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A Further Examination of Cumulative Prospect Theory Parameterizations

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Abstract

Recent experimental studies have focused on fitting parameterized functional forms to cumulative prospect theory's weighting function. This paper examines the behavioral implications of the functional forms and the estimated parameters. We find that none of the parameterizations can simultaneously account for gambling on unlikely gains and the Allais paradox behavior or other strong choice patterns from experiments. Parameter estimates that lead to reasonable amounts of insurance and gambling behavior tend to also generate large risk premia. Taken as a whole, the analysis suggests that the functional forms proposed in the literature are not suitable for generalization to applied settings.

Keywords: cumulative prospect theory, probability weighting function, expected utility theory, Allais paradox, rank-dependent utility, risk attitudes

JEL Classification: D81

Of the many models that have been proposed to accommodate violations of expected utility, Tversky and Kahneman's (1992) cumulative prospect theory with rank-dependent probabilities has received the most recent attention.[1] In fact, in his overview of the field of behavioral economics, Camerer (1998) states that it is time to abandon expected utility in favor of this more general alternative. There are at least two good reasons for doing so. First, the model has already been applied fruitfully to some interesting settings, such as explaining the equity premium puzzle (Epstein and Zin, 1990), the behavior of options traders (Fox, Rogers, and Tversky, 1996), and the low incidence of tax evasion (Bernasconi, 1998). Second, researchers have already specified parameterized versions of the model (Tversky and Kahneman, 1992; Prelec, 1998), and the parameters have been estimated from experimental data (Tversky and Kahneman, 1992; Camerer and Ho, 1994; Wu and Gonzalez, 1996). These parameterized versions are sufficiently simple, typically adding only one or two parameters to the usual expected utility specifications, to lend themselves to further applications. Before accepting these parameterizations of the model, though, they should be explored more carefully.

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Cumulative prospect theory (with rank-dependent probabilities) has three principal components. First, there is a value function defined over (monetary) gains, similar to the utility function in expected utility theory. Second, there is a loss aversion function that transforms utilities over gains into utilities over corresponding losses. This function allows individuals to be risk averse over gains but risk seeking over losses, and for losses to matter more than gains. Finally, there is a weighting function used to transform probability distributions, which allows the model to accommodate such violations of expected utility as the Allais paradox. The loss aversion function was new to the original prospect theory (Kahneman and Tversky, 1979), while the weighting function is new to cumulative prospect theory but is similar to that used in rank-dependent expected utility theory (Quiggin, 1993). The research that parameterizes cumulative prospect theory concentrates on estimating the parameters of the weighting function, focusing little attention on the other functions. These studies have explored the robustness of the weighting function's parameter estimates, but none have explored the robustness, or appropriateness, of any combination of the parameterized components together. That is the purpose of this study.

We look at the commonly-used single-parameter specifications of the value and weighting functions introduced by Tversky and Kahneman (1992) and Prelec (1998) and look at the behavioral implications of different parameter combinations. We begin with insurance and gambling behavior. For some parameter combinations, the model implies that individuals insure against sufficiently unlikely losses. Loss aversion and the reflection effect (behavior in the loss domain is the opposite of behavior in the gains domain) imply that people should also find fair bets with sufficiently unlikely gains attractive. We look at parameter combinations that imply "reasonable" threshold probabilities below which individuals find bets on unlikely gains attractive, and compare them with parameter combinations which imply the standard Allais paradox behavior. The sets of parameter combinations overlap very little, and none of the parameter combinations estimated from experimental work can accommodate both types of behavior.[2] Since this result may stem from the fact that the Allais paradox payoffs are extremely large, we repeat the analysis using evidence from choices with small payoffs with the same result.

We next look more closely at the threshold probabilities that differentiate risk avoidance behavior from risk taking behavior. This is a useful exercise on its own, because while the implication that individuals insure against sufficiently unlikely losses and bet on sufficiently unlikely gains is well-known, the values of "sufficiently unlikely" are not. We find that these threshold probabilities are quite sensitive to the choice of parameter values, and that only Tversky and Kahneman's (1992) estimates are compatible with independent evidence regarding these threshold values. We also examine the magnitudes of the risk premia implied by the parameterizations and find that many of the estimated parameters imply extreme risk attitudes for low- and high-probability gains.

The paper proceeds as follows. Section 1 reviews the cumulative prospect theory model, paying particular attention to the functional forms typically used for the utility function and the probability transformation function. These functional forms are parametric, and the parameter estimates from experimental studies are also reviewed. Section 2 investigates the robustness of the parameters to insurance and gambling behavior; it also investigates the compatibility of the parameter estimates with Allais-type behavior. We find that the sets of parameter values that are consistent with insurance and gambling are, for the most part, inconsistent with Allais-type behavior. Section 3 looks at the threshold probabilities separating insurance and gambling behavior. It is shown that for the functional forms and parameter estimates offered by Tversky and Kahneman (1992), individuals insure against a loss or bet on a gain when the probability is below about 0.24, thereby calibrating the notion of "sufficiently unlikely." Other researchers' parameter estimates, however, lead to virtually no insurance against unlikely losses or gambling on unlikely gains. Section 4 considers risk premia, and shows that the proposed functional forms lead to very large risk premia. Section 5 presents our conclusion—given their inability to accommodate combinations of important behavioral patterns with a single set of parameters, the fact that the parameters estimated from most experimental evidence are incompatible with gambling and insurance, and the size of the implied risk premia, the currently-used functional forms are inappropriate for empirical work.

1. Cumulative prospect theory

As constructed by Tversky and Kahneman (1992), cumulative prospect theory treats gains and losses separately. Suppose a gamble is composed of $m + n + 1$ monetary outcomes, $x_{-m} < \cdots < x_0 < \cdots < x_n$, which occur with probabilities $p_{-m}, \ldots, p_n$, respectively. The corresponding gamble can be denoted by the pair $(\mathbf{x}; \mathbf{p})$, where $\mathbf{x} = (x_{-m}, \ldots, x_n)$ and $\mathbf{p} = (p_{-m}, \ldots, p_n)$. Using the familiar expected utility theory, the expected utility of this gamble would simply be the summation of the utility of each outcome multiplied by its respective probability, or $\mathrm{EU}(\mathbf{x}; \mathbf{p}) = \sum p_i u(x_i)$. Cumulative prospect theory is somewhat different, however. Define

$$V^+(\mathbf{x}; \mathbf{p}) = g(p_n)u(x_n) + \sum_{k=1}^{n}\left[g\left(\sum_{j=0}^{k} p_{n-j}\right) - g\left(\sum_{j=0}^{k-1} p_{n-j}\right)\right]u(x_{n-k}), \tag{1}$$

and

$$\begin{aligned} V^-(\mathbf{x}; \mathbf{p}) &= g(p_{-m})u(x_{-m}) \\ &\quad + \sum_{k=1}^{m}\left[g\left(\sum_{j=0}^{k} p_{-(m-j)}\right) - g\left(\sum_{j=0}^{k-1} p_{-(m-j)}\right)\right]u(x_{-(m-k)}). \end{aligned} \tag{2}$$

The preference value of the gamble $(\mathbf{x}; \mathbf{p})$ is given by

$$V(\mathbf{x}; \mathbf{p}) = V^+(\mathbf{x}; \mathbf{p}) + V^-(\mathbf{x}; \mathbf{p}). \tag{3}$$

The expression V^+ measures the contribution of gains, and V^- measures the contribution of losses. The function $g(p)$ is a probability weighting function assumed to be increasing with $g(0) = 0$ and $g(1) = 1$, and $u(x)$ is a utility (or value) function assumed to be increasing with $u(0) = 0$. Note that if all outcomes are losses, the cumulative prospect preference function in (2) is identical to the rank-dependent expected utility preference function (e.g., Quiggin, 1993). Both apply the probability weighting function to the probability of the lowest outcome first. If all outcomes are gains the preference function in (1) differs from the rank-dependent expected utility preference function by applying the weighting function to the probability of the highest outcome first. In general, cumulative prospect theory weights extreme outcomes first.

In this paper we are concerned primarily with two- and three-outcome gambles with only gains or only losses. From (1)–(3), the preference value of a two-outcome gamble with either $x_2 < x_1 \leq 0$ or $x_2 > x_1 \geq 0$ is

$$V(x_1, x_2; p_1, p_2) = g(p_2)u(x_2) + (1 - g(p_2))u(x_1), \tag{4}$$

while the preference value of the three-outcome gamble with either $x_3 < x_2 < x_1 \leq 0$ or $x_3 > x_2 > x_1 \geq 0$ is

$$V(\mathbf{x}; \mathbf{p}) = g(p_3)u(x_3) + [g(p_3 + p_2) - g(p_3)]u(x_2) + [1 - g(p_3 + p_2)]u(x_1). \tag{5}$$

The value (utility) function proposed by Tversky and Kahneman is

$$u(x) = \begin{cases} x^{\alpha} & \text{for } x \geq 0, \\ -\lambda(-x)^{\alpha} & \text{for } x < 0. \end{cases} \tag{6}$$

For $\alpha < 1$, the value function exhibits risk aversion over gains and risk seeking over losses. Furthermore, if λ, the loss-aversion coefficient, is greater than one, individuals are more sensitive to losses than gains.

Tversky and Kahneman (1992) use the weighting function

$$g(p) = \frac{p^{\gamma}}{\left(p^{\gamma} + (1 - p)^{\gamma}\right)^{1/\gamma}}. \tag{7}$$

Figure 1 displays this weighting function. Note that relative to the expected utility model, with this weighting function cumulative prospect theory overweights extreme outcomes when their probabilities are low and underweights them when their probabilities are high. When probability distributions have only two outcomes and the extreme outcome is underweighted, the less extreme outcome is, by default, overweighted, which is consistent with risk aversion over gains and risk-seeking over losses. In contrast, overweighting the extreme outcome is consistent with risk seeking over gains and risk aversion over losses. As shown in the Figure 1, decreasing γ causes the weighting function to become more curved and to cross the 45° line farther to the right.

Prelec (1998) proposes an alternative specification for the weighting function:

$$g(p) = e^{-(-\ln p)^{\gamma}}. \tag{8}$$

Its shape is nearly identical to that of Tversky and Kahneman's weighting function. The key difference is that Prelec's specification is based on behavioral axioms rather than the

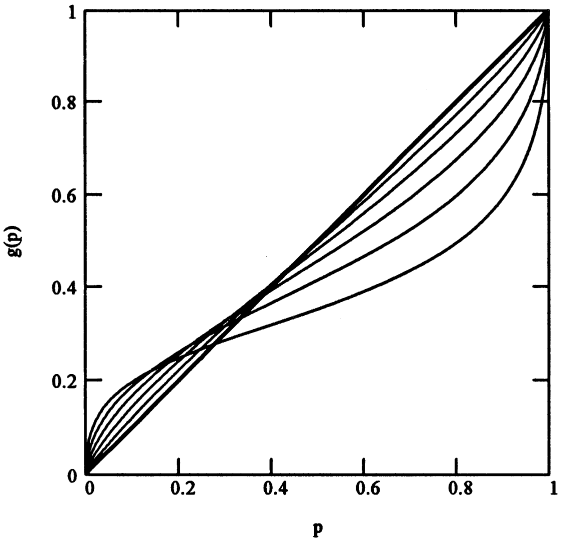


Figure 1. Kahneman and Tversky's weighting function.

convenience of the functional form, and it must cross the 45° line at $p = 1/e \approx 0.37$. A decrease in γ causes Prelec's weighting function to become more concave to the left of $1/e$ and more convex to the right of $1/e$, as shown in Figure 2.[3]

Several studies provide estimates of the parameters of the different functions. Tversky and Kahneman (1992) gave subjects a series of pairwise choices in an effort to derive

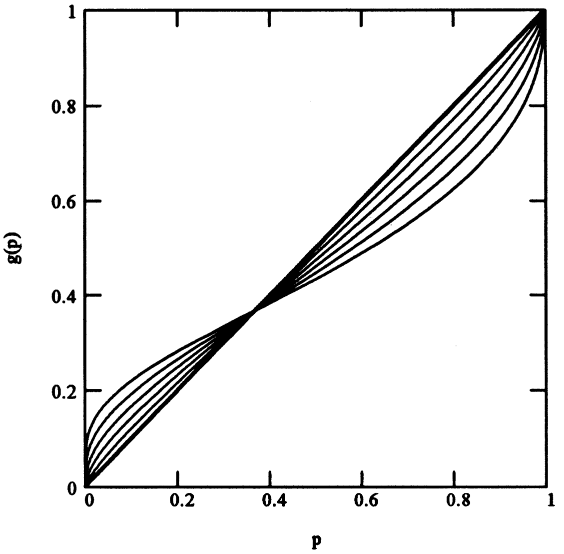


Figure 2. Prelec's weighting function.

certainty equivalents for gambles. Using this data to fit Equations (6) and (7), they estimated $\alpha = 0.88$, $\lambda = 2.25$, $\gamma = 0.61$ for gains, and $\gamma = 0.69$ for losses. Camerer and Ho (1994) used data from nine studies that have pairwise choice tasks related to betweenness. Their overall estimates of Equations (6) and (7) yield $\alpha = 0.32$ and $\gamma = 0.56$. Wu and Gonzalez (1996) give subjects pairwise choices from five different "ladders," each containing eight "rungs" which differ from each other by a common consequence. Their estimates of equations (6) and (7) yield $\alpha = 0.52$ and $\gamma = 0.74$. They also fit the data to Prelec's weighting function (8), yielding $\alpha = 0.48$ and $\gamma = 0.74$. The fact that these parameter estimates are robust to the use of different data from different tasks leads one to believe that these parameterizations are useful for estimation with non-experimental data. In the remainder of the paper we investigate out-of-sample properties of these parameterizations in an attempt to determine whether or not they really are suitable for broad application.

2. Gambling on unlikely gains and the Allais paradox

The specifications of cumulative prospect theory reviewed in the preceding section have three parameters: the risk attitude coefficient α, the loss aversion coefficient λ, and the weighting function coefficient γ. This paper is primarily concerned with investigating the interaction of the parameters α and γ, and accordingly we restrict attention to settings in which either all outcomes are gains or all are losses. We begin the investigation with an exploration of parameter combinations that generate certain behavioral patterns. Specifically, we look at parameter combinations that generate certain gambling patterns and that exhibit the standard Allais paradox behavior.

One implication of the functional forms specified in Equations (6) through (8) is that individuals bet on sufficiently unlikely gains, even though the utility function is concave over gains. Note that in Figures 1 and 2 if the probability of a gain is sufficiently low, that probability is overweighted, which tends to make the individual risk seeking. If this overweighting is enough to overcome the risk aversion of the utility function, the individual is willing to take a fair gamble that has a sufficiently unlikely gain. More concretely, suppose that an individual faces a gamble which entails winning $\$G$ with probability p or getting \$0 otherwise. He is indifferent to playing the gamble or receiving the expected value for sure when

$$g(p)u(G) + (1 - g(p))u(0) = u(pG). \tag{9}$$

Figure 3 shows combinations of values of α and γ that imply identical values of p in Equation (9), using Tversky and Kahneman's weighting function (7).[4] For every parameter combination in a level set, then, individuals have the same threshold probability above which they are risk averse over gains and below which they are risk seeking. Combinations below the level sets correspond to lower threshold probabilities, which translates into less gambling on unlikely gains. The figure shows that most of the parameter space

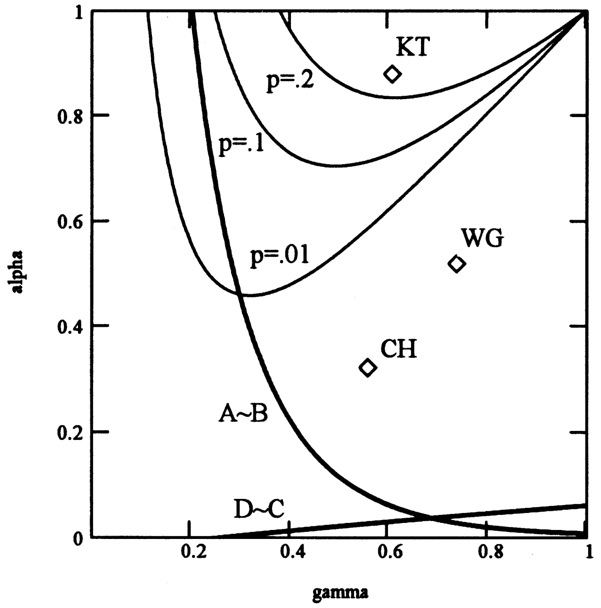


Figure 3. Tversky and Kahneman parameter combinations.

implies almost no gambling on unlikely events. Furthermore, for high values of α and medium values of γ, the threshold probability is very sensitive to the parameters.

Figure 3 also shows combinations of parameter values that are consistent with the behavior in the Allais paradox. In the Allais paradox, subjects typically choose lottery $A = (\$1\text{M}; 1)$ over lottery $B = (\$0, \$1\text{M}, \$5\text{M}; 0.01, 0.89, 0.10)$, where "M" denotes millions. Subjects also choose $D = (\$0, \$5\text{M}; 0.9, 0.1)$ over $C = (\$0, \$1\text{M}; 0.89, 0.11)$. This behavior is inconsistent with expected utility, and it is the example that initiated the search for alternative models. The region between the curves labeled "A $\backsim$ B" and "C $\backsim$ D" in Figure 3 shows the parameter combinations that are consistent with the Allais paradox behavior. Above the region individuals are not sufficiently risk averse and choose B over A, and below the region individuals are too risk averse to choose D over C.

One surprising result is that, for the Tversky and Kahneman weighting function specification, cumulative prospect theory has difficulty accommodating both gambling on unlikely gains and the Allais paradox. There are parameter combinations that generate both the Allais behavior and some gambling, but only for threshold probabilities less than 0.07. A large portion of the Allais-consistent combinations imply only negligible gambling on unlikely gains.

Figure 3 also shows parameter estimates based on various experimental data. Tversky and Kahneman (1992) estimate $\alpha = 0.88$ and $\gamma = 0.61$, which is in the region where individuals tend to gamble on unlikely gains, but is inconsistent with Allais behavior. Camerer and Ho (1994) estimate $\alpha = 0.32$ and $\gamma = 0.56$, which is near the edge of the Allais region but is inconsistent with gambling on unlikely gains.[5] Wu and Gonzalez (1996) estimate $\alpha = 0.52$ and $\gamma = 0.74$, which is outside of both regions.

Figure 4 shows the same information for Prelec's weighting function (8). The primary difference between this weighting function and the Tversky and Kahneman weighting function is that in Figure 4 the level sets for the threshold gambling probabilities are upward sloping, whereas in Figure 3 they are somewhat U-shaped. As with the Tversky and Kahneman weighting function, though, the set of combinations consistent with some gambling on unlikely gains and the set of combinations consistent with Allais behavior barely overlap. For Prelec's weighting function, Wu and Gonzalez (1996) estimate $\alpha = 0.48$ and $\gamma = 0.74$, which lies outside both the gambling and Allais regions.

One can speculate as to why the different parameterized versions of cumulative prospect theory fail to capture both types of behavior. To be willing to pay the expected value for a gamble with an unlikely gain, the individual must be somewhat risk seeking. This means that the overweighting of the gain by the weighting function must overcome the risk aversion of the utility function. But, to exhibit the Allais behavior the individual must take \$1M for sure over a risk with an expected value of \$1.39M and a small probability of exceeding this amount. Such a choice requires quite a bit of risk aversion. All parameter combinations that are consistent with the Allais paradox choices have high degrees of risk aversion for the utility function, too high for the weighting function to overcome, so that the individual is risk seeking over unlikely gains.

One possible reason for this negative result stems from the size of the Allais paradox payoffs. Rabin (2000) shows that, at least for expected utility preferences, a utility function calibrated for low-stakes gambles implies unreasonable behavior for high-stakes gambles, and vice-versa.[6] If attention is concentrated on either the gains domain or the

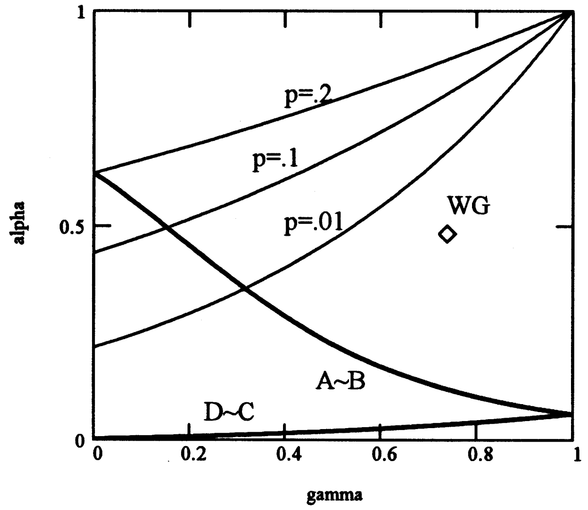


Figure 4. Prelec parameter combinations.

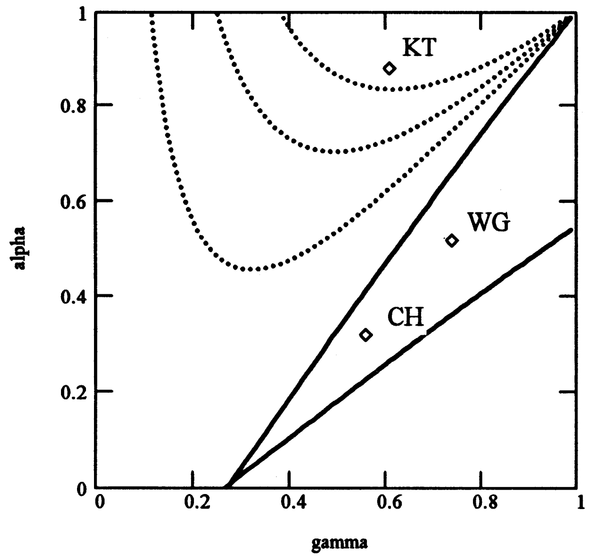


Figure 5. Tversky and Kahneman parameter combinations with Battalio, Kagel, and Jiranyakul (1990) choice sets.

loss domain, so that loss aversion effects are avoided, the same result should hold for cumulative prospect theory preferences. To counter this explanation, we identify parameter combinations that are compatible with choices over lotteries with small monetary outcomes and find that these, too, are disjoint from the set of combinations that imply "reasonable" threshold probabilities between gambling and insurance.

Battalio, Kagel, and Jiranyakul (1990), identify three 2-outcome lotteries for which at least 80% of the subjects made the same choices in the real-payoff treatments. Letting (x, p) denote a lottery which pays $\$x$ with probability p and \$0 otherwise, they find that 81% of subjects prefer $(\$12, 1)$ to $(\$20, 0.6)$, 80% of subjects prefer $(\$12, 0.20)$ to $(\$20, 0.12)$, and 84% of subjects prefer $(\$27, 0.16)$ to $(\$18, 0.20)$. All parameter combinations predict the first preference, while Figure 5 shows the parameter combinations that are consistent with the other two patterns.[7] Note that these parameter combinations imply that there is no gambling over unlikely gains, contrary to other evidence. Consequently, the failure of the model to accommodate both choice behavior and gambling-insurance probability thresholds is not simply an artifact of the large payoffs in the Allais paradox.

3. Unlikely gains and losses

Figures 3 and 4 show that for some parameter combinations individuals are willing to pay for bets with unlikely gains. This is consistent with the argument made above that individuals with cumulative prospect theory preferences bet on sufficiently unlikely gains and insure against sufficiently unlikely losses. In this section we explore this gambling and insurance behavior in more detail. The results are useful for judging the appropriateness of the proposed parameterizations of cumulative prospect theory, but they are also interesting in and of themselves. While it is well understood that cumulative prospect theory implies betting on sufficiently unlikely gains and insuring against sufficiently unlikely losses, no one has determined what is meant by "sufficiently unlikely." This section gives the threshold probability values for different parameter combinations.

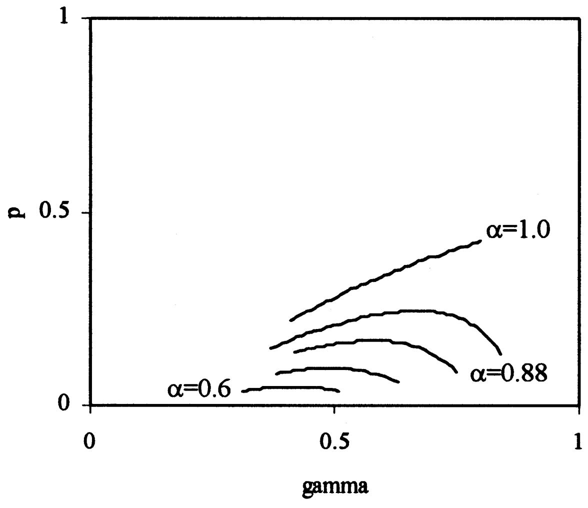
The threshold probability at which an individual is indifferent between taking a gamble with an unlikely gain of G and its expected value for sure is given by Equation (9) above. Under the assumptions that $u(x)$ is the power function and that $u(0) = 0$, (9) reduces to $g(p) = u(p)$, with the individual preferring the gamble when $g(p) > u(p)$ and preferring the expected value for sure when $g(p) < u(p)$. In the loss case, the threshold probability at which an individual is indifferent between facing the possible loss of L and losing the expected value for sure satisfies

$$g(p)u(-L) + (1 - g(p))u(0) = u(-pL). \tag{10}$$

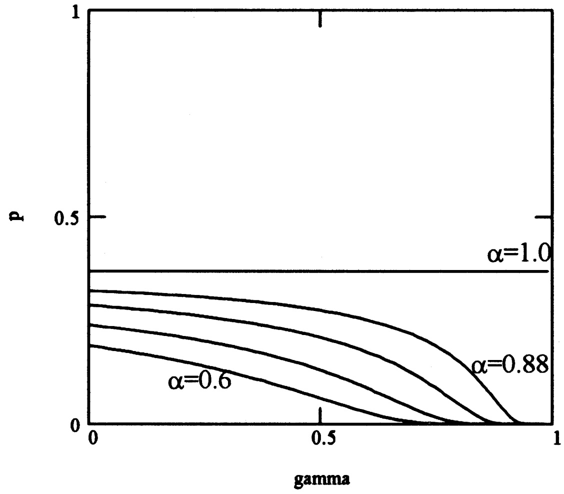
Once again, with the assumptions made on $u(x)$ this reduces to $g(p) = u(p)$, with the individual preferring the expected value when $g(p) < u(p)$ and preferring the gamble when $g(p) > u(p)$. Note that in both cases the equation determining the threshold probability is independent of the amount of the gain or loss.

Figure 6 plots the threshold probabilities for several different parameter specifications. Figure 6(a) uses Tversky and Kahneman's weighting function, and Figure 6(b) uses Prelec's. The figure reconfirms what is shown in Figures 3 and 4—for the individual to bet on unlikely gains or insure against unlikely losses, the utility function parameter must be high. When α is low, the convexity of the weighting function cannot overcome the concavity of the utility function. The α estimates obtained by Camerer and Ho (1994) and Wu and Gonzalez (1996) are too low. The Tversky and Kahneman estimates ($\alpha = 0.88$, $\gamma = 0.61$ for gains, $\gamma = 0.69$ for losses) allow for some betting on unlikely gains and some insurance against unlikely losses, with a threshold probability of about 0.24.

There is some independent evidence with which this threshold probability is consistent, although it deserves further study. Beginning with the loss domain, in their preference reversal experiments, MacDonald, Huth and Taube (1992) find that 51% of the subjects in the real-payoff treatments prefer a .25 chance of losing $4L$ to a sure loss of L, where L is either \$7 or \$8. The remainder of the subjects preferred taking the risk. A threshold probability of 0.24 fits this aggregate data pretty well. In the gains space, as part of their preference reversal study Cox and Grether (1996) elicit certainty equivalents for a lottery with a 7/36 chance of winning \$9.00 and a 29/36 chance of losing \$0.50, and a lottery with a 11/36 chance of winning \$16.00 and a 25/36 chance of losing \$1.50. For both cases, and with a number of different incentive-compatible elicitation mechanisms, they find that the average certainty equivalents exceed the means of the gambles. So, in the gains space, the threshold probability is higher than 11/36.



(a)



(b)

Figure 6. (a) Threshold probabilities using Tversky and Kahneman's weighting function. (b) Threshold probabilities using Prelec's weighting function.

4. Risk premia

The preceding section used threshold probabilities to address the suitability of different cumulative prospect theory parameterizations and to illustrate the sensitivity of implied behavior to changes in the parameter values. Threshold probabilities are just one aspect of risk-seeking or risk-avoiding behavior. They show when individuals are willing to forego sure payoffs to take risks, but not how much they are willing to forego. That is the subject of this section.

Assume an individual faces a gamble in which he may win \$$G$ with probability p and nothing otherwise. We are interested in the payment he would accept in place of playing the gamble. In particular, we are interested in calculating the amount π above the expected value which leaves him indifferent to gambling or not and can be determined by

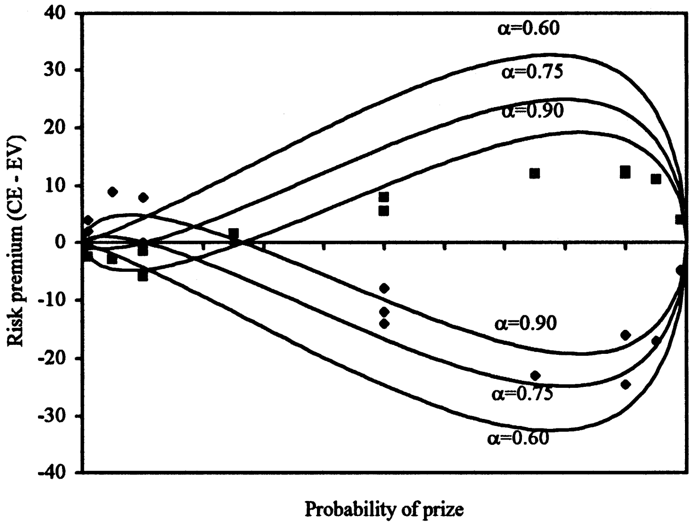
$$g(p)u(G) = u(\pi + pG). \tag{11}$$

Note that $\pi + pG$ is the certainty equivalent for the gamble, and π is the premium required by the individual to *forego* the risk. With this construction, a positive premium signifies that the individual is risk seeking, and a negative premium signifies that he is risk averse.

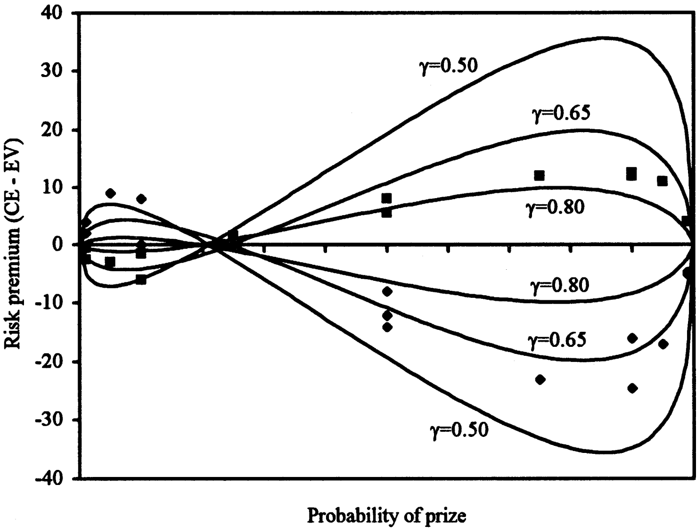
Figure 7 shows different values of the risk premium for different probabilities of a \$100 gain using the Tversky and Kahneman parameterization. Figure 7(a) holds γ constant at 0.65 and uses $\alpha = 0.9, 0.75$, and 0.60. Figure 7(b) holds α constant at 0.88 and uses $\gamma = 0.50, 0.65$, and 0.80. In both panels the plotted points are the average risk premia from Tversky and Kahneman's (1992) experiment, and the curves and points that are above the axis for large probabilities and below the axis for small probabilities correspond to gambles over losses. The remaining curves and points corresponding to gambles over gains. Tversky and Kahneman's preferred parameter values of $\alpha = 0.88$ and $\gamma = 0.61$ yield some large risk premia. For example, when $p = 0.1$, $\pi = \$4.82$; this means that the individual is indifferent between receiving \$14.82 for sure or playing the gamble with an expected value of \$10. When $p = 0.8$, $\pi = -\$23.25$; in other words, the individual is indifferent between a sure \$56.75 and the gamble that has an expected value of \$80. In both cases, the risk premia are large fractions of the expected value, suggesting that these preferences have rather extreme risk attitudes.[8]

The points in the figure correspond to the average risk premia of Tversky and Kahneman's (1992) experimental subjects. Looking first at Figure 7(a), for any value of α it appears that the predicted risk premia for gains are too small when probabilities are low and too large when the probabilities are high. Increasing α improves the fit for high probabilities but worsens the fit for low probabilities. A similar problem exists for gambles over losses. As shown by the figure, increasing α causes risk premia to rise throughout the range for gains and fall throughout the range for losses, consistent with the notion that higher values of α correspond to "more risk neutral" preferences.

Figure 7(b) shows that for any value of γ, the predicted risk premia for gains are again too small when probabilities are low and too large when probabilities are high. This does not appear to be as much of a problem for losses. Decreasing γ, which increases the curvature of the weighting function, causes risk premia to rise in magnitude. Camerer and Ho (1994) estimate $\gamma = 0.56$, and Wu and Gonzalez (1996) estimate $\gamma = 0.74$. According to the figure, then, the data Camerer and Ho use are consistent with more extreme risk premia, while the data used by Wu and Gonzalez are consistent with less extreme risk premia than the data used by Tversky and Kahneman.



(a)



(b)

Figure 7. (a) Risk premia with different values of α. (b) Risk premia with different values of γ.

The shape of the weighting function might be the cause of the model's failure to fit Tversky and Kahneman's gains data. For any value of γ, the largest (in magnitude) risk premium for large probabilities is five to ten times larger than the maximal (in magnitude) risk premium for small probabilities. Tversky and Kahneman's data suggest that smaller multiples are more appropriate, and the current parameterizations are unable to accommodate these patterns.

We repeated the above exercises using Prelec's specifications; however, we find similar conclusions, and therefore, it is not useful to discuss them here.

5. Conclusions

The exercises undertaken in this paper demonstrate that we are not yet ready to generalize laboratory work on relatively narrow stimuli to the wide range of stimuli embodied by applied work, at least not with the functional forms investigated so far. The problem is that parameterizations based on experimental results tend to be too extreme in their implications. The preference function estimated by Tversky and Kahneman (1992) implies an acceptable amount of risk seeking over unlikely gains and risk aversion over unlikely losses, but can accommodate neither the strongest choice patterns from Battalio, Kagel, and Jiranyakul (1990) nor the Allais paradox, and implies some rather large risk premia. The preference functions estimated by Camerer and Ho (1994) and Wu and Gonzalez (1996) imply virtually no risk seeking over unlikely gains and virtually no risk aversion over unlikely losses, so that individuals will purchase neither lottery tickets nor insurance. The troubles run deeper than just these estimates, though. We show that there are no parameter combinations that allow for both the desired gambling/insurance behavior *and* a series of choices made by a strong majority of subjects *and* reasonable risk premia. So, while the proposed functional forms might fit the experimental data well, they have poor out-of-sample performance.

The most obvious conclusion of our work is that alternative functional forms are needed. The results also show that parameterizations that work well for choices over likely gains (that is, in the subset of the gains space where preferences are risk averse) do not work well for choices over unlikely gains (the subset where preferences are risk-seeking), and vice-versa. This is especially clear in Figures 7(a) and 7(b), where parameterizations that fit well with unlikely non-zero payoffs do poorly with likely non-zero payoffs, and vice-versa. Therefore, it might be fruitful to further segment the choice set beyond its separation into the sets of lotteries over gains, lotteries over losses, and mixed lotteries. The set of lotteries over gains could be further segmented into those with likely gains and those with unlikely gains, with different estimated parameters for each.

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Notes

1. Rank-dependent expected utility (Quiggin, 1993) has essentially the same functional form, but cumulative prospect theory makes additional assumptions on the utility function and has been the subject of more experimental work.

2. It should be noted that Hershey, Kunreuther, and Schoemaker (1982) argue that response mode effects, probability and outcome level effects, aspiration level effects, inertia effects, and context effects all influence and/or distort the elicited preference function, and therefore one must be cautious in using a single preference function to explain different choices. Their quite general argument suggests that no single preference function can ever be expected to accommodate a wide range of actual choices. In light of this, the failure of a single parameterized model to fit a variety of evidence is hardly surprising. Viewed in this context, though, our work shows where the failures of the preference parameterizations occur, and illustrate their inability to accommodate choice patterns that may be important when applied to real-world (as opposed to experimental) data.
3. The final weighting function we consider is one investigated by Gonzalez and Wu (1999), given by

   $$g(p) = \frac{\delta p^\gamma}{\delta p^\gamma + (1-p)^\gamma},$$

   where γ controls the curvature and the δ controls the height of the function. It generates behavior similar to that generated by Prelec's weighting function, so we omit graphical analyses with this function.
4. Note that the use of the power function makes the choice of G irrelevant, since (9) has G^α on both sides.
5. The estimate of Camerer and Ho's α can be found in Wu and Gonzalez (1996).
6. See also Hershey, Kunreuther, and Schoemaker (1982).
7. The corresponding figure based on Prelec's weighting function is almost identical, so is omitted.
8. Risk premia that are large fractions of the expected value may be appropriate. In their preference reversal experiments, Cox and Grether (1996) find certainty equivalents that are 1.5 to 2.5 times the expected value for gambles with unlikely gains.

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