Some mixed results on boundary effects

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Recent experimental evidence suggests that expected utility theory performs well when all lotteries have the same number of probable outcomes. A straightforward model designed to accommodate this evidence also allows Allais paradox behavior but predicts violations of stochastic dominance preference.

1. Introduction

Experiments have shown that the expected utility model of behavior under uncertainty fails in systematic ways, and this evidence has led to the introduction of many new models of behavior. Most of the evidence involves choices over lotteries with at most three outcomes, and, typically, the choices have different numbers of possible outcomes. For example, of the four lotteries involved in the Allais paradox, one places positive probability on three outcomes (three probable outcomes), two place positive probability on two outcomes (two probable outcomes) and the fourth has one probable outcome. Recently, however, Camerer (1992), Conlisk (1989), Harless (1992) and Sopher and Gigliotti (1990) have shown that if one takes sets of lotteries which have generated violations of expected utility, such as the Allais lotteries, and slightly perturbs all of the lotteries so that they all have the same number of probable outcomes, then expected utility fits the data much better than before. In fact, Harless and Camerer (1991), in their analysis of a large number of experimental data sets, conclude that the expected utility model should be used when all lotteries have the same number of probable outcomes, but a different model must be used when lotteries have different numbers of probable outcomes. The purpose of this paper is to explore the behavior implied by this evidence by using a model which is a straightforward extension of the experimental results.

In the model an individual behaves as an expected utility maximizer when all lotteries have the same number of probable outcomes, but the utility functions are different for different numbers of probable outcomes. The results of this model are mixed. A nice feature, which is somewhat surprising, is that many of the usual violations of expected utility, including the Allais paradox, can be explained within the context of the model as a preference for fewer probable outcomes, which is the boundary effect hypothesis first hinted at by Conlisk (1989). A bad feature of the model is that it violates stochastic dominance preference. Violations of stochastic dominance preference can be explained within the context of the model as a preference for fewer probable outcomes, which is the boundary effect hypothesis first hinted at by Conlisk (1989). A bad feature of the model is that it violates stochastic dominance preference. 1

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1 Another model with separate treatment of distributions with different numbers of probable outcomes is Viscusi's (1989) prospective reference theory, which weights probabilities (but in a manner which preserves stochastic dominance preference).
avoided if an editing phase similar to that in prospect theory [Kahneman and Tversky (1979)] is added. Nevertheless, the main contribution of the paper is not the model, but the interaction between boundary effects and violations of expected utility.

2. Expected cardinality-specific utility

Let $P$ denote the set of probability distributions defined over the set $X = \{x_1, \ldots, x_m\}$ of outcomes, and let $p$ denote a typical element of $P$. Outcomes are measured as gains or losses from some reference wealth level. Define $n(p)$ to be the number of outcomes assigned strictly positive probability by $p$, and an outcome assigned positive probability by $p$ is called a probable outcome of $p$. The individual is an expected cardinality-specific utility (ECSU) maximizer if there exists a set of utility functions $u_i(x)$ such that the individual prefers $p$ to $q$ if and only if

$$\sum_{i=1}^{m} p_i u_{n(p)}(x_i) \geq \sum_{i=1}^{m} q_i u_{n(q)}(x_i).$$

(1)

The utility function over which expectations are taken is determined by the number of probable outcomes in the lottery under consideration. The expected utility hypothesis holds if $u_i(x) = \cdots = u_m(x)$ for all $x \in X$, that is, if the same utility function is used to evaluate all lotteries. An alternative hypothesis is given below.

Boundary Effect Hypothesis. $u_i(x) > \cdots > u_m(x)$ for all $x > 0$, $u_i(0) = \cdots = u_m(0)$, and $u_i(x) < \cdots < u_m(x)$ for all $x < 0$.

Boundary effects can be interpreted as a preference for fewer probable outcomes. The property that people overweight outcomes which are certain relative to outcomes which are merely probable (as opposed to possible) was labelled the certainty effect by Kahneman and Tversky (1979). ECSU preferences with boundary effects possess an even stronger property. The preference ordering will place a higher value on an outcome when it is in a distribution with a small number of other outcomes relative to when the distribution contains a large number of other outcomes, and the certainty effect arises as a special case. This should not be interpreted as the outcome itself yielding different levels of utility when the number of alternative probable outcomes changes, but that the individual's ex ante utility of the lottery changes when the number of probable outcomes changes.

Loewenstein (1987) presents a model of savoring and dread of anticipated consumption which can be used to justify the ECSU model with boundary effects. In his model the individual's preference ordering relies not only on the utility from consumption of an outcome, but also on the anticipation of consumption of that outcome. The individual savors potential gains and dreads potential losses. Utility from anticipation of an outcome is assumed to be proportional to utility from consumption of that outcome. If attention is limited, and if each outcome must compete for a share of total attention, then adding probable outcomes will dilute the attention placed on each individual outcome. As a result, utility from anticipation is diluted when more outcomes become probable. If utility from consumption of an outcome $x$ is $u(x)$, utility from anticipation of a probable outcome $x$ in the distribution $p$ is $\alpha(n(p))u(x)$, and total utility is

$$u_{n(p)}(x) = (1 + \alpha(n(p)))u(x).$$

(2)

If $\alpha$ is decreasing in $n$, then utility from anticipation is diluted, and boundary effects are exhibited.
Camerer (1992), Conlisk (1989), Harless (1992), and Sopher and Gigliotti (1990) have all found that expected utility performs well when all choices have three probable outcomes, but that expected utility fails when different choices have different numbers of probable outcomes. The ECSU model was designed specifically to accommodate this evidence, and so attention will be turned to the second set of evidence. A common choice problem is shown in fig. 1 for the set of three-outcome lotteries, where \( x_1 < x_2 < x_3 \), and subjects must make choices from pairs \((A_1, B_1)\). If all outcomes are gains, then a common choice pattern is \( B_1 \) over \( A_1 \), \( A_2 \) over \( B_2 \), and \( B_3 \) over \( A_3 \), or \( BAB \) for short. If all outcomes are losses, then \( ABA \) is common. Expected utility allows only \( AAA \) or \( BBB \). As will be shown, the ECSU model with boundary effects allows the three patterns \( AAA \), \( BBB \), and \( BAB \) for gains, and the first two and \( ABA \) for losses.

To prove this, consider the following generalized common consequence framework. Define \( Q \) to be the probability distribution which assigns the outcome \( x_2 \) probability one, and define \( R \) to be the probability distribution which offers \( X_1 \) with probability \( r > 0 \) and \( x_3 \) otherwise. Let \( A_i \) be the probability distribution composed of distribution \( Q \) with probability \( p \) and \( x_i \) otherwise. Similarly, let \( B_i \) be the probability distribution offering \( R \) with probability \( p \) and \( x_i \) otherwise. An expected utility maximizer prefers \( A_i \) to \( B_i \) for all \( i \) if \( u_i(x_2) > u_i(x_1) + (1 - r)u_i(x_3) \). A sufficient condition for the pattern \( BAB \) to arise in the ECSU framework with boundary effects is \( u_1(x_2) > ru_2(x_1) + (1 - r)u_2(x_3) > u_2(x_2) \). The pattern \( ABA \) arises in loss space if the opposite inequalities hold.

A strong negative implication of the ECSU model with boundary effects is the violation of stochastic dominance preference. It is easy to see how this occurs. If \( u_1(x) > u_2(x) \), then it is possible, by choosing a large enough value of \( r \), to mix the outcome \( x \) with some larger outcome \( y \) in such a way that \( u_1(x) > ru_2(x) + (1 - r)u_2(y) \), even though the mixture is a first-order stochastically dominating distribution. In fact, violations of stochastic dominance preference will occur at every distribution (except the best distribution) which has fewer than the maximal number of probable outcomes. These violations are a direct result of the discontinuity imposed by the assumption that expected utility is satisfied on subsets of probability space in which all distributions have the same number of probable outcomes.

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2 The Allais paradox lotteries are an example for \((A_1, B_1)\) and \((A_2, B_2)\).
3 This formulation is similar to the formulation of the common consequence effect in Machina (1987).
4. Conclusion

The ECSU model with boundary effects is a simple model which can accommodate a large amount of evidence, but it has one major drawback: violations of stochastic dominance. This problem can be solved using an editing phase similar to that in prospect theory [Kahneman and Tversky (1979)]. In the editing phase choice candidates are scanned to detect stochastically dominated lotteries, and those candidates are discarded without further evaluation. In the second phase choices are made among the remaining lotteries using the ECSU model with boundary effects. In fact, the ECSU model has many features in common with prospect theory, including certainty effects, discontinuities when probabilities are zero, and reflection effects, but achieves them without probability weights.

References