Impulsive Actions and Agonizing Decisions

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Abstract

Rather than having precise preferences consistent with a single utility function over commodity bundles, individuals are assumed to have imprecise preferences with a set of possible utility functions, one of which is the true one. When attention is devoted to the task the set of utility functions contracts, making preferences more precise. This increases the individual's willingness to pay for a good, and when attention is exogenously determined this can lead to impulse shopping. When attention is endogenously determined, more attention is required to choose between bundles that are closer to indifferent, making these harder choices.

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1 Introduction

Sometimes people behave impulsively, acting without much apparent thought, while in other instances they seem to take an inordinatley long time to make a decision. The former decisions appear easy, while the latter appear hard. One way to think about easy versus hard decisions is through decision costs, as in Smith and Walker (1993). The individual searches through various options, incurring a search cost for each one. As with all sequential search models, the optimal search strategy employs a stopping rule, taking the first encountered option that is "good enough," which leads to the satisficing behavior introduced by Simon (1955). In such a model decisions are made quickly when the differences in the amount of utility generated by the different alternatives are small relative to the decision costs, so that the benefits from considering one more alternative are outweighed by the costs of doing so, while decisions are made slowly when the utility differences between the alternatives are large relative to the decision costs. Consequently, holding decision costs constant, the satisficing model predicts that choices are quick and easy when the alternatives are nearly indifferent, but difficult when the alternatives are far from indifferent because then finding the best alternative has a higher payoff.

This result, however, conflicts with what most people would consider a hard decision. For most people, an agonizing decision arises when the two (or more) choices are nearly indifferent, not far from indifferent, and it is difficult to ascertain which of the choices will generate the most utility. The satisficing model says that, since the two outcomes generate almost the same amount of utility (relative to decision costs), either choice is fine. But this is not how people behave.

The present paper constructs an alternative decision model that predicts that hard decisions are the ones where the alternatives are nearly indifferent. It also generates impulsive behavior, in particular impulse shopping behavior. Impulse shopping is making an unplanned purchase, such as grabbing a candy bar at the checkout line at a grocery store, and it is thought to be quite prevalent. In their study of purchases in South African supermarkets, Abratt and Goodey (1990) found that 22.5% of items purchased were impulse purchases, with half of all book and magazine purchases and 40% of snack food purchases made on impulse.[1] Often these decisions are triggered by an

[1] As detailed in Abratt and Goodey (1990), the 1977 Popai/Du Pont Consumer Buying

external stimulus, such as proximity to a display of the good in question.[2]

The decision model has two key components, imprecise preferences and attention. The idea is that initially a consumer is not sure how much she values a good, but knows that the subjective value is in some (possibly large) range. She buys the good if the price is below the lower endpoint of this value range, in which case she is certain to value the good more than the price. Consequently, the lower endpoint of the subjective value range is the consumer's willingness to pay (WTP). After devoting attention to the usefulness or desirability of the good, her preferences become more precise, narrowing the range of possible values. This narrowing can only increase the lower endpoint of the range, so reflection increases the consumer's WTP, and further reflection can only lead to further increases. If the WTP rises above the price of the good, the consumer buys it, but if the WTP does not rise enough to exceed the price of the good the consumer opts not to make the purchase. This can explain impulse shopping. When a good is prominently displayed and attracts the customer's attention, that attention increases the buyer's WTP. If WTP increases enough, the buyer purchases the good. Displays that capture more and longer attention generate larger increases in WTP, which is why displays and product placement are such important considerations in marketing.[3]

The above scenario relies on attention being "captured," which makes it exogenous. That attention can be captured by a stimulus is consistent with the psychology literature (e.g. Pashler, 1998), but Kahneman (1973) also argues that attention can be controlled volutarily, making it endogenous.

Habits Study categorized buying decisions into four categories: planned purchases in which the item and brand are known in advance, purchases in which the general item is planned but the brand is chosen at the store, planned purchases in which the item is known in advance but the consumer switches from a planned brand to a different brand while at the store, and unplanned purchases in which the consumer had no intention of buying any brand of that item before entering the store. Purchases in the last category are impulse purchases.

[2]In the Abratt and Goodey (1990) study, 60% of respondents who made unplanned purchases cited signs or special displays as the reason for their decisions. Hoch and Loewenstein (1991), and Dholakia (2000) both discuss how physical proximity to a good can trigger impulsive decisions. Rook (1987, p. 193) notes that the sudden urge to buy is "likely to be triggered by a visual confrontation with a product or by some promotional stimulus."

[3]A similar story can explain purchases on television home shopping channels. Park and Lennon (2004) document impulse shopping behavior among consumers who purchased apparel through television shopping networks.

If, in the above example, the posted price is close to the consumer's true WTP, so that the required payment and the good in question are nearly indifferent, she must voluntarily devote more attention in order to make the "correct" decision than if the posted price were father from the true WTP. In this sense, then, harder decisions are the ones that are closer to indifferent because they require more endogenous attention in order to decide.

The model relies on the concept of incomplete preferences, which arose in the 1960s and 1970s from efforts to test the robustness of general equilibrium theory to the underlying assumptions over preferences (e.g. Aumann, 1962; Schmeidler, 1969; Mas-Colell, 1974). More recently, interest has resurged because of its applicability to behavioral economics. For example, Bewley (2002) shows how incomplete preferences over probability distributions can capture ambiguity aversion,[4] Mandler (2004) and Masatlioglu and Ok (2005) demonstrate that incomplete preferences over probability distributions can lead to differences between selling and buying prices of lotteries, the so-called endowment effect, and Masatlioglu and Ok (2005) also demonstrate that incomplete preferences over consumption bundles can lead to endowment effects in goods markets. For preferences over probability distributions, preferences that are "more complete" correspond to preferences that are less ambiguity averse. As yet, however, no one has formalized a notion of "more complete" preferences over commodity bundles. Doing so is an additional contribution of this paper.

Section 2 provides a model of imprecise preferences, and Section 3 presents a model of attention. Section 4 considers a setting in which attention is involuntary, and finds how WTP and willingness-to-accept (WTA) measures are affected by attention. Section 5 considers a setting in which attention is deliberate, and identifies the characteristics of choice problems that require more attention than others. Section 6 offers a brief conclusion, including a discussion of satisficing in light of the new model.

2 Imprecise preferences

Instead of a single utility function u defined over commodity bundles $\mathbb{R}^n_+$, at time $t = 1, 2, ...$ the individual possesses a set of utility functions U_t

[4] Gilboa and Schmeidler (1989) use a different approach to generate a similar representation.

with typical element $u : \mathbb{R}^n_+ \to \mathbb{R}$.[5] The individual knows that one of the utility functions in U_t is the correct one, but does not know which one. She also knows that more of each good is better, but does not know the precise tradeoff between two goods. Thus every $u \in U_t$ exhibits the property that $u(x) > u(y)$ whenever $x > y$.

The strict preference relation $\succ_t$ is defined by $x \succ_t y$ if and only if $u(x) > u(y)$ for all $u \in U_t$. The properties of the utility functions in U_t induce several restrictions on the preference relation $\succ_t$. In particular, $\succ_t$ must be transitive, irreflexive, and monotone. Transitivity follows from the fact that if $x \succ_t y$ and $y \succ_t z$ then for every $u \in U_t$ it must be the case that $u(x) > u(y) > u(z)$. Monotonicity, the property that $x > y$ implies $x \succ_t y$, follows from the fact that $x > y$ implies $u(x) > u(y)$ for all $u \in U_t$.

Other properties of the utility functions in U_t also induce corresponding restrictions on the preference ordering $\succ_t$. For example, if every $u \in U_t$ is quasiconcave then the better-than sets $\{x|x \succ_t y\}$ are convex. To see why, take any y and let z and z^* be two elements of $\{x|x \succ_t y\}$. Then for any utility function $u \in U_t$ and any $\lambda \in [0, 1]$, the quasiconcavity of u implies that $u(\lambda z + (1 - \lambda)z^*) \geq u(z) > u(y)$, and so $\lambda z + (1 - \lambda)z^* \succ_t y$ and the better-than set is convex.

One property that does not readily extend from the elements of U_t to $\succ_t$ is continuity. The continuity of u implies that $\{x|u(x) > u(y)\}$ is open, but this is not enough to guarantee that $\{x|x \succ_t y\}$ is open because $\{x|x \succ_t y\} = \bigcap_{u \in U_t}\{x|u(x) > u(y)\}$, and arbitrary intersections of open sets need not be open. Consequently, continuity of $\succ_t$ must be assumed directly, and we do so for the remainder of the paper.

A second property of the elements of U_t that does not extend to $\succ_t$ is completeness, which is not assumed in this paper. A formal discussion of completeness requires a notion of indifference, and define the indifference relation $\sim_t$ by $x \sim_t y$ if $u(x) = u(y)$ for all $u \in U_t$. Note that the indifference relation is reflexive and transitive. A preference ordering is a combination of the strict preference relation $\succ_t$ and the indifference relation $\sim_t$. The preference ordering is complete if for any pair x and y one of the following statements is true: $x \succ_t y$, $y \succ_t x$, or $x \sim_t y$. Thus, completeness requires that for every pair x and y, either $u(x) > u(y)$ for every $u \in U_t$, $u(x) < u(y)$ for every $u \in U_t$, or $u(x) = u(y)$ for every $u \in U_t$. One can readily show that the preference ordering is complete if and only if every element of U_t is an

[5] How the set U_t of utility functions changes over time is the subject of the next section.

increasing transformation of every other element of U_t.[6] If none of $x \succ_t y$, $y \succ_t x$, or $x \sim_t y$ hold we say that x and y are *incomparable*.

For the remainder of this paper we make the following assumptions on preferences:

(1) At time t there exists a set U_t of continuous, increasing utility functions;

(2) $x \succ_t y$ if and only if $u(x) > u(y)$ for all $u \in U_t$ and $x \sim_t y$ if and only if $u(x) = u(y)$ for all $u \in U$; and

(3) For each y, $\{x|x \succ_t y\}$ is open.

As noted, assumptions (1) and (2) imply that $\succ_t$ is transitive, irreflexive, monotone, and convex. When combined with (3), the assumptions are consistent with the models of incomplete preferences analyzed by Aumann (1962), Schmeidler (1969), and Mas-Colell (1974).[7] Ok (2002) and Evren and Ok (2007) provide an axiomatic basis for the existence of an incomplete preference ordering with properties (1) through (3) above. Models with more structure exist when attention is restricted to preferences over lotteries. Dubra and Ok (2002) and Dubra, Macccheroni, and Ok (2004) explore the foundations of a model in which individuals possess sets of von Neumann-Morgenstern utility functions and prefer one lottery to another if and only if the first lottery generates higher expected utility according to every utility function in the set. Butler and Loomes (2006,2007) use this sort of model to explain preferences reversals and violations of the independence axiom. The approach with a set of utility functions is dual to the one used by Gilboa and Schmeidler (1989), Bewley (2002), and Rigotti and Shannon (2005), whose models assume a single von Neumann-Morgenstern utility function and a set of potential probability vectors over states rather than a set of utility functions over commodity bundles. Chambers and Melkonyan (20006) explore how refined beliefs through information acquisition can contract the set of potential probability vectors over states in the Gilboa-Schmeidler setting.

[6] If every element of U_t is an increasing transformation of every other element, then U_t can be treated as a singleton $\{u\}$, and completeness holds. On the other hand, if U_t contains two elements, u and v, which are not increasing transformations of each other, then there exist points x and y such that $u(x) > u(y)$ but $v(x) < v(y)$.

[7] Because of its consistency with Schmeidler (1969), the model presented here implies the usual upper secmicontinuous demand correspondences. Note that convexity of $\succ_t$ is not required because the incompleteness of preferences induces sufficient convexity in small neighborhoods.

3 Attention

The interpretation of the preferences in the preceding section is that at time t the decision maker knows that her true utility function is in the set U_t but does not know which utility function is the true one. The purpose of this section is to place some structure on the dynamic nature of U_t, and the structure is attained through the process of attention.

Psychologists have explored attention for decades, and two primary foci of the research have been the autonomous, as opposed to deliberate, nature of attention and the ability of people to parallel-process or multitask. Two natural byproducts of this research have relevance for this study. First, attention can either be consciously focused by the individual or it can be attracted by external stimuli. This means that individuals have some control over how their attention is spent, but that they do not have complete control. Second, devoting more attention to a task leads to improved performance.

The use of attention in this paper is to contract the set of possible utility functions. Consider a finite collection of sets of utility functions, $U^1 \supset ... \supset U^m = \{u^*\}$, where u^* is the individual's true utility function for consumption purposes. At any time $t = 1, 2, ...$ her collection of utility functions is $U_t \in \{U^1, ..., U^m\}$. Attention is a binary process, being either on or off, and is captured by the binary function $\theta(t)$ which takes the value 1 if the individual devotes attention to the the decision problem in period t and takes the value 0 otherwise. The time path of the sets of utility functions is

$$U_1 = U^1;$$

$$U_t = U^k \text{ and } k < m \Longrightarrow U_{t+1} = \begin{cases} U^{k+1} & \theta(t) = 1 \\ & \text{if} \\ U^k & \theta(t) = 0 \end{cases} \tag{1}$$

$$U_t = U^m \Longrightarrow U_{t+1} = U^m$$

According to this formulation, the individual begins with the set of utility functions U^1, and devoting attention to the choice problem leads to a one-step contraction in the set of utility functions. No further attention can be devoted once the set U^m is reached because at that point no further contractions in the set of utility functions are possible.

In keeping with the psychology literature, the value of $\theta(t)$ may be determined either exogenously or endogenously. It is exogenous if an external stimulus captures the individual's attention, while it is endogenous if the individual chooses to devote attention to the task. These two cases are treated separately below.

4 Exogenous attention and impulse shopping

Attention is exogenous when it is triggered involuntarily by some stimulus. Different stimuli may capture attention for different lengths of time. In accordance with the dynamic attention process is (1), though, the post-attention set of utility functions V is a subset of the pre-attention set U. The purpose of this section is to find the impact of this contraction on behavior, and to relate it to impulse shopping.

For purely notational purposes, let $(x; a, i; b, j) = (x_1, ..., x_i + a, ..., x_j + b, ..., x_n)$ be the commodity bundle that differs from the bundle x by adding a to good i and adding b to good j. Define $WTP_{ij}^U(b; x)$ to be the largest amount of good i the individual is willing to give up to gain b units of good j when the set of utility functions is U and the endowment is x. One can interpret $WTP_{ij}^U(b; x)$ as the willingness to pay (in terms of good i) for $b > 0$ units of good j. For the measure to be meaningful, the initial endowment x must be such that $x_i > 0$ so that the individual has some of good i to pay with, and $b > 0$ must be sufficiently small that $x \succ (x; -x_i, i; b, j)$ so that the individual has enough of good i to pay with. When these conditions hold we call $(b; x)$ a *feasible willingness-to-pay problem.* Define $WTA_{ij}^U(b; x)$ to be the smallest amount of good i the individual is willing to accept to give up b units of good j when the set of utility functions is U and the endowment is x. This measure can be interpreted as the willingness to accept (in terms of good i) for giving up $b > 0$ units of good j, and for it to be meaningful b must be less than x_j and good i must be sufficiently desirable to offset the loss of good j; that is, there must be some value $\alpha > 0$ for which $(x; \alpha, i; -b, j) \succ x$. When these conditions hold we call $(b; x)$ a *feasible willingness-to-accept problem.* Let $WTP_{ij}^u(b; x)$ and $WTA_{ij}^u(b; x)$ be the values of $WTP_{ij}^U(b; x)$ and $WTA_{ij}^U(b; x)$, respectively, when the set of utility functions is $\{u\}$.

Proposition 1 *(1)* $WTP_{ij}^U(b; x) = \inf\{WTP_{ij}^u(b; x) | u \in U\}$ *for every feasible willingness-to-pay problem.*

(2) $WTA_{ij}^U(b; x) = \sup\{WTA_{ij}^u(b; x) | u \in U\}$ *for every feasible willingness-to-accept problem.*

Proof. (1) Let $(b;x)$ be a feasible willingness-to-pay problem. $WTP^u_{ij}(b;x)$ is defined by the amount a that solves $u(x;-a,i;b,j) = u(x)$. By hypothesis, $u(x;-x_i,i;b,j) < u(x)$ for all u, and monotonicity guarantees that $u(x;0,i;b,j) > u(x)$, so that $WTP^u_{ij}(b;x)$ is uniquely defined in $(0,x_i)$ for all u. Define $a^* = \inf\{WTP^u_{ij}(b;x)|u \in U\}$. Then $u(x;a^*,i;b,j) \geq u(x)$ for all $u \in U$. Also, because of the continuity assumption that $\{x|x \succ y\}$ is open for each y, $u(x;a^*,i;b,j) = u(x)$ for somel $u \in U$. Consequently, if $a > a^*$ there is some $u \in U$ for which $u(x;a,i;b,j) < u(x)$, and a^* is the most of good i the individual is willing to give up for b additional units of good j.

(2) Let $(b;x)$ be a feasible willingness-to-accept problem. $WTA^u_{ij}(b;x)$ is defined by the amount a that solves $u(x;a,i;-b,j) = u(x)$. By hypothesis, $u(x;\alpha,i;-b,j) > u(x)$ for all u, and monotonicity guarantees that $u(x;0,i;-b,j) < u(x)$ and that $WTA^u_{ij}(b;x)$ is uniquely defined in $(0,\alpha)$ for all u. Define $a^* = \sup\{WTA^u_{ij}(b;x)|u \in U\}$. Then $u(x;a^*,i;-b,j) \geq u(x)$ for all $u \in U$. Also, because of the continuity assumption that $\{x|x \succ y\}$ is open for each y, $u(x;a^*,i;-b,j) = u(x)$ for somel $u \in U$. Consequently, if $a < a^*$ there is some $u \in U$ for which $u(x;a,i;b,j) < u(x)$, and a^* is the least amount of good i the individual is willing receive in exchange for giving up b units of good j. ■

The mathematical constructions of willingness to pay and willingness to accept in Proposition 1 lead to our main result for exogenous attention, which states that as the set of possible utility functions shrinks, willingness to pay increases and willingness to accept decreases.

Corollary 1 *If $V \subset U$ then $WTP^V_{ij}(b;x) \geq WTP^U_{ij}(b;x)$ and $WTA^V_{ij}(b;x) \leq WTA^U_{ij}(b;x)$ for all goods i and j and all feasible willingness-to-pay and willingness-to-accept problems.*

Proof. The conclusions follow immediately from Proposition 1 after noting that $\{WTP^u_{ij}(b;x)|u \in V\} \subset \{WTP^u_{ij}(b;x)|u \in U\}$ and $\{WTA^u_{ij}(b;x)|u \in V\} \subset \{WTA^u_{ij}(b;x)|u \in U\}$. ■

The corollary establishes that attention to a good can only increase the willingness to pay for that good and can only decrease the willingness to accept for that good. Moreover, it also establishes the stronger result that attention to a good can only increase the willingness to pay and decrease the willingness to accept for *any* good, not just the one that is being considered. An obvious implication of the corollary is that WTA-WTP disparities should be large for goods the decision maker is either unfamiliar with or has trouble valuing, but should be smaller when the decision maker is familiar with the good and has thought about the tradeoff.

Of more interest for impulse shopping, though, is a second implication of the corollary that, because attention can only increase the willingness to pay for a good, product placement in stores is important. End cap or cash register displays, if they can capture consumers attention and get them to consider the possible tradeoffs, can increase consumers' willingness to pay for the goods and lead to unplanned purchases or impulse buying.[8]

The converse of Corollary 1 is obviously not true, because it could be that $WTP_{ij}^{V}(b;x) \geq WTP_{ij}^{U}(b;x)$ and $WTA_{ij}^{V}(b;x) \leq WTA_{ij}^{U}(b;x)$ for all goods i and j and all commodity bundles x but some utility functions in the "interior" of V are missing from U. To get around this problem and formulate a converse to Corollary 1 it is necessary to restrict the class of sets from which U and V can be chosen. We say that U is *fully covering* if, for every $x, y \in \mathbb{R}_{+}^{n}$ with neither $x \geq y$ nor $y \geq x$, there exists a utility function $u \in U$ such that $u(x) = u(y)$. We say that U is *uniquely covering* if there are no combinations of distinct $x, y \in \mathbb{R}_{+}^{n}$ and $u, v \in U$ such that $u(x) = u(y)$ and $v(x) = v(y)$. In two dimensions this is a single-crossing property for indifference curves. Rigotti and Shannon (2004) restrict attention to fully and uniquely covering sets of utility functions over state-payoff combinations by considering a single von Neumann-Morgenstern utility function and allowing the probabilities of states to vary over $(0,1)^{n}$. A set U is *radially convex* if for any combination of functions $u, v \in U$ and points $x, y, z \in \mathbb{R}_{+}^{n}$ such that $u(y) = u(x)$, $v(z) = v(x)$, and $y \geq z$, there exists for every $\beta \in (0,1)$ a function $w \in U$ such that $w(\beta y + (1-\beta)z) = w(x)$. Radial convexity says that if both y and z are incomparable with x and if y dominates z in the sense of containing at least as much of every commodity, then convex combinations of y and z are also incomparable with x. In two dimensional space this requires that the sets of incomparable points are "thick" without holes. The next proposition establishes that restricting attention to uniquely and fully covering sets of utility functions allows one to order sets of utility functions in an intuitively plausible way.

Proposition 2 *Suppose that* $\mathcal{U}$ *is uniquely and fully covering, and that* $U, V \subset$

[8] See the related literature on slotting allowances, e.g. Shaffer (1991), Marx and Shaffer (2004).

$\mathcal{U}$ *are radially convex.* $V \subset U$ *if and only if, for all* $x \in \mathbb{R}^n_+$,

$$\{y|u(y) > u(x) \text{ for all } u \in U\} \subset \{y|v(y) > v(x) \text{ for all } v \in V\} \quad (2)$$

and

$$\{y|u(y) < u(x) \text{ for all } u \in U\} \subset \{y|v(y) < v(x) \text{ for all } v \in V\}. \quad (3)$$

Proof. Suppose that $V \subset U$. Then, by construction, if $u(y) > u(x)$ for all $u \in U$, it must follow that $u(y) > u(x)$ for all $u \in V$. Consequently, if $y \in \{y|u(y) > u(x)$ for all $u \in U\}$, it must follow that $y \in \{y|u(y) > u(x)$ for all $u \in V\}$, and (2) holds. Similarly, if $y \in \{y|u(y) < u(x)$ for all $u \in U\}$, it must follow that $y \in \{y|u(y) < u(x)$ for all $u \in V\}$, and (3) holds.

Suppose that (2) and (3) hold. Define

$$I_U(x) = \mathbb{R}^n_+ \backslash (\{y|u(y) > u(x) \text{ for all } u \in U\} \cup \{y|u(y) < u(x) \text{ for all } u \in U\})$$

and similarly for $I_V(x)$. Then $I_V(x) \subset I_U(x)$ for all x. Now for any $v \in V$ one can find some pair x, y such that $v(x) = v(y)$. Then $y \in I_V(x)$, and so $y \in I_U(x)$. Because $\mathcal{U}$ is uniquely covering and because U and V are radially convex, v is the only element of V with $v(y) = v(x)$. Because $I_V(x) \subset I_U(x)$, v is also in U, and $V \subset U$. ■

The restriction to uniquely and fully covering sets of utility functions allows one to establish the converse of Corollary 1.

Proposition 3 *Suppose that* $\mathcal{U}$ *is uniquely and fully covering, and that* $U, V \subset \mathcal{U}$ *are radially convex. If either*

(1) $WTP^V_{ij}(b; x) \geq WTP^U_{ij}(b; x)$ *for all goods* i *and* j *and all feasible willingness-to-pay problems, or*

(2) $WTA^V_{ij}(b; x) \leq WTA^U_{ij}(b; x)$ *for all goods* i *and* j *and all feasible willingness-to-accept problems,*

then $V \subset U$.

Proof. Suppose not, that is, suppose that V is not a subset of U. In light of Proposition 2 there exist points $x, y \in \mathbb{R}^n_+$ and a function $v \in V$ such that either (i) $u(x) < u(y)$ for all $u \in U$ but $v(x) > v(y)$, or (ii) $u(x) > u(y)$ for all $u \in U$ but $v(x) < v(y)$. Furthermore, x, y, and v can be chosen so that x and y differ on only two dimensions. Otherwise, in case (i) for example, one can construct a sequence of points $z^1, ..., z^s$ such that z^t and z^{t+1} differ on only two dimensions for each $t = 1, ..., s-1$ and $u(z^{t+1}) > u(z^t)$ for all $u \in U \cup \{v\}$. Transitivity then implies that $v(y) > v(x)$, contrary to condition (i). Consequently, assume that $x_i = y_i$ for $i \neq j, k$.

Case (i). Assume, without loss of generality, that $y_j < x_j$ (otherwise switch the indices j and k). For $u(y) > u(x)$ it must then follow that $y_k > x_k$. By construction,

$$\begin{aligned} WTA^V_{kj}(x_j - y_j; x) &\geq WTA^v_{kj}(x_j - y_j; x) \\ &> y_k - x_k \\ &> WTA^U_{kj}(x_j - y_j; x), \end{aligned}$$

where the first inequality comes from Proposition 1, the second folows from the fact that $v(y) < v(x)$, and the third follows from $u(y) > u(x)$. This contradicts condition (2) of the proposition. Also,

$$\begin{aligned} WTP^V_{jk}(y_k - x_k; x) &\leq WTP^v_{jk}(y_k - x_k; x) \\ &< x_j - y_j \\ &< WTP^U_{jk}(y_k - x_k; x) \end{aligned}$$

which contradicts condition (1).

Case (ii). Assume, without loss of generality, that $y_j < x_j$, which implies $y_k > x_k$. By construction,

$$\begin{aligned} WTA^V_{jk}(y_k - x_k; y) &\geq WTA^V_{jk}(y_k - x_k; y) \\ &> x_j - y_j \\ &> WTA^U_{jk}(y_k - x_k; y) \end{aligned}$$

which contradicts condition (2). Also,

$$\begin{aligned} WTP^V_{kj}(x_j - y_j; y) &\leq WTP^V_{kj}(x_j - y_j; y) \\ &> y_k - x_k \\ &> WTP^U_{jk}(x_j - y_j; y) \end{aligned}$$

which contradicts condition (1). ■

5 Endogenous attention and hard choices

The question to be addressed in this section is, how much attention is required for an individual to make a correct decision, that is, one that is consistent with maximizing the true utility function? This bypasses the issue of whether correctness of the decision is the appropriate goal, focusing instead on correctness of the decision as a means of motivating attention.

Most endogenous activities have costs associated with them. This may not be true for attention. The psychological research finds that attention can be captured exogenously, which suggests that it can be effortless, and the research also finds that attention can be shared among multiple tasks, and so there may not be opportunity costs associated with devoting attention to a particular task. In keeping with these findings I model attention as if it is costless, and comment on the effects of attention costs in the conclusion.

As in Section 3, there is a finite collection of sets of utility functions, $U^1 \supset ... \supset U^m = \{u^*\}$, with u^* the individual's true utility function. Consequently, the choice of x over y is correct if and only if $u^*(x) \geq u^*(y)$. At time t her set of utility functions is $U_t \in \{U^1, ..., U^m\}$. She is sure of a correct decision between x and y at time t if either $u(x) > u(y)$ for all $u \in U_t$ or $u(x) < u(y)$ for all $u \in U_t$. Accordingly, let X be the set of distinct pairs of commodity bundles, so that X excludes pairs of the form (x, x), and define X^k, $k = 1, ..., m$, by

$$X^k = \{(x, y) \in X | u(x) > u(y) \text{ for all } u \in U^k \\ \text{or } u(x) < u(y) \text{ for all } u \in U^k\}.$$

Thus X^k is the set of pairs of commodity bundles over which the individual can choose correctly even though preferences are imprecise and given by the set of utility functions U^k.

Now let

$$\begin{aligned} Y^1 &= X^1, \\ Y^k &= X^k \setminus X^{k-1}, \, k = 2, ..., m. \end{aligned}$$

If the pair of commodity bundles (x, y) is in the set Y^k, then the individual can choose between them when the set of utility functions is U^k or smaller, but not when the set is U^{k-1} or larger.

The sets $Y^1, ..., Y^m$ do not cover the set of all pairs X. In particular, if

x and y are truly indifferent, none of the sets can yield a strict preference for one over the other. To remedy this, let $Y^{m+1} = \{(x,y) \in X | u^*(x) = u^*(y)\}$ be the set of truly-indifferent, distinct pairs. Then $\bigcup_{k=1}^{m+1} Y^m = X$.

The indicator function θ used in (1) allows us to model a "unit" of attention, with one unit of attention corresponding to one time period in which $\theta(t) = 1$. According to the construction of $Y^1, ..., Y^m$, pairs of alternatives in Y^1 require no attention for a decision, while pairs in Y^2 require one unit of attention, pairs in Y^3 require two units, and so on with pairs in Y^m requiring $m-1$ units of attention. Pairs in Y^{m+1} also require $m-1$ units of attention because no futher refinements of the set of utility functions is possible beyond $U^m = \{u^*\}$. One can think of this as a ranking of the difficulty of the decision, with decisions in Y^k being harder than those in Y^{k-1} because they require more precision.

The above construction characterizes pairs of alternatives according to the amount of attention they require, with a decision task in Y^k harder than one in Y^j if $k > j$. The next step is to provide an alternative and, as it turns out, equivalent characterization of choice problems that is based solely on the properties of the two alternatives, x and y, and the true utility function u^*, and which sheds further light on which problems are hard.

Define (x, y') as being *closer to indifferent according to* u than (x, y) if one of the following two scenarios holds: (i) $u(x) > u(y)$, $u(x) \geq u(y')$, and $y' > y$, which implies that $u(x) - u(y') < u(x) - u(y)$; or (ii) $u(x) < u(y)$, $u(x) \leq u(y')$, and $y > y'$, which implies $u(y') - u(x) < u(y) - u(x)$. Simply put, one pair is closer to indifferent than another according to a specific utility function if the first pair generates a smaller utility difference than the second. In scenario (i), x is the best of the three commodity bundles and y' is unambiguously better than y, that is, better according to any increasing utility function. In scenario (ii), x is the worst of the three bundles and y' is unambiguously worse than y. In both cases y' is closer to x as measured by the utility function u.

We wish to characterize the sets $Y^1, ..., Y^m$ in terms of closeness to indifference according to the true utility function u^*. To this end, let A and B be two sets of pairs of commodity bundles, that is, $A, B \subseteq \mathbb{R}^n_+ \times \mathbb{R}^n_+$. We say that A is *outside* of B *according to* u if for every $(x, y) \in A$ there exists $(x, y') \in B$ such that (x, y') is closer to indifferent according to u than (x, y).

Proposition 4 *Suppose that* $U^1 \supset ... \supset U^m = \{u^*\}$ *are all uniquely covering and radially convex, and construct* $Y_0^1, ..., Y^{m+1}$ *as above. Then* Y^k *is outside* *of* Y^{k+1} *according to* u^**for* $k = 1, ..., m$.

Proof. Let $I_x(U^k)$ be the set of points that are indifferent or incomparable to x according to U^k. Because strictly-better-than and strictly-worse-than sets are open by maintained assumption, it follows that $I_x(U^k)$ is closed. Because all of the U^k's are uniquely covering, Y^k is the closure of $I_x(U^{k-1}) \setminus I_x(U^k)$.

Fix $x \in \mathbb{R}^n_+$. For each y assign $z(y)$ such that $u^*(z) = u^*(x)$ and $y = \lambda z$ for some scalar $\lambda > 0$. Construct the set

$$R^k(x, y) = \{w \in I_x(U^k) | w = \alpha z(y) \text{ for some scalar } \alpha > 0\},$$

so that $R^k(x, y)$ is the intersection of the set of points incomparable to x according to U^k and a ray running from the origin through y. Because $I_x(U^k)$ is closed so is $R^k(x, y)$. Because U^k is radially convex, $R^k(x, y)$ is connected. Construct

$$C^k(x, y) = \{u^*(w) - u^*(x) | w \in R^k(x, y)\},$$

which is a closed interval containing zero. Because $I_x(U^k) \subset I_x(U^{k-1})$ it follows that $C^k(x, y) \subset C^{k-1}(x, y)$. Define $S^k(x, y)$ to be the closure of $R^{k-1}(x, y) \setminus R^k(x, y)$. By construction, $S^k(x, y) \subset Y^k$. Let

$$D^k(x, y) = \{u^*(w) - u^*(x) | w \in S^k(x, y)\}.$$

Then $D^k(x, y)$ is the closure of $C^{k-1}(x, y) \setminus C^k(x, y)$. Partition $D^k(x, y)$ into two components, with $D^k_+(x, y) = D^k(x, y) \cap \mathbb{R}_+$ and $D^k_-(x, y) = D^k(x, y) \cap \mathbb{R}_-$. Then $\sup D^k_+(x, y) = \inf D^{k-1}_+(x, y)$ and $\inf D^k_-(x, y) = \sup D^{k-1}_-(x, y)$, establishing that Y^k is outside of Y^{k+1} according to u^*. ■

Proposition 4 shows decisions between alternatives that are closer to indifferent are harder decisions in that they require more attention and further precision.

6 Conclusion

This paper formalizes the following story. When an individual is either asked to value an unfamiliar good or faced with an exchange possibility involving an unfamiliar good, she does not possess a single utility function that prescribes the choice. Instead, she possesses a set of utility functions, and she makes an exchange only when the exchange is preferred by all of the utility functions in the set. Increases in familiarity with a good, attention to a good, and reflection about a good can all lead to contractions in the set of utility functions. When attention is captured exogenously by a prominently displayed good, her set of utility functions contracts and her willingness to pay increases. This can lead to impulse shopping behavior.

The model leads to implications for when a decision is easy or difficult. According to the model, if the decision maker wishes to get the decision right she must devote more attention to it the closer the true preference is to indifferent. This contrasts with the predicitons of satisficing theories, which says that when the individual is sufficiently close to indifferent she will stop devoting attention to the decision, and so under satsificing the hard decisions are the ones that are far from indifferent.

One difference between the model proposed here and satisficing behavior is that the latter imposes a cost to attention while in the former attention is costless. Making attention costly would lead to a model that is in-between the one proposed here and satisficing. Costly attention means that the individual pays a cost for moving from the set of utility functions U^k to the smaller set U^{k+1}. As these sets contract, the potential gains from further contractions may fall below the cost of achieving them, in which case the individual will opt to cease paying attention before she gets to the true preference function. Consequently, there may be some positive integer $K < M$ such that the individual devotes attention until either she has a clear preference or she reaches set U^K. She will get closer to U^K when the choice is closer to indifferent, and so the same ordering of "more difficult" choices arises, but one still obtains satisficing behavior because the individual may choose to stop before reaching the true optimum.

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