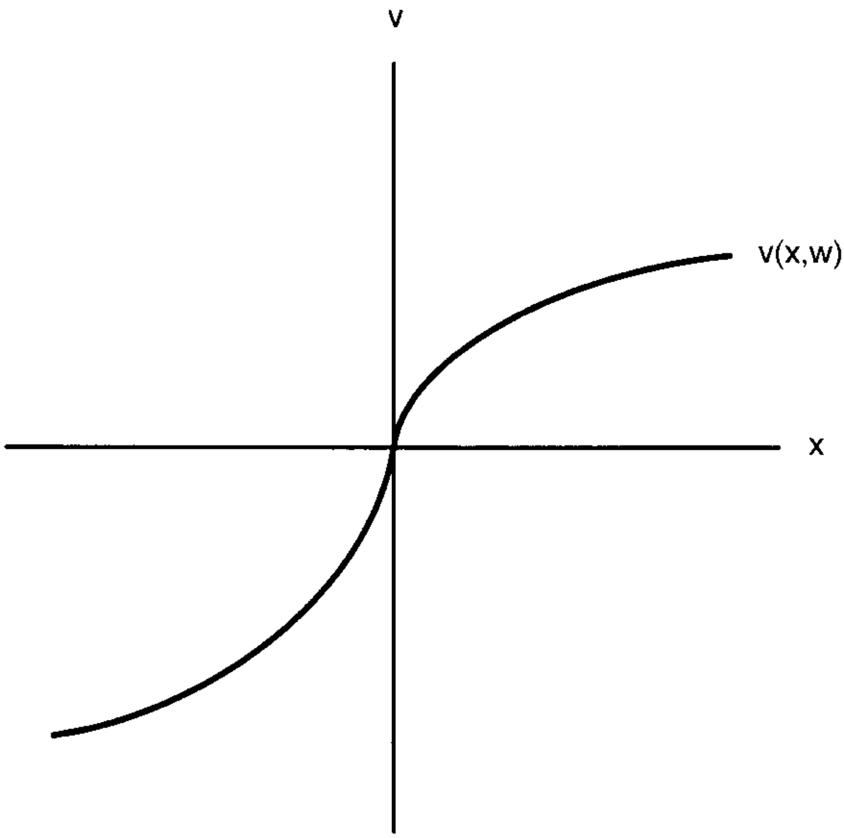

Figure 1. Two-argument utility function

new payoffs means that the individual evaluates the new gamble entirely in gains space. Since he is willing to take the fair gamble, his utility function cannot be concave over gains. Similar intuition applies to initial losses.

Since the house money effect rules out a two-argument utility function where initial gains are added to current payoffs, we now turn attention to a utility function in which initial gains are immediately incorporated into reference wealth. Here there are three sources of inconsistency. First, if the initial outcome is a gain, the house money effect is inconsistent with loss aversion because after a gain the individual is willing to take some fair bets. Second, if the initial outcome is a loss, the break-even effect is inconsistent with loss aversion because after a loss the individual is willing to take some fair bets. Neither of these results refer to S-shaped utility, however, and the failure of asset integration is one of the best-accepted tenets of behavior toward risk (Harless and Camerer (1994)), while loss aversion was not verified in the studies by Battalio, Kagel, and Jiranyakul (1990) and Thaler and Johnson (1990). To this end, it is shown that the house money and break-even effects together are inconsistent with Property 1.

Proposition 2. Suppose that an individual's preferences satisfy the house money effect and that prior income is added to reference wealth. Then the individual's utility function cannot exhibit loss aversion.

Proposition 3. Suppose that an individual's preferences satisfy the break-even effect and that prior income is added to reference wealth. Then the individual's utility function cannot exhibit loss aversion.

Proposition 4. Suppose that an individual's preferences satisfy both the house money and break-even effects and that prior income is added to reference wealth. Then the individual's utility function cannot be S-shaped.

Proposition 4 has the following intuition. Suppose that there is an initial loss. The house money effect says that the individual is averse to additional small fair bets, but the break-even effect says that the individual is willing to take fair bets that entail sufficiently small losses and sufficiently large gains. If the house money effect is satisfied, the utility function must be concave over some interval containing zero. But, for the break-even effect to be satisfied, the utility function must then be convex to the right of that interval, and therefore cannot be S-shaped.

3. A Three-Argument Utility Function

The preceding section established that a two-argument utility function is unable to accommodate evidence of the house money and break-even effects. In this section it is demonstrated that the evidence can be handled if a third argument measuring income to date from the sequence is added to the utility function.

Suppose that an expected utility maximizer with reference wealth w faces a sequence of n related risky choices, where the outcomes of the risky choices are measured as changes in wealth. We ignore any dynamic effects in making sequential choices by assuming that individuals ignore the future when making current choices.[7] At the time the individual makes the t-th risky choice, he knows the outcomes of the previous $t - 1$ risky choices. Let $x_{1,\ldots,}x_{t-1}$ denote the outcomes of the previous decisions and define y_t to be the total payoff from previous decisions, that is, $y_t = \sum_{j=1}^{t-1} x_j$. For the case of the first risky decision, $y_1 = 0$. The value y_t is given the term *sequence-specific income*. The expected utility functional takes the form

$$V(F, y_t, w) = \int u(x, y_t, w)\, dF(x), \qquad (1)$$

where F is the probability distribution over gains and losses from *accumulated wealth*, which is defined as $w + y_t$ and x denotes an outcome of the probability distribution F, which is a gain or loss relative to accumulated wealth.

A major contribution of this paper is to distinguish between changes in reference wealth, w, and accumulations of sequence-specific income, y_t. We explicitly assume that reference wealth can change only prior to the first choice or after the last choice in a sequence. This is consistent with the notion that a sequence ends when the outcomes of the sequence are incorporated into reference wealth, while allowing reference wealth to change at the beginning of a sequence allows for framing effects. After reference wealth is set at the beginning of the sequence, it remains constant throughout the sequence, and only sequence-specific income changes. After the last choice in the sequence is made, sequence-specific income is added to reference wealth.

When making the first risky decision, the value of y_1 is set to zero, and the only relevant consideration for the shape of the utility function is the value w. Thus, the utility function $u(x,0,w)$ is isomorphic to the utility function $v(x,w)$ considered in Section 2, and should satisfy Properties 1 and 2.

To show how the house money and break-even effects can be accommodated by the three-argument utility function, suppose that the individual has already made some decisions in the sequence and has accumulated a sequence-specific loss, so that $y < 0$. The house money effect states that there exists some number $\alpha(y)$ such that the individual is averse to taking a 50:50 chance of winning or losing x when $0 < x < \alpha(y)$. Thus, to accommodate the house money effect, the utility function must be concave over $[-\alpha(y), \alpha(y)]$, as in Figure 2. The break-even effect states that there exist numbers $\beta(y) < 0$ and $\gamma(y) > 0$ such that the individual is willing to accept fair bets over the payoffs x and z, where $\beta(y) < x < 0$ and $z > \gamma(y)$. In Figure 2, $\beta(y)$ is taken to be identical to $-\alpha(y)$, so that the utility function is concave over $[\beta(y),0]$.[8] The figure shows a line passing through the point $(\beta(y), u(\beta(y),y,w))$ and the origin. For the break-even effect to be accommodated, the utility function must rise above this line and stay above it, as shown, and $\gamma(y)$ can be found where the utility function intersects the line from below. Any fair bet offering $x \in [\beta(y),0]$ and $z \in [\gamma(y),\infty)$ is preferred to getting zero for sure, satisfying the break-even effect.

The utility function depicted in Figure 2 is consistent with the finding that gamblers are more prone to bet on longshots at the end of the racing day (McGlothin, 1956). When a bettor is behind for the day, so that $y < 0$, the break-even effect suggests that the bettor is willing to take bets that provide some chance of breaking even. But, as Figure 2 shows, the bettor is unwilling to bet a large amount on a likely winner; the distributions the bettor is willing to take are low-cost bets with high potential payoffs, not high-cost bets with high potential payoffs.

Next consider the case of an initial gain, so that $y > 0$. In this case the house money effect states that the individual is willing to take a 50:50 chance of winning or losing x when $0 < x < \alpha(y)$. Thus, to accommodate the house money effect, the utility function must be convex over $[-\alpha(y), \alpha(y)]$, as in Figure 3. Also, following the initial gain, the break-even effect states that the individual is unwilling to accept fair bets over the payoffs x and z, where $0 < x < \beta(y)$ and $z < \gamma(y) < 0$. Following the method used to draw Figure 2, a line is drawn through the point $(\beta(y), u(\beta(y),y,w))$ and the origin in Figure 3. To accommodate the break-even effect, the utility function must lie below this line on $(-\infty, \gamma(y)]$.

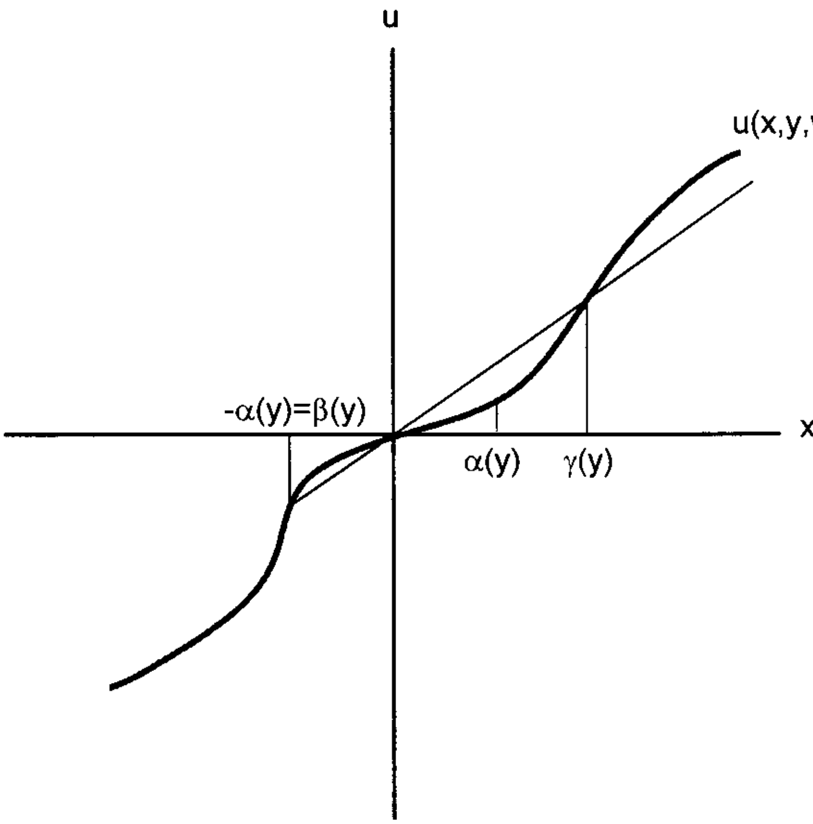

Figure 2. Utility function following an initial loss

In both Figures 2 and 3, the utility function has two convex and two concave regions, leading to the following property:

Property 3. Sequence sensitivity. There exist functions $a(y)$, $b(y)$, and $c(y)$ such that

(i) $u(x,y,w)$ is convex in x for $x \in (-\infty, a(y))$,
(ii) $u(x,y,w)$ is concave in x for $x \in (a(y), b(y))$,
(iii) $u(x,y,w)$ is convex in x for $x \in (b(y), c(y))$,
(iv) $u(x,y,w)$ is concave in x for $x \in (c(y), \infty)$.

In both Figures 2 and 3 the inflection point $a(y)$ is negative and the inflection point $c(y)$ is positive. The position of the middle inflection point, $b(y)$, depends on whether y is a gain or a loss.[9] In Figure 2, y is a loss, and the middle inflection point is positive. In contrast, in Figure 3, y is a gain, and the middle inflection point is negative. In both cases the middle inflection point is crucial for accomodating the house money effect. As dis-

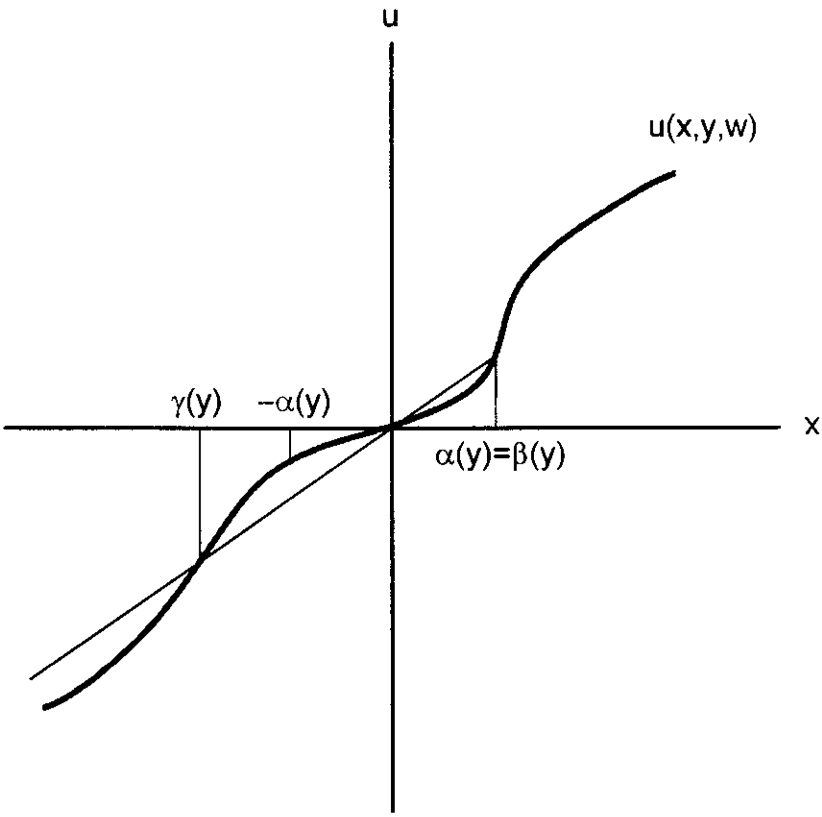

Figure 3. Utility function following an initial gain

cussed in Section 2, though, when $y = 0$ there should only be one inflection point at the origin, as in Figure 1. These considerations lead to the final property of the utility function:

Property 4. Sequence scaling. $a(y)$ is quasiconcave and nonpositive, $b(y)$ is nonincreasing, $c(y)$ is quasiconvex and nonnegative, and $a(0) = b(0) = c(0) = 0$.

The property that $a(0) = b(0) = c(0) = 0$ ensures that the utility function of Figure 1 is the one used for making the first decision in the sequence. In conjunction with this, the property that $a(y)$ is quasiconcave and nonpositive means that $a(y)$ moves closer to zero when y moves closer to zero. Similarly, the property that $c(y)$ is quasiconvex and nonnegative means that $c(y)$ moves closer to zero when y moves closer to zero. Finally, when coupled with Property 3 the requirement on $b(y)$ means that the individual is risk averse at $x = 0$ when $y < 0$ and risk seeking at $x = 0$ when $y > 0$. This captures the house money effect.

Note that according to Property 3 (and in both Figures 2 and 3) the utility function is convex for large losses and concave for large gains. There are three reasons for doing this. The first is the presumption that if all losses in the current gamble are sufficiently large, the usual risk seeking behavior will obtain, and that if all gains are sufficiently large, the usual risk averse behavior will hold. The second reason is that when $y = 0$, the three inflection points $a(y)$, $b(y)$, and $c(y)$ all collapse to the origin (by Property 4), and the resulting utility function is as shown in Figure 1. This allows the same utility specification to govern behavior in both the first period when there is no sequence-specific wealth and in later periods. The third reason is to capture the general property that individuals exhibit decreasing sensitivity to payoff changes as the payoffs move farther from the reference point, the same property that underlies the shape of the utility function in Figure 1.

4. Comparison with Other Models

The utility functions shown in Figures 2 and 3 and described by Properties 3 and 4 have two closely-related counterparts in the literature. One counterpart can be derived from the editing process outlined by Thaler and Johnson (1990) to accommodate their evidence of the house money and break-even effects. The other is the utility function proposed by Markowitz (1952).

Interestingly, the three-argument utility function described in the preceding section is almost exactly the same as the one proposed by Markowitz. He proposed a utility function defined over gains and losses from "customary wealth" that is convex for large losses, concave for small losses, convex for small gains, and concave for large gains. There are, however, two key differences between the utility function of Properties 3 and 4 and the utility function proposed by Markowitz. First, his utility function is based on only two arguments, not three.[10] By using three arguments in the utility function, individuals can be risk averse over gains and risk seeking over losses when there is no sequence-specific income, but can exhibit the house money effect and the break-even effect when there is sequence-specific income. Since Markowitz uses only two arguments in his utility function, he must give up one of these patterns. Since he assumes that the utility function is convex then concave then convex then concave (moving from left to right), he gives up the basic premise of risk aversion over gains and risk seeking over losses for the first decision, which we listed as Property 1 in Section 2. Nevertheless, the same sort of reasoning that leads us to the utility function shown in Figures 2 and 3 led Markowitz to his specification.

The second important difference from Markowitz' utility function is that here the inflection points depend on the amount of sequence-specific income, where Markowitz assumed that they are independent of sequence-specific income, but may depend on the level of "customary wealth," which corresponds to reference wealth w here. Making the inflection points dependent on sequence-specific income makes the house money and break-even effects more sensible, in that they can depend on the size of the payoffs relative to sequence-specific income. So, for example, larger prior gains make the individual willing to take larger fair bets, as proposed in Property 4.

Thaler and Johnson (1990) adopt a different approach in response to the evidence of the house money and break-even effects. Rather than construct a utility function that can accommodate the evidence, they propose an editing process based on a two-outcome utility function, but their editing process generates a utility function that can be compared to the three-argument utility function proposed here. Suppose that an individual has earned sequence specific income of y, and faces an additional gain or loss of x. Thaler and Johnson argue that there are two ways that this combination can be coded with the two-outcome utility function shown in Figure 1. One is to *integrate* the two outcomes and use the utility function $v(x + y,w)$. The other is to *segregate* the two outcomes by treating x as an additional gain or loss beyond y, and use the utility function $v(y,w) + v(x,w)$.

The *hedonic* editing process (Thaler, 1985) proposes that the individual codes the combination in the way that yields the highest utility; that is, the individual integrates the two outcomes if and only if $v(x + y,w) \geq v(y,w) + v(x,w)$. Thus, the hedonic editing process yields a three-outcome utility function $u(x,y,w) = \max \{v(x + y,w), v(y,w) + v(x,w)\}$. The evidence in Thaler and Johnson (1990) contradicts certain aspects of the hedonic editing hypothesis, so they propose an alternative, the *quasi-hedonic* editing process. It is summarized in Table 3. Figure 4a shows the resulting utility function when y is a loss and Figure 4b shows the resulting utility function when y is a gain. The utility functions are constructed from the two underlying two-argument utility functions, $v(y + x,w)$ and $v(x,w)$, which are also shown in the figures. The resulting three-argument utility function is depicted by the thick curve in each figure.[11]

The most important difference between the utility function in Figure 4a and its counterpart in Figure 2 is the discontinuity of the former at $x = -y$. The discontinuity is caused by loss aversion for the function v, which implies that when $y < 0$, $v(y,w) + v(-y,w) < v(0,w)$. This discontinuity also has the consequence of making the utility function convex at $x = -y$, and it therefore replaces the rightmost convex section of the utility function in Figure 2, enabling the Thaler-Johnson formulation to accommodate the break-even effect. However, as drawn in the figure, the utility function also generates some troubling behavior. After suffering an initial loss of y, the individual with the preferences shown would be willing to accept any two-outcome fair gamble that offers a chance of winning $-y > 0$ and has a sufficiently large loss. This is an extreme form of the break-even effect, since it implies that the individual will go to any lengths in an effort to break even, and it differs from the formal characterization in Section 1 in which the individual is only willing to

Table 3. Utility function derived from the quasi-hedonic editing process

Conditions on x and y	Segregate or integrate	$u(x,y,w)$
$x, y > 0$	Segregate	$v(y,w) + v(x,w)$
$x, y < 0$	Segregate	$v(y,w) + v(x,w)$
$x > 0, y < 0, \lvert x\rvert < \lvert y\rvert$	Segregate	$v(y,w) + v(x,w)$
$x > 0, y < 0, \lvert x\rvert > \lvert y\rvert$	Integrate	$v(y + x,w)$
$x < 0, y > 0, \lvert x\rvert < \lvert y\rvert$	Integrate	$v(y + x,w)$
$x < 0, y > 0, \lvert x\rvert > \lvert y\rvert$	Segregate	$v(y,w) + v(x,w)$

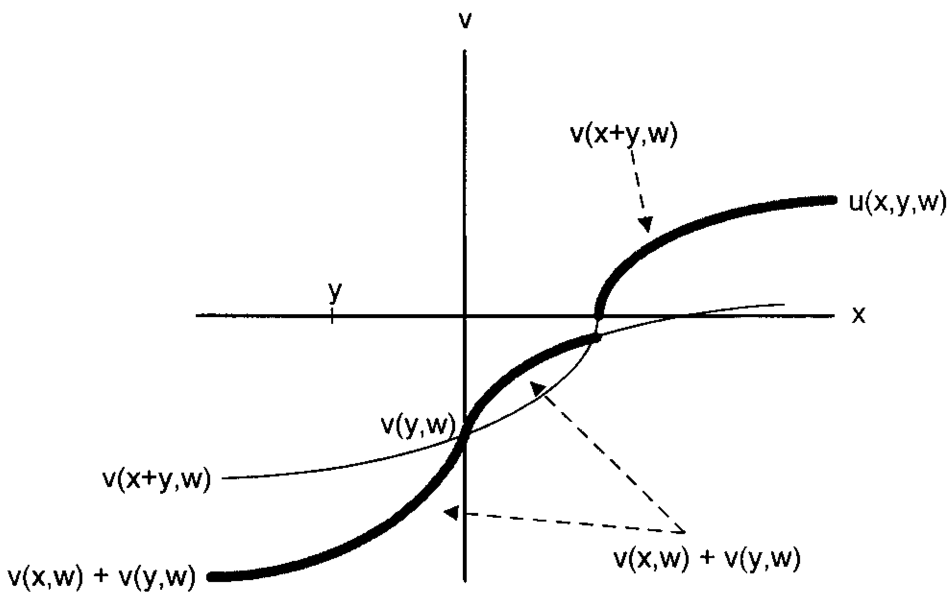

Figure 4a. Thaler and Johnson's quasi-hedonic editing after an initial loss

take fair gambles that offer a chance of breaking even if the potential loss is sufficiently small. In contrast, in Figure 4a the individual is willing to take fair gambles that offer a chance of breaking even if the potential loss is sufficiently large.

The case of an initial gain is shown in Figure 4b. It differs from the utility function in Figure 3 in two ways: it has a discontinuity and the rightmost convex section in Figure 3 is replaced by a kink between concave sections in Figure 4b. The discontinuity is not as troubling with an initial gain. According to the preferences shown in the figure, the individual is unwilling to take any fair gamble that entails a possible loss of y and a sufficiently large gain. This degree of risk aversion is not problematic in the same way that the degree of risk seeking was for the utility function depicted in Figure 4a, since it is also true of the Kahneman-Tversky utility function depicted in Figure 1.

The other difference between the model proposed here and the one proposed by Thaler and Johnson is that in their model the inflection points are fixed at $x = -y$ and $x = 0$, whereas in the model proposed here the inflection points are not fixed. Property 4 just requires that the inflection points vary with y and that they all converge to zero when $y = 0$. The extra generality entailed in Property 4 may or may not be warranted, and there is as yet no experimental evidence that sheds light on the location of the inflection points.

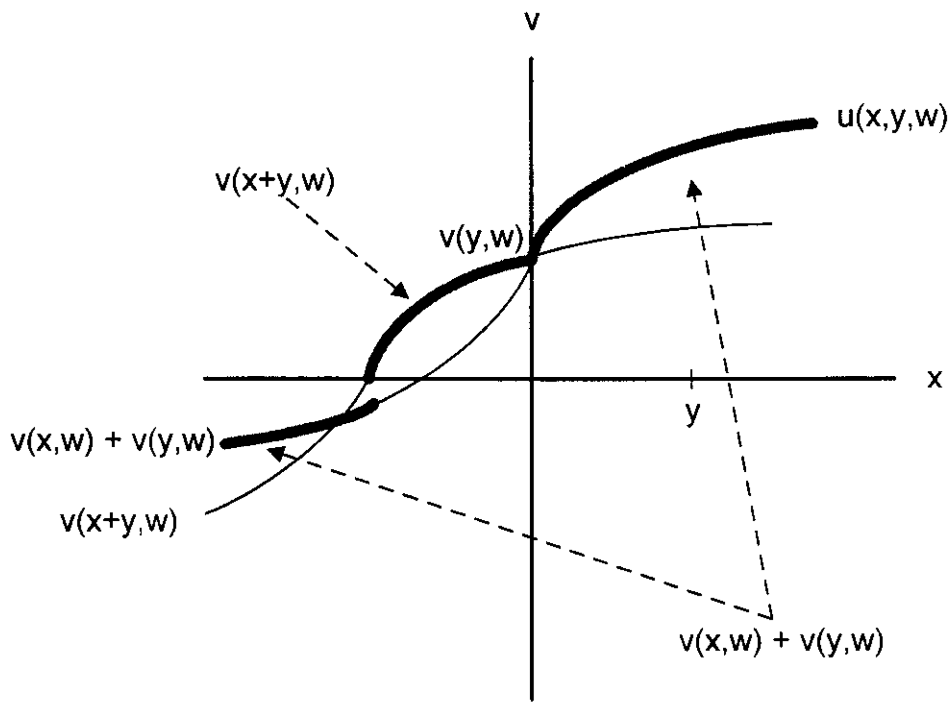

Figure 4b. Thaler and Johnson's quasi-hedonic editing after an initial gain

5. Dynamic Consistency and Consequentialism

For the purposes of fitting the experimental evidence, the driving feature of the preference function in (1) is the dependence of the utility function on past outcomes in the sequence. This suggests that tastes change during the sequence, raising concerns about dynamic consistency. In particular, and following the definition in Karni and Schmeidler (1991), preferences are *dynamically consistent* if the continuation of the optimal plan formulated at the outset of the sequence agrees with the optimal continuation plan at any point within the sequence. In other words, dynamic consistency requires that when we use the preference function (1) to predict behavior through the entire sequence, the individual's actual behavior matches the predicted behavior at every point in the sequence. Preferences are dynamically inconsistent if there is some point within the sequence at which the individual would wish to act differently than predicted.

Most arguments that preferences are dynamically inconsistent, especially those pertaining to nonexpected utility preferences, assume consequentialism (see Machina, 1989). Preferences satisfy *consequentialism* if choices are independent of history, and depend only on the payoffs at hand. More specifically, in terms of the preference function (1), consequentialism requires that the utility function over payoffs used at time t must be an increasing affine transformation of the utility function over payoffs used at time *t'*. But Propositions 1–4 already establish that consequentialism must be discarded to accommodate the house money and break-even effects. The preferences discussed here are explicitly history-dependent, and, in fact, the preference function itself depends on both reference wealth and sequence-specific income.

To see that the preferences in (1) are dynamically consistent, it is necessary to describe an optimal plan formulated at the outset of the sequence and compare it to actual behavior at any given point in the sequence. The decision planned for the first period can be determined using the utility function $u(x,0,w)$. The decision planned for the second period must be contingent on the outcome of the first decision, because $y_2 = x_1$. Thus, with the preferences discussed here, optimal plans are complete contingent plans, with the contingencies based on the outcomes from prior periods. In general, the decision planned for period t is contingent on the outcomes of the previous $t - 1$ decisions, and is selected using the utility function $u(x,y_t,w)$.[12] Once the individual actually reaches period t, he will have earned some level of sequence-specific income, y_t, and will make a decision according to the utility function $u(x,y_t,w)$. This is necessarily the same decision as predicted by the complete contingent plan.[13]

6. Application: Disposition Effects

Models with reference wealth and S-shaped utility have been used to explain the disposition effect, that is, the observed tendency of investors to hold losers too long and sell winners too soon (see Shefrin and Statman (1985) and Odean (1998)). The basic idea is the following. Initially the investor is risk averse because of loss aversion. Nevertheless, he chooses to invest in a risky asset. If reference wealth is not updated, an initial loss moves the investor into loss space where he is risk seeking, and he is thus willing to continue to bear the initial risk. On the other hand, an initial gain moves him into gains space, where he is risk averse, and if he is sufficiently more risk averse he may choose not to hold the asset any longer.

The model developed here makes some additional predictions regarding the disposition effect. To illustrate them, consider the setting investigated by Weber and Camerer (1998). Suppose that there is an asset whose value can gain or lose x in each period. After a loss in the first period, holding the asset in the second period can result in a loss of $2x$ or a return to zero. After two losses in a row, holding the asset in the third period can result in a loss of $3x$ or a loss of x. The house money effect states that an initial loss makes the individual risk averse over gambles whose payoffs are small relative to the initial loss, and the break-even effect states that an initial loss makes the individual risk seeking over gambles that allow a chance of recovering a loss. In Weber and Camerer's setting, the stakes are too large for the house money effect (if $\alpha(y) < |y|$), and the break-even effect suggests that the investor will hold the asset in the second period (as in Problem 3 in Table 2). After further uninterrupted losses, the stakes are too small for the break-even effect (if

$\gamma(y) \approx -y$), and the house money effect makes the individual more risk averse (as in Problem 4 in Table 2). Thus, the model implies that more people should sell the asset after two straight losses than sell the asset after a single loss.

Now suppose that there was an initial gain. Since holding the asset entails a high probability that the investor will lose all of his initial gains, the break-even effect suggests that he will not hold the asset. After two or more straight gains, the stakes are too small for the break-even effect to apply, and the house-money effect suggests that the investor becomes risk seeking and is more likely to hold the risky asset. Consequently, the model implies that fewer subjects should sell the asset after two straight gains than after a single gain.

The results of Weber and Camerer's experiment can be found in Table 4. The data shows that the volume of sales of the asset following a gain in period $t - 1$ was more than double the volume following a loss in period $t - 1$, which is the disposition effect. This evidence also supports the model's prediction that reference wealth is not updated during the sequence of investment decisions. Our model makes two additional predictions. First, sales should be higher after two losses than after a gain then a loss. The data does not support this conclusion, since the sales volumes were roughly equal in the two cases. Second, sales should be lower after two gains than after a loss then a gain. The data shows 3815 units sold after two straight gains and 5265 units sold after a loss then a gain, supporting the model's prediction. This finding matches that of Keasey and Moon (1996), whose experimental data confirmed the existence of the house money effect after a gain but not after a loss.

7. Conclusion

The main goal of this paper is to show that a utility function with two arguments—reference wealth and changes in wealth—cannot accommodate all of the behavioral patterns suggested by experimental evidence. In particular, a two-argument utility function can handle either the pattern that individuals are risk averse over gains and risk seeking over losses, or it can handle the house money and break-even effects. It cannot do both. To fit all of these patterns, we analyze sequences of choices using a utility function with three arguments—reference wealth, income from the sequence of choices, and deviations from current wealth—and show that the three-argument utility function is able to conform to all of the desired patterns.

Table 4. Disposition effect evidence from Weber and Camerer (1998)

Gain or loss in $t - 2$	Gain or loss in $t - 1$	Units sold
Gain	Gain	3815
Loss	Gain	5265
Gain	Loss	2067
Loss	Loss	2002

According to the model, for the first decision individuals are risk averse over gains and risk seeking over losses. For later decisions in the sequence, if income from the sequence so far is positive, individuals are risk seeking over small fair bets but risk averse over losses approximately the same size as their winnings to date. Put another way, after winning some money individuals are willing to bet with the "house money" but they are unwilling to risk losing all of their winnings. In contrast, if individuals have suffered a loss in the sequence, they are averse to small fair bets but are willing to take fair bets that give them some chance of winning their money back. To use the model, then, it is important to define which choices are in the relevant sequence, and it is important to identify sequence-specific gains and losses.

This model has an important implication for experimental attempts to discern risk attitudes over losses. In experiments using real payoffs, it is common to give subjects some fixed amount of money at the beginning and then let them choose between lotteries whose payoffs are losses. This way the subjects do not lose money during the experiment. It is commonly found in this type of experiment that subjects exhibit risk seeking over losses, that is, a risky gamble with negative payoffs is preferred to losing the expected value of the gamble for sure. The model developed here suggests an alternative explanation: subjects treat the initial payment as the first payoff in a sequence, and then behave in a risk seeking manner because of the house money effect. If the model proposed here is accurate, then economists do not have any information about risk attitudes over first-period losses with real payoffs. We have information about risk attitudes over first-period losses using hypothetical payoffs, which avoid the initial positive payment, and we have information about the second choice in a sequence following an initial gain. Alternative experimental procedures are needed to identify risk attitudes over losses in the first choice in a sequence.

Appendix

Proof of Proposition 1. Given y, choose $x \in (0,\alpha(y))$ with $|x| < |y|$. By the house money effect,

$$\frac{1}{2} v(y + x,w) + \frac{1}{2} v(y - x,w) \lesseqgtr v(y,w)$$

as y $\gtreqless$ 0, which violates the concavity of v over positive values of its first argument and convexity of v over negative values of its first argument.

Proof of Proposition 2. Fix $y > 0$ and choose $x \in (0,\alpha(y))$ with $x < y$. By the house money effect,

$$\frac{1}{2} v(x,w + y) + \frac{1}{2} v(- x,w + y) > v(0,w + y)$$

which violates loss aversion.

Proof of Proposition 3. Fix $y < 0$ and choose $x \in (\beta(y), 0)$ and $z > \gamma(y)$. Let $p = z/(z - x)$. By the break-even effect,

$$pv(x,w + y) + (1 - p)v(z,w + y) > v(0,w + y)$$

which violates loss aversion.

Proof of Proposition 4. Fix $y < 0$. Choose $x \in (-\alpha(y),0) \cap (\beta(y),0)$ and $z > \gamma(y)$. Define $p(x) = z/(z - x)$. Then by the house money effect and the break-even effect,

$$\frac{1}{2}v(x,w + y) + \frac{1}{2}v(-x,w + y) < v(0,w + y)$$
$$< p(x)v(x,w + y) + (1 - p(x))v(z,w + y).$$

This implies that the line segment connecting the point $(-x,v(-x,w + y))$ to the point $(x,v(x,w + y))$ is flatter than the line segment connecting the point $(-x,v(-x,w + y))$ to the point $(z, v(z,w + y))$, which means that

$$\frac{v(-x,w + y) - v(x,w + y)}{2x} < \frac{v(z,w + y) - v(-x,w + y)}{z - x}. \quad (1)$$

Now let $x \to 0$. The left-hand term in (2) goes to the first partial derivative $v_1(0,w + y)$ if v is differentiable in its first argument at zero, and it is no less than the right-hand partial derivative $v_1^+(0,w + y)$ if v is not differentiable in its first argument. The right-hand term in (2) goes to $[v(z,w + y) - v(0,w + y)]/z$. Since the slope of the utility function at the origin is less than the slope of the chord connecting the origin to z, the function cannot be concave over $[0,z]$, which violates the assumption that the utility function is S-shaped.

A similar argument establishes a violation which $y > 0$.

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Notes

1. See, for example, Kahneman and Tversky (1979), Battalio, Kagel, and Jiranyakul (1990) and Tversky and Kahneman (1991).
2. Such a model is used successfully by Bowman, Minehart, and Rabin (1996), for example.
3. These two effects were also described by Markowitz (1952).
4. Battalio, Kagel, and Jiranyakul were not the first to test these notions. For example, Kahneman and Tversky (1979) tested many of the same hypotheses using hypothetical payoffs.
5. The assumption of expected utility maximization is not the best one for use with a utility function defined over changes in wealth, because probability weighting schemes are needed to explain the purchase of insurance against losses. Nevertheless, the expected utility assumption allows for the usual graphical analysis of lotteries, and so is employed here for expositional purposes.
6. The characterization of loss aversion is dependent on the individual being an expected utility maximizer. For evidence supporting loss aversion, see Tversky and Kahneman (1991), although it should be noted that the experiments of Battalio, Kagel, and Jiranyakul (1990) do not support loss aversion.
7. Allowing for the anticipation of future decisions would only complicate the mathematics without changing any of the results, which all pertain to how past outcomes and reference wealth affect current decisions.
8. This convenience of $\beta(y) = -\alpha(y)$ does not fit Thaler and Johnson's evidence, and is therefore unwarranted, but it makes the figure less cluttered. Their evidence suggests that $\beta(y) < -\alpha(y)$.
9. Note that the inflection points $a(y)$, $b(y)$, and $c(y)$ may differ from the boundaries for the house money and break-even effects, $\alpha(y)$, $\beta(y)$, and $\gamma(y)$.
10. Markowitz does allow for a distinction between "present wealth," which is $w + y$ in our model, and "customary wealth," which is w in our model. In his construction, total wealth at the middle inflection point equals w, whereas for the utility function constructed here total wealth at the middle inflection point differs from both w and $w + y$ as long as $y \neq 0$. In our notation, Markowitz' utility function takes the form $v(x + y, w)$, and the origin corresponds to the point where $x + y = 0$.
11. The utility functions generated by the hedonic editing process are the upper envelope of the component utility functions shown in Figures 4a and 4b.
12. Treating y_t as a state variable makes the process Markovian in the sense that the utility function in period t depends on the value of y in period $t - 1$ but not on earlier values of y.
13. This argument is very similar to Machina's (1989) argument as to why nonexpected utility preferences are dynamically consistent: Once the assumption of consequentialism is dropped, dynamic consistency problems disappear.

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