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A mixed fan hypothesis and its implications for behavior toward risk

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In order to accommodate recent experimental evidence which questions the validity of the fanning out hypothesis, a mixed fan hypothesis is proposed. The hypothesis combines fanning out for the less-preferred region of probability triangles with fanning in for the more preferred region, and is defined over general probability spaces. The relevance of fanning hypotheses for economic analysis is illustrated with an asset demand example, and a new method for testing for fanning behavior is proposed.

1. Introduction

Over the past decade or so researchers have proposed weaker alternatives to the expected utility model because the expected utility model is violated by certain empirical evidence.[1] The new models and new behavioral hypotheses which have been suggested have implications for the actions of economic agents. It is the purpose of this paper to construct a new behavioral hypothesis based on new experimental evidence [Battalio et al. (1990), Conlisk (1989) and Sopher and Gigliotti (1990)], and to examine its effect on behavior. The construction will be done in a simple decision framework based on the work of Dekel (1986). Initially, attention will be restricted to a single probability triangle (that is, the set of lotteries with only three possible outcomes) because most of the available evidence concerns behavior within single probability triangles. The analysis will then be extended to a more general probability space.

Dekel's implicit expected utility model is used for two reasons.[2] First,

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[1]For an introduction to the problems, see Fishburn (1988). For a review of recent experimental work, see Camerer (1989b).

[2]The implicit weighted utility model of Chew (1985, 1989) is equivalent.

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since it is weaker than the expected utility model, one can regard the expected utility model as a particular behavioral hypothesis applied to the implicit expected utility model. Second, the nature of the model allows fanning effects to be isolated from other violations of expected utility. The model also provides an uncomplicated generalization of the expected utility model when used in the probability triangle: It entails replacing a single value with a function.

One drawback with working in the probability triangle is the restrictiveness of the set of problems which can be discussed.[3] The most interesting asset demand problems, for example, require larger probability spaces. However, the experimental studies of Battalio et al. (1990), Camerer (1989a, b), Conlisk (1989), Harless (1991) and Sopher and Gigliotti (1990) all pertain to probability triangles, and so any behavioral hypothesis proposed as an alternative to the expected utility hypothesis must be compatible with the observed behavior in probability triangles.

To accommodate the recent experimental evidence, a mixed fan hypothesis will be proposed as an alternative to both the expected utility hypothesis and the fanning out hypothesis [Machina (1982)]. Fanning out implies a particular pattern of indifference curves in the probability triangle, as will be shown below, and the hypothesis states that first order stochastically dominating shifts make utility functions more risk averse at each outcome. The mixed fan hypothesis combines fanning out for the less-preferred region of the triangle with fanning in for the more-preferred region. Section 2 reviews the implicit expected utility model for the probability triangle and the general probability space, discusses the evidence and proposes a mixed fan hypothesis for probability triangles. Section 3 extends the mixed fan hypothesis to the general probability space, shows that it generates the desired behavior for probability triangles, and gives a behavioral interpretation. In section 4 attention is turned to the impact of fanning properties on economic behavior. Situations exist in which fanning properties provide different predictions from the expected utility model, and a specific asset demand example is given. The paper concludes in section 5 with the proposal of a new experimental procedure for determining fanning properties. The procedure is based on the BDM mechansim for eliciting certainty equivalents of lotteries [Becker et al. (1964)].

2. The model and evidence

Let $D[0, M]$ be the space of probability distributions over $[0, M]$ endowed with the topology of weak convergence, as in Machina (1982). It will be assumed that the decision maker maximizes implicit expected utility, as in

[3]See Neilson (1990).

Dekel (1986), so that there exists an implicit utility function $u(x,v)$ increasing in x with $u(0,v)=0$ and $u(M,v)=1$ for all v, and the individual's preference function $V(F)$ is given by the solution to the implicit equation

$$v=\int u(x,v)\,\mathrm{d}F(x). \tag{1}$$

It will be assumed throughout that V is continuously Frechet differentiable,[4] and we will call the level $V(F)$ the 'preference value' of the distribution F. An implicit expected utility maximizer satisfies the expected utility hypothesis if $u(x,v)$ is a constant function of v for all x.

Since the preference value is an argument of the utility function u, then for fixed v, $u(x,v)$ can be interpreted as a 'local' utility function. Changes in the probability distribution which make the individual better or worse off cause changes in v, and therefore change the shape of the local utility function. On the other hand, changes in F which leave the individual indifferent do not change the shape of the local utility function. The simplest way to interpret (1), then, is that there is a different local utility function corresponding to each indifference set, and the individual maximizes expected local utility.

One particular subset of $D[0,M]$ which is convenient for comparing various behavioral hypotheses is the set of three-outcome lotteries. Let $0\leqq x_1<x_2<x_3\leqq M$, and consider the set of lotteries with support in $\{x_1,x_2,x_3\}$. These lotteries can be represented by triples of the form (p_1,p_2,p_3) where $p_1+p_2+p_3=1$. Using the transformation $p_2=1-p_1-p_3$, the set of lotteries can be represented in (p_1,p_3) space by the set $\{(p_1,p_3)|\ p_1+p_3\leqq 1\}$, which forms a triangle with vertices at the origin, $(0,1)$, and $(1,0)$, as in fig. 1. An implicit expected utility maximizer makes choices over lotteries in probability triangles by maximizing $V(p)$, where $V(p)$ solves $v=\sum p_i u(x_i,v)$. It is straightforward to show that implicit expected utility preferences generate indifference sets which are straight lines in (p_1,p_3) space and have slope $\mu(v)\equiv[u(x_2,v)-u(x_1,v)]/[u(x_3,v)-u(x_2,v)]$. If the expected utility hypothesis holds, then $\mu(v)$ is constant, and so indifference lines must be parallel.

The most frequently cited violation of expected utility theory is the Allais Paradox. Decision makers are asked to make choices in two pairs of lotteries, and all four lotteries involve the payoffs \$0, \$1 million, and \$5 million. Letting the triple $(p_{\$0},\ p_{\$1\mathrm{M}},\ p_{\$5\mathrm{M}})$ denote the probabilities on the payoffs \$0, \$1 million and \$5 million, respectively, lottery A can be written as $(0,1,0)$ and lottery B as (0.01, 0.89, 0.10). In the other pair, lottery C is (0.89, 0.11, 0) and lottery D is (0.9, 0, 0.1). The commonly observed modal choice pair is A over B and D over C. Expected utility theory predicts that the

[4]Dekel (1986) proves that implicit expected utility preferences need not be smooth, and so this is an additional, though weak, assumption on preferences.

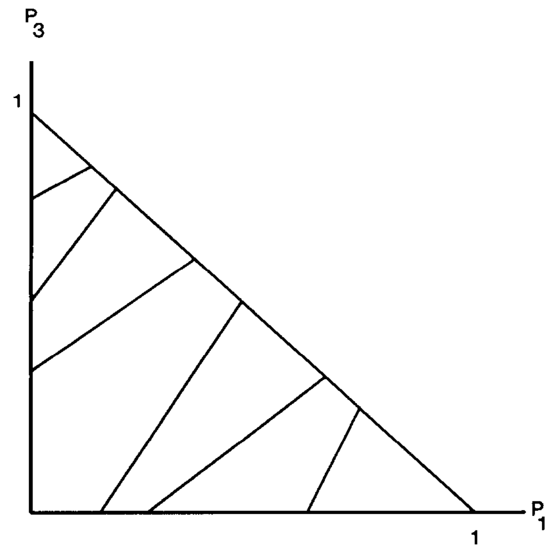

Fig. 1. An implicit expected utility indifference map.

choice pair must be either A and C if $0.1u(\$5\text{M})-0.11u(\$1\text{M})+0.01u(\$0)<0$, or B and D if $0.1u(\$5\text{M})-0.11u(\$1\text{M})+0.01u(\$0)>0$. The empirical evidence violates the expected utility hypothesis.

The four lotteries of the Allais Paradox are shown in the probability triangle in fig. 2 (not to scale), where $x_1=\$0$, $x_2=\$1$ million, and $x_3=\$5$ million. The four points form a parallelogram, and therefore if preferences satisfy the expected utility hypothesis, so that indifference curves are parallel straight lines, the individual must prefer either A and C or B and D. The modal choice pair is A and D, however, and so parallel indifference lines cannot describe preferences. An indifference map which chooses lotteries A and D is shown in fig. 2, and the indifference lines fan out from the origin. The Allais Paradox and other, similar, evidence has led to the development of the fanning out hypothesis for probability triangles, which states, in terms of the implicit expected utility model, that $\mu(v)$ is an increasing function. If $\mu(v)$ is increasing, then movements in the direction of increasing preference make indifference lines steeper.

Battalio et al. (1990) ran an experiment to test the fanning out hypothesis in the upper left corner of the probability triangle. The Allais Paradox, and most other tests of the expected utility hypothesis, had used the lower right corner of the triangle to get violations of independence, and the upper left corner had not been investigated. Battalio et al.'s evidence violates both the expected hypothesis and the fanning out hypothesis. One experiment

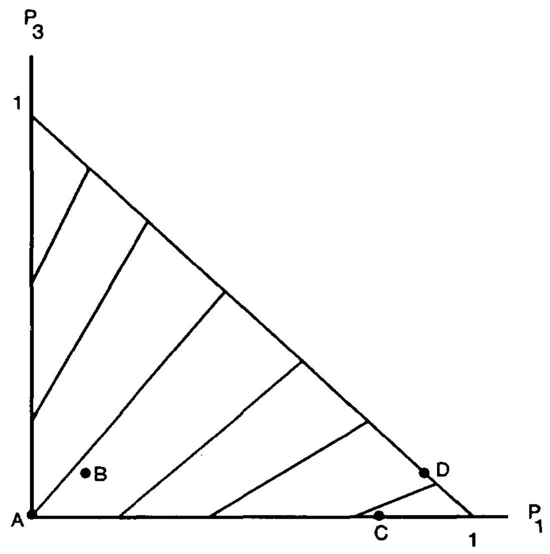

Fig. 2. The Allais lotteries and fanning out.

required subjects to make choices in two pairs of lotteries. Using triples of the form $(p_{\$0}, p_{\$18}, p_{\$27})$, lottery A can be written as (0.1, 0.9, 0), B as (0.28, 0, 0.72), C as (0.06, 0.2, 0.74) and D as (0.1, 0, 0.9). Expected utility would predict either the choice of A and C (if $0.72u(\$27)-0.9u(\$18)+0.18u(0)<0$) or B and D (if $0.72u(\$27)-0.9u(\$18)+0.18u(0)>0$). Choosing B and C would suggest fanning out. Of the 36 subjects in this experiment, 6 chose A and C, 10 chose B and D, 4 chose B and C, and 16 chose A and D. The modal choice is inconsistent with both expected utility theory and the fanning out hypothesis.

Conlisk (1989) finds a similar pattern by testing a different version of the Allais Paradox. The payoffs remain the same ($0, $1 million, and $5 million) and lotteries A and B are the same as in the Allais Paradox, but lotteries C and D are shifted from the lower right corner of the triangle to the upper left. Specifically, C is (0.01, 0.11, 0.88) and D is (0.02, 0, 0.98). 45% of the subjects violated expected utility theory, and 82% of those also violated the fanning out hypothesis, that is, they chose lotteries A and D.[5] Sopher and Gigliotti (1990) performed almost the same test, but with $C=(0, 0.11, 0.89)$ and $D=(0.01, 0, 0.99)$, and found that 45% of the subjects violated expected utility theory, with 74% of those violating fanning out.

[5]Strictly speaking, Conlisk's results, and those of Battalio et al. can only reject the joint hypothesis of betweenness and fanning out, which is the joint hypothesis being considered in this paper.

The new experimental evidence is implied by a fanning in hypothesis for behavior in probability triangles, which simply asserts that indifference lines fan in from the origin, or, alternatively, that $\mu(v)$ is a decreasing function. Movements into the better-than set cause indifference lines to become flatter. In Battalio et al.'s experiment outlined above, the fanning in hypothesis explained 44% of the subjects' choices, and 80% of the violations of expected utility theory.

Combining the evidence, it appears that preferences satisfy the fanning out hypothesis in the lower right corner of the triangle, and the fanning in hypothesis in the upper left corner.[6] The implicit expected utility model makes it convenient to construct a behavioral hypothesis which combines these two patterns of behavior.

Mixed fan hypothesis (triangle version). There exists a $\bar{v}\in(0,1)$ such that $\mu(v)$ is increasing for $v<\bar{v}$ and decreasing for $v>\bar{v}$.

Since the lower right corner of the probability triangle corresponds to low values of v and the upper left corner corresponds to high values of v, we need $\mu(v)$ to be increasing for low preference values and decreasing for high preference values.[7] An example of preferences which satisfy the mixed fan hypothesis is shown in fig. 3.

3. Extension to $D[0,M]$

Let $r(x,v)=-u_{11}(x,v)/u_1(x,v)$, the Arrow–Pratt measure of absolute risk aversion.[8] In Pratt (1964, theorem 1) it is demonstrated that the slope of an expected utility maximizer's indifference line in a probability triangle is related to $r(x)$, where the second argument is suppressed because of the assumption of expected utility maximization. If u and u^* are two utility functions, then the condition $r^*(x)\geqq r(x)$ for all x is equivalent to the condition that $[u^*(x)-u^*(w)]/[u^*(z)-u^*(y)]\geqq[u(x)-u(w)]/[u(z)-u(y)]$ for all $w<x\leqq y<z$. A straighforward application of this result is that u^* must generate steeper indifference lines than u.

Machina (1982, theorem 5) used this fact to find a behavioral hypothesis for general probability spaces which implies the triangle version of fanning

[6]The evidence in Camerer (1989a) does not reject the fanning out hypothesis, but neither does it reject fanning in for the northwest region of the triangle. One of the stylized facts with which Camerer (1989b) concludes his survey of recent experimental work is that the fanning out hypothesis is violated.

[7]If payoffs are losses instead of gains, then effects tend to be reversed [see, for example, Battalio et al. (1990)]. Consideration of losses would require nontrivial extensions to the theory being proposed here, and will not be discussed in this paper.

[8]For notational purposes, let $f_i(\cdot,\cdot)$ denote the partial derivative of f with respect to its ith argument, and let $f_{ij}(\cdot,\cdot)$ denote the appropriate second partial derivative.

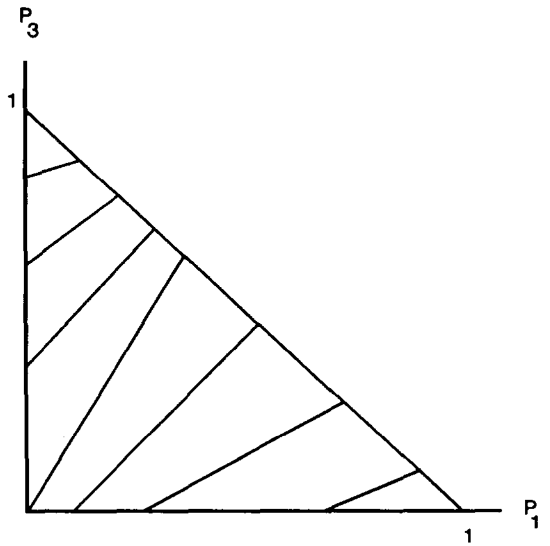

Fig. 3. An indifference map obeying the mixed fan hypotheses.

out. Stated in conjunction with the implicit expected utility model used here, it states that $r(x,v)$ is nondecreasing in v for all $x \in [0, M]$, and strictly increasing for some x in every interval. The fanning in hypothesis is similar with $r(x,v)$ nonincreasing in v for all $x \in [0, M]$, and strictly decreasing for some x in every interval. These can be combined in the following hypothesis.

Mixed fan hypothesis (general version). $r(x,v)$ is concave in v for all x, and strictly concave for some x in every interval.

We wish to demonstrate that if the general version of mixed fanning holds, then mixed fan behavior is exhibited in every triangle. An intermediate result is Proposition 1.

Proposition 1. Let $r(x,v)$, $r_2(x,v)$ and $r_{22}(x,v)$ be continuous and bounded. If the mixed fan hypothesis holds, then for all $0 \leqq x_1 < x_2 < x_3 \leqq M$, $\mu(v)$ is quasiconcave in v.

Proof. See appendix.

Proposition 1 establishes that indifference lines in probability triangles will fit one of three patterns: Either $\mu(v)$ is nondecreasing throughout the triangle; it is nonincreasing throughout the triangle; or it is nondecreasing for low levels of v and nonincreasing for high levels of v. To get a mixed fan in every probability triangle, some additional assumptions are needed. The following notational convention and terminology are helpful. Let δ_x be the degenerate distribution placing probability one on outcome x, and let the term 'increasing' mean that the function is nondecreasing and strictly increasing in every interval, so that, for example, the fanning out hypothesis states that $r(x, v)$ is increasing in v.

Proposition 2. Suppose that all of the assumptions of Proposition 1 hold. If, for every $x \in [0, M]$, $r(x, v)$ *is increasing in* v *for* $v < V(\delta_x)$ *and decreasing in* v *for* $v > V(\delta_x)$ *then the triangle version of the mixed fan hypothesis holds in every triangle.*

Proof. See appendix.

The extra assumptions in Proposition 2 are only needed to match the general version of the mixed fan hypothesis to the evidence, which only involves probability triangles. They will not be used in the remainder of the paper.

The various fanning hypotheses concern changes in the degree of risk aversion at specific outcomes. To interpret the different fanning hypotheses behaviorally, think of risk aversion at the outcome x as a '(consumption) good', and think of first order stochastically dominating shifts as 'increases in income'. The fanning out hypothesis states that risk aversion at x is a 'normal' good, since first order stochastically dominating shifts make the individual more risk averse at every outcome. Similarly, fanning in states that risk aversion at x is an 'inferior' good. The mixed fan hypothesis states that risk aversion at x is normal for 'low' income levels and inferior for 'high' income levels, with the distinction between 'low' and 'high' depending on x.

Looked at this way, the mixed fan hypothesis begins to make some sense. All ordinary consumer goods are normal at extremely low levels of income, since consumption must be zero when income is zero. Risk aversion is costly in the sense that the individual must be willing to give something up to avoid risks, and so one would expect risk aversion at positive outcomes to be a normal good at least at low levels of income. When income becomes sufficiently high, however, risk aversion becomes an inferior good. Proposition 2 surmises that income becomes sufficiently high when the individual is able to consume outcome x with certainty, and that risk aversion at a higher outcome is 'less inferior' than risk aversion at a lower outcome.

Other theoretical constructs can also accommodate mixed fanning in triangles. Gul (1990) shows that if preferences exhibit disappointment aversion, they they must also exhibit mixed fanning in every triangle. Chew's (1985, 1989) semi-weighted utility theory can also accommodate the mixed fan hypothesis. Since both models are based on stronger assumptions than those used here,[9] the mixed fan hypothesis is still needed for use in more general settings. Furthermore, it is useful for analysis of some common economic problems, as demonstrated in the next section.

4. An asset demand example

The fanning hypotheses discussed above are based on evidence using pairwise choices, but they are also relevant for richer choice problems. Just as in the Allais Paradox, when there is a probability mixture of an event in which a decision matters and an event in which it does not, then a change in either the mixing probability or the wealth distribution in the second event can affect the individual's choice. Therefore, one must be careful when modelling behavior when the expected utility hypotheses does not hold, because the choice variable is affected by occurrences which are independent of the choice. This is illustrated with a simple asset demand problem.

Suppose that an individual must invest his pension money with a specific investment firm which sponsors a variety of mutual funds. The individual has freedom to allocate his money among the funds in any way he wishes. There is some probability p that the firm will go bankrupt, in which case the individual can only recover $\$R$, which may depend on his contributions to the pension fund but not on their earnings. If the investor is an expected utility maximizer, then a change in p or R will not affect his portfolio decision. If, on the other hand, one of the fanning hypotheses holds, then changes in p or R do affect the individual's investment decision.

The normative rationale for the expected utility hypothesis holds here. If the firm happens to go bankrupt, then the individual's portfolio decision is moot. If the firm does not go bankrupt, then the individual should choose his mutual fund portfolio so as to maximize his preference function conditional on the event that the firm remains solvent. Since the wealth distributions in these two events are independent, the individual should not let what happens in one event affect his decision in the other event. This rationale is denied by fanning behavior.

Suppose that the individual has a choice between two mutual funds: A riskless one which yields a gross return of y and a risky one which yields a gross return of $y+\tilde{x}$ where $\mathrm{E}[\tilde{x}]\geq 0$. If the individual has $\$w$ to invest in these two funds, the value of his portfolio is $w(y+\alpha\tilde{x})$, where α is the share of w which is invested in the risky fund. Let F be the distribution of x. There is also a probability $0<p<1$ that the firm will go bankrupt, in which case the

[9] It must be noted that Gul's model is purposefully restrictive, so much so that risk aversion implies mixed fanning.

individual recovers \$R. If the individual is an expected utility maximizer, then he chooses α to maximize

$$(1-p)\int u(w(y+\alpha x))\,\mathrm{d}F(x)+pu(R). \tag{2}$$

The first order condition for maximization can be written

$$\int u'(w(y+\alpha x))wx\,\mathrm{d}F(x)=0. \tag{3}$$

Since the first order condition does not depend on p or R, the choice of α is independent of p and R.

Now suppose that the individual is an implicit expected utility maximizer. Let $G(\cdot,\alpha)$ be the distribution of the random wealth variable $w(y+\alpha\tilde{x})$, and let $K(\cdot,\alpha,p,R)$ be the distribution function for the probability mixture $(1-p)G+p\delta_{\mathrm{R}}$. Then the individual chooses α to maximize

$$(1-p)\int u[w(y+\alpha x),\ V(K(\cdot,\alpha,p,R))]\,\mathrm{d}F(x)+pu[R,V(K(\cdot,\alpha,p,R))]. \tag{4}$$

The first condition for maximization[10] can be written

$$\int u_1[w(y+\alpha x),\ V(K(\cdot,\alpha^*,p,R))]\,wx\,\mathrm{d}F(x)=0, \tag{5}$$

and the second order condition for maximization is that $V(K(\cdot,\alpha,p,R))$ is strictly quasiconcave in α. In (5) the choice of α depends on the implicit utility function under consideration, which in turn depends on p and R.

To study choice behavior, note that by Arrow (1974) and Pratt (1964), if one utility function has a higher Arrow–Pratt measure of risk aversion than another at every wealth level, then the first utility function will choose a smaller α than the second one. Therefore, if the individual satisfies the fanning out hypothesis, then as p decreases or R increases the demand for the risky fund decreases. This is because shifts in K which make the individual better off make the individual more risk averse. Similarly, if the individual satisfies the fanning in hypothesis, then decreases in p or in increases in R cause the demand for the risky fund to increase.

Mixed fanning behavior is more complicated. The idea behind the mixed fanning hypothesis is fanning out for low preference values and fanning in for high preference values. Consequently, if $V(K)$ is low, then fanning out dominates, and we would expect increases in R or decreases in p to cause demand for the risky fund to decline, and if $V(K)$ is high, demand for the risky fund will increase. These arguments are made formal below.

[10]See Machina [1982, eq. (8)].

Proposition 3. *Suppose that $u_1(z,v)>0$ for all v, and that $V(K(\cdot,\alpha,p,R))$ is strictly quasiconcave in α for all p,R. Let $F(\cdot)$ and p be given, and let $R<w$, then*:

(*a*) *If the fanning out hypothesis holds, then α^* is decreasing in R;*
(*b*) *If the fanning in hypothesis holds, then α^* is increasing in R;*
(*c*) *If the mixed fan hypothesis holds, then α^* is quasiconvex in R.*

Proof. See appendix.

Proposition 4. *Suppose that $u_1(z,v)>0$ for all v, and that $V(K(\cdot,\alpha,p,R))$ is strictly quasiconcave in α for all p,R. Let $F(\cdot)$ and R be given, and let $R<w$, then*:

(*a*) *If the fanning out hypothesis holds, then α^* is increasing in p;*
(*b*) *If the fanning in hypothesis holds, then α^* is decreasing in p;*
(*c*) *If the mixed fan hypothesis holds, then α^* is quasiconcave in p.*

Proof. Similar to the proof of Proposition 3 (see appendix).

Regarding Proposition 3, two points can be made. First, if α^* is quasiconvex in R, then there are three possibilities: Either α^* is increasing for all R; decreasing for all R; or decreasing for $R<\bar{R}$ and increasing for $R>\bar{R}$ for some recovery level $\bar{R}$. Turning points such as this will be common for mixed fanning behavior, as one might expect.[11] Second, the results still hold if R is replaced by a probability distribution which is independent of α. All that is required is a state in which the choice of α does not matter. Certainty is not necessary.

5. Conclusion

The mixed fan hypothesis presented above was designed to account for experimental data concerning choices in probability triangles. The hypothesis was then applied to an asset demand problem in a more general probability space. However, there is no data on fanning properties in larger probability spaces, and, in fact, experimental evidence suggests that fanning effects disappear in the interior of probability triangles [Harless (1991), Conlisk (1989), Camerer (1989b), Sopher and Gigliotti (1990)]. To see if fanning behavior arises in general probability spaces, a new test for fanning hypotheses is proposed.

Becker et al. (1964) propose an incentive compatible means for determining the certainty equivalent of a lottery $\tilde{x}$. The subject specifies an amount of money c and then observes the realization y of a random variable $\tilde{y}$. If $y\geqq c$

[11] For example, see Proposition 2.

he receives y as his reward. If $y<c$, he receives a randon reward $\tilde{x}$ having some given probability distribution F with support $[x_1, x_2]$, The random variables $\tilde{x}$ and $\tilde{y}$ are assumed to be independent. It can be shown that an expected utility maximizer will choose c so that $u(c)=\int u(x)\,\mathrm{d}F(x)$, that is, so that c is the certainty equivalent of $\tilde{x}$.

To understand the reason for this, let G be the distribution function for $\tilde{y}$ and $[y_1, y_2]$ its support. Then the object is to choose c to maximize

$$\int_{y_1}^{c}\int_{x_1}^{x_2} u(x)\,\mathrm{d}F(x)\,\mathrm{d}G(y)+\int_{c}^{y_2} u(y)\,\mathrm{d}G(y). \tag{6}$$

An application of Leibniz' rule yields the desired result. However, if the individual is an implicit expected utility maximizer, then the above expression must change. Let $H(c)$ denote the distribution function for the final lottery. Then the individual must choose c to maximize

$$\int_{y_1}^{c}\int_{x_1}^{x_2} u[x, V(H(c))]\,\mathrm{d}F(x)\,\mathrm{d}G(y)+\int_{c}^{y_2} u[y, V(H(c))]\,\mathrm{d}G(y). \tag{7}$$

Another application of Leibniz' rule yields

$$\int u[x, V(H(c^*))]\,\mathrm{d}F(x)-u[c^*, V(H(c^*))]=0. \tag{8}$$

One feature of this result is that c^* is still the certainty equivalent for the random variable $\tilde{x}$ for the relevant implicit utility function. However, when $\tilde{y}$ changes, $H(c)$ changes for given c, and therefore $u(\cdot, H(c^*))$ changes. Therefore, a change in the alternative random variable $\tilde{y}$ affects the certainty equivalent which is elicited. This suggests an alternative test for fanning properties.[12]

Begin with a random variable $\tilde{y}_1$ and use it to determine the certainty equivalent c_1 of $\tilde{x}$. Then select another random variable $\tilde{y}_2$ such that either $\mathrm{Prob}\{\tilde{y}_2 \leqq c_1\} < \mathrm{Prob}\{\tilde{y}_1 \leqq c_1\}$, or, alternatively, there is a first order stochastically dominating shift in the portion of the distribution which is above c_1. Either of these (or a combination) will cause a first order stochastically dominating shift in the distribution of $H(c_1)$, which will make the individual better off. If the new elicited certainty equivalent $c_2 \neq c_1$, then there must be some type of fanning behavior. If $c_2 < c_1$, then the individual has become more risk averse, which is consistent with fanning out. If $c_2 > c_1$, then the evidence is consistent with fanning in. This mechanism would be able to

[12] Safra et al. (1990) propose an essentially similar test, but for a different purpose.

generate evidence of fanning behavior, if it exists, in spaces more useful than probability triangles.

Appendix: Proofs of propositions 1, 2 and 3

Proof of Proposition 1. Using the fact that $r(x,v)=\mathrm{d}/\mathrm{d}x[-\log u_1(x,v)]$, we get

$$\int_{w_1}^{w_2} r(x,v)\,\mathrm{d}x=\log\frac{u_1(w_1,v)}{u_1(w_2,v)}, \tag{A.1}$$

for any $w_1<w_2$. By the mean value theorem there exists $0\leqq x_1<x_2<x_3\leqq M$ such that

$$\frac{u(x_2,v)-u(x_1,v)}{u(x_3,v)-u(x_2,v)}=\frac{u_1(w_1,v)(x_2-x_1)}{u_1(w_2,v)(x_3-x_2)}, \tag{A.2}$$

so that

$$\int_{w_1}^{w_2} r(x,v)\,\mathrm{d}x=\log\frac{u(x_2,v)-u(x_1,v)}{u(x_3,v)-u(x_2,v)}-\log\frac{x_2-x_1}{x_3-x_2}. \tag{A.3}$$

By the mixed fan hypothesis, $r_{22}(x,v)\leqq 0$, and, by Fleming (1977, p. 237, Lemma 2),

$$\frac{\partial^2}{\partial v^2}\int_{w_1}^{w_2} r(x,v)\,\mathrm{d}x=\int_{w_1}^{w_2} r_{22}(x,v)\,\mathrm{d}x\leqq 0. \tag{A.4}$$

Therefore, the left hand side of (A.3) is concave in v, and so the right hand side must be concave in v, which implies that $\log\mu(v)$ is quasiconcave in v, and therefore $\mu(v)$ is quasiconcave in v.

Proof of Proposition 2. Given $0\leqq x_1<x_2<x_3\leqq M$, it can be demonstrated that there exists a $\bar{v}\in[V(\delta_{x_1}),V(\delta_{x_3})]$ such that $\mu(v)$ is increasing in v for $v\leqq\bar{v}$ and decreasing in v for $v\geqq\bar{v}$. The assumptions of the proposition imply that $r(x,v)$ is increasing in v at $V(\delta_{x_1})$ for all $x\in[x_1,x_3]$, and strictly increasing for some x in every interval. By Pratt (1964, Theorem 1), $[u(x,v)-u(w,v)]/[u(z,v)-u(y,v)]$ is increasing in v at $V(\delta_{x_1})$ for all $x_1\leqq w<x\leqq y<z\leqq x_3$. Let $z=x_3$, $x=y=x_2$, and $w=x_1$. Then $\mu(v)$ is increasing in v at $V(\delta_{x_1})$. A similar argument establishes that $\mu(v)$ is decreasing in v at $V(\delta_{x_3})$. The desired result then follows from Proposition 1.

Proof of Proposition 3. First show that $V(K(\cdot,\alpha,p,R))$ is increasing in R for fixed α and p. Note that if $w>R_1>R_2$, then for any given α and p, $K(\cdot,\alpha,p,R_1)$ first order stochastically dominates $K(\cdot,\alpha,p,R_2)$, and therefore $K(\cdot,\alpha^*(R_1),p,R_1)$ first order stochastically dominates $K(\cdot,\alpha^*(R_2),p,R_2)$. By Dekel (1986, Property 1), $V(K(\cdot,\alpha^*(R_1),p,R_1)>V(K(\cdot,\alpha^*(R_2),p,R_2))$.

Next, note that α^* solves (5), and therefore it also solves

$$\int u_1[w(y+\alpha x),\ V(K(\cdot,\alpha^*,p,R))]\,wx/u_1[wy,V(K(\cdot,\alpha^*,p,R))]\,\mathrm{d}F(x)=0, \tag{A.5}$$

since $u_1(z,v)>0$. Let

$$\eta(z_1,z_2,v)\equiv\int_{z_1}^{z_2} r(x,v)\,\mathrm{d}x. \tag{A.6}$$

By eq. (A.1),

$$\int wx\,\frac{u_1(w(y+\alpha x),v)}{u_1(wy,v)}\,\mathrm{d}F(x)=\int\frac{wx}{\eta(wy,w(y+\alpha x),v)}\,\mathrm{d}F(x). \tag{A.7}$$

If a change in v causes

$$\int u_1[w(y+\alpha x),\ V(K(\cdot,\alpha^*,p,R))]\,wx/u_1[wy,V(K(\cdot,\alpha^*,p,R))]\,\mathrm{d}F(x), \tag{A.8}$$

to increase for fixed α^*, then α^* must also increase because of the strict quasiconcavity of $V(K(\cdot))$ in α.

If the fanning out hypothesis holds, then r is increasing in v. $\eta(wy, w(y+\alpha x),v)$ is increasing in v when $x>0$ and decreasing in v when $x<0$, so x/η is decreasing in v. Therefore the expression in (A.8) is decreasing in v. Since v decreases when R decreases, the expression in (A.8) must be decreasing in R, which implies that α^* must be decreasing in R.

If the fanning in hypothesis holds, then r is decreasing in v, so that $\eta(wy,w(y+\alpha x),v)$ is decreasing in v when $x>0$ and increasing when $x<0$. Therefore x/η is increasing in v, and so is the expression in (A.8). Since v increases when R increases, the expression in (A.8) must be increasing in R, which implies that α^* must be increasing in R.

If the mixed fan hypothesis holds, r is concave in v. η is quasiconcave in v when $x>0$ and quasiconvex in v when $x<0$, and therefore x/η is quasiconvex in v, which implies that the expression in (A.8) is quasiconvex in v. Since v is increasing in R, and since α^* increases when the expression in (A.8) increases, α^* must be quasiconvex in R.

References

Arrow, Kenneth J., 1974, Essays in the theory of risk-bearing (North-Holland, Amsterdam).
Battalio, Raymond C., John H. Kagel and Komain Jiranyakul, 1990, Testing between alternative

models of choice under uncertainty: Some initial results, Journal of Risk and Uncertainty 3, 25–50.

Becker, Gordon M., Morris H. DeGroot and Jacob Marschak, 1964, Measuring utility by a single-response sequential method, Behavioral Science 9, 226–232.

Camerer, Colin F., 1989a, An experimental test of several generalized utility theories, Journal of Risk and Uncertainty 2, 61–104.

Camerer, Colin F., 1989b, Recent tests of generalizations of expected utility theory, Manuscript (University of Pennsylvania, PA).

Chew, Soo Hong, 1985, Implicit-weighted and semi-weighted utility theories, M-estimators, and non-demand revelation of second-price auctions for an uncertain auctioned object, Working paper no. 155 (Johns Hopkins University, Baltimore, MD).

Chew, Soo Hong, 1989, Axiomatic utility theories with the betweenness property, Annals of Operations Research 19, 273–298.

Conlisk, John 1989, Three variants on the Allais example, American Economic Review 79, 392–407.

Dekel, Eddie, 1986, An axiomatic characterization of preferences under uncertainty: Weakening the independence axiom, Journal of Economic Theory 40, 304–318.

Fishburn, Peter C., 1988, Nonliear preference and utility theory (Johns Hopkins University Press, Baltimore, MD).

Fleming, Wendell, 1977, Functions of several variables (Springer-Verlag, New York).

Gul, Faruk, 1990, A theory of disappointment aversion, Econometrica, forthcoming.

Harless, David, 1991, Predictions about indifference curves inside the unit triangle: A test of variants of expected utility theory, Journal of Economic Behavior and Organization, forthcoming.

Machina, Mark J., 1982, 'Expected utility' analysis without the independence axiom, Econometrica 50, 277–323.

Neilson, William S., 1990, Use of probability triangles in the analysis of behavior toward risk, Working paper 90-04 (Texas A&M University, College Station, TX).

Pratt, John W., 1964, Risk aversion in the small and in the large, Econometrica 32, 122–136.

Safra, Zvi, Uzi Segal and Avia Spivak, 1990, The Becker–De Groot–Marschak mechanism and non-expected utility, Journal of Risk and Uncertainty 3, 177–190.

Sopher, Barry and Gary Gigliotti, 1990, A test of generalized expected utility theory, Manuscript (Rutgers University, New Brunswick, NJ).