Ambiguity Aversion: An Axiomatic Approach Using Second Order Probabilities

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Abstract

Assume that reduction of compound lotteries fails so that, in the Anscombe-Aumann framework, individuals treat horse lotteries separately from the roulette lotteries. Applying the von Neumann-Morgenstern axioms to roulette lotteries and the Savage axioms to horse lotteries yields a second-order expected utility specification. Specifically, the decision maker forms a subjective second-order probability distribution over objective probability distributions and possesses two von Neumann-Morgenstern utility functions, one governing risk attitudes and one governing ambiguity attitudes.

Keywords: Ambiguity; Savage axioms; second-order probabilities
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[*] This paper is a revised and updated version of a 1993 unpublished working paper with the same title. I am grateful to Peter Klibanoff for encouraging me to resurrect it.

1 Introduction

One of the famous problems that highlights the difference between risk and ambiguity (or uncertainty) is the two-color Ellsberg problem (see Ellsberg, 1961). A decision maker is faced with two urns. The first urn contains 50 red and 50 yellow balls, and the second contains 100 balls but in an unknown mixture of red and yellow. The decision maker will be paid $10 if she can draw a yellow ball and must choose from which urn to draw. The first urn generates a known payoff distribution, so it is risky, but the second urn generates an unknown payoff distribution, so it is ambiguous. Subjects systematically avoid the ambiguous urn in favor of the risky urn, thereby creating the need for a model of choice behavior which can accommodate the distinction between risk and ambiguity.

Researchers have used two methods to generalize the subjective expected utility model to allow for ambiguity aversion. One approach is based on non-additive probabilities, and examples can be found in Gilboa (1987), Schmeidler (1989), Gilboa and Schmeidler (1989), and Sarin and Wakker (1992). The alternative approach, which has generated considerable recent interest, is based on second-order probabilities, or subjective probability distributions defined over objective probability distributions. An early example of this research can be found in Hazen (1987), and more recent work (conducted independently from the current paper) can be found in Ergin and Gul (2004), Klibanoff, Marinacci, and Mukerji (2005), Maccheroni, Marinacci, and Rustichini (2006), Nau (2006), Strzalecki (2007), Ahn (2008), and Chew and Sagi (2008). At its most basic level, the second-order probability approach yields a preference function representation of the form[1]

$$W(\phi) = \int w\left(\int u(x)d(\phi(s)(x))\right)d\mu(s), \tag{1}$$

where the state of nature s determines the objective probability distribution $\phi(s)$ over payoffs x, μ is the subjective probability distribution over states of nature, u is a von Neumann-Morgenstern utility function governing attitudes toward risk, and w is another utility function, this time governing attitudes toward ambiguity. The overall preference function W is defined over the object of choice ϕ, which is an assignment of objective probability distributions to states of nature. We call preferences that have the representation in (1) second-order expected utility preferences.

The purpose of this paper is to provide an axiomatic framework for models of ambiguity aversion using second order probabilities. It adds to the literature cited above by presenting a particularly simple axiomatization. The primitives for the model are the horse lotteries and roulette lotteries of Anscombe and Aumann (1963), and the paper applies Savage's (1954) axioms to the Anscombe-Aumann horse lotteries in order to obtain the probability measure μ over states.[2]

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[1] As noted by others, this functional specification is also proposed by Kreps and Porteus (1978) for the analysis of dynamic choices under risk.

[2] Roulette lotteries are lotteries in which the probability distribution is known. Horse lotteries are lotteries in which the distribution of outcomes is unknown and the outcomes

One of the key assumptions of this model is that horse lotteries and roulette lotteries are treated separately by decision makers. Similar assumptions are made by Segal (1987), Hazen (1987), Sarin and Wakker (1992), Ergin and Gul (2004), Klibanoff, Marinacci, and Mukerji (2005), and Nau (2006). In the framework used here, the individual faces a horse lottery whose outcomes are roulette lotteries, or risks. If the individual forms subjective probabilities over the possible roulette lotteries, then it is possible to reduce the compound horse/roulette lottery to a single subjective probability distribution. This reduction returns us to the Anscombe and Aumann subjective expected utility model, though, and therefore cannot accommodate ambiguity aversion. Throughout the paper, then, it is necessary to assume that compound horse/roulette lotteries are not reduced.[3] Because horse lotteries and roulette lotteries are treated separately by decision makers, risk attitudes and ambiguity attitudes can be different.

This paper is similar to one by Hazen (1987), who uses a different set of axioms to produce a more general preference function which he calls subjective weighted linear utility. Its relation to the model proposed here is the same as the relation between Chew's (1983) weighted utility model and the expected utility model of decisions toward risk. Hazen (1989) shows how his model can be used to discuss ambiguity aversion, and Hazen and Lee (1991)

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are roulette lotteries. Acts are lotteries in which the distribution of outcomes is unknown and the outcomes are (monetary) payoffs. Savage (1954) originally applied his axioms to acts.

[3] For more on reduction of compound lotteries, see Segal (1990).

discuss comparative ambiguity aversion. The main difference between the work presented here and Hazen's work is that here the subjective expected utility functional form is retained (albeit in a modified form), thus simplifying the extensions of results from the subjective expected utility literature to situations in which decision makers are ambiguity averse.

The paper is also similar to one by Klibanoff, Marinacci, and Mukerji (2005), which derives the functional form in (1) by assuming that the decision maker has one preference ordering over risks (or first-order lotteries), another preference ordering over second-order lotteries, and that the two preference orderings agree when the second-order lotteries are degenerate so that they are simply objective probability distributions. Their axiomatic setup, which was constructed independently, differs from the one used here. Their paper also explores how ambiguity attitudes can be formulated in the second-order expected utility model, and those results apply directly to the preferences modeled here.

Klibanoff, Marinacci, and Mukerji (2005) is the first publication in a recent spate of papers concerning second-order probabilities. Ergin and Gul (2004) and Nau (2006) expand the state space beyond the Anscombe-Aumann framework and derive functional forms of which (1) is a special case. Maccheroni, Marinacci, and Rustichini (2006) and Strzalecki (2007) construct axiomatizations of variational preferences and multiplier preferences, respectively, both of which are based on the robust control theory of Hansen and Sargent (2001). Strzalecki (2007) shows that multiplier preferences are a special case of (1) in which the function w takes a particular form, and that multiplier preferences comprise the intersection of the set of variational preferences and the set of second-order expected utility preferences.

2 The representation theorem

To model risk, let D denote the space of probability distributions over some bounded interval X. The set D is the set of risky alternatives, which Anscombe and Aumann (1963) refer to as *roulette lotteries.* To model ambiguity, let S be the set of states of the world, with generic element s. Let Σ be the set of all subsets of S, with generic element E, which is interpreted as an event. Savage (1954) defines an *act* as a function from S to X. Anscombe and Aumann define a *horse lottery* ϕ as a function from S to D, that is, ϕ assigns a probability distribution to each state in S. The resolution of an act is an outcome in the payoff space, while the resolution of a horse lottery is a roulette lottery, which is a probability distribution over payoffs.

Begin with behavior toward risk. Let F and G denote elements of D, and let $\succeq_A$ be a preference relation defined on D. The following axioms are standard for $\succeq_A$.

A1: (A-Ordering) – $\succeq_A$ is complete, reflexive, and transitive.

A2: (Continuity) – The sets $\{G \in D | G \succeq_A F\}$ and $\{G \in D | F \succeq_A G\}$ are closed.

A3: (Independence) – $F \succeq_A F'$ if and only if $\alpha F + (1-\alpha)G \succeq_A \alpha F' + (1-\alpha)G$ for all $G \in D$ and all $\alpha \in (0,1)$.

These axioms are the standard axioms for expected utility toward risk, and if axioms (A1) - (A3) hold there is an expected utility representation of $\succeq_A$ (see, for example, Fishburn, 1970). In particular, there exists a utility function $u : X \to \mathbb{R}$, unique up to affine transformations, such that $F \succeq_A G$ if and only if

$$\int u(x)dF(x) \geq \int u(x)dG(x). \tag{2}$$

The three axioms above are more than are needed to guarantee the existence of a functional representation of preferences. In fact, (A1) and (A2) are enough (Debreu, 1954). If a functional representation of $\succeq_A$ exists, let it be denoted by V, and let v be a generic value of V.

Turning now to ambiguity, let Φ denote the set of all horse lotteries defined over D, and let $\succeq_B$ denote the preference relation over Φ. The goal is to derive subjective probabilities over states from preferences over Φ, and therefore axioms similar to either the Savage axioms or the Anscombe-Aumann axioms are needed. The axioms that follow are similar in form to the Savage axioms as presented in Fishburn (1970), except that here the axioms apply to horse lotteries, and in Savage's work they apply to acts. In all cases, $\phi, \phi', \psi, \psi' \in \Phi$ are horse lotteries, $F, F', G, G' \in D$ are roulette lotteries, and $E, E', E_i \in \Sigma$ are events. The horse lottery Δ_F is the degenerate lottery which yields F in every state, that is, $\Delta_F(s) = F$ for all $s \in S$, and these lotteries are called *constant lotteries.* The set E^c is the complement of E in S, that is, $S \setminus E$. A set E is *null* if $\phi \sim_B \psi$ whenever $\phi(s) = \psi(s)$ for all $s \in E^c$. It is said that $\phi = \psi$ *on* E if $\phi(s) = \psi(s)$ for all $s \in E$. It is said that $\phi \succeq_B \psi$ *given* E if and only if $\phi' \succeq_B \psi'$ whenever $\phi(s) = \phi'(s)$ for $s \in E$, $\psi(s) = \psi'(s)$ for $s \in E$, and $\phi'(s) = \psi'(s)$ for all $s \in E^c$.

B1: (B-Ordering) – $\succeq_B$ is complete, reflexive, and transitive.

B2: (Sure-thing principle) – If $\phi = \phi'$ and $\psi = \psi'$ on E, and $\phi = \psi$ and $\phi' = \psi'$ on E^c, then $\phi \succeq_B \psi$ if and only if $\phi' \succeq_B \psi'$.

B3: (Eventwise monotonicity) – If E is not null and if $\phi = \Delta_F$ and $\psi = \Delta_G$ on E, then $\phi \succeq_B \psi$ given E if and only if $F \succeq_A G$.

B4: (Weak comparative probability) – Suppose that $F \succeq_A G$, $\phi = \Delta_F$ on E, $\phi = \Delta_G$ on E^c, $\psi = \Delta_F$ on E', and $\psi = \Delta_G$ on E'^c, and suppose that $F' \succeq_A G'$, $\phi' = \Delta_{F'}$ on E, $\phi' = \Delta_{G'}$ on E^c, $\psi' = \Delta_{F'}$ on E', and $\psi' = \Delta_{G'}$ on E'^c. Then $\phi \succeq_B \psi$ if and only if $\phi' \succeq_B \psi'$.

B5: (Nondegeneracy) – $F \succ_A G$ for some $F, G \in D$.

B6: (Small event continuity) – If $\phi \succ_B \psi$, for every $F \in D$ there is a finite partition of S such that for every E_i in the partition, if $\phi' = \Delta_F$ on E_i and $\phi' = \phi$ on E_i^c then $\phi' \succ_B \psi$, and if $\psi' = \Delta_F$ on E_i and $\psi' = \psi$ on E_i^c then $\phi \succ_B \psi'$.

B7: (Uniform monotonicity) – For all $E \in \Sigma$ and for all $F \in \psi(E)$, if $\phi \succeq_B \Delta_F$ given E, then $\phi \succeq_B \psi$ given E. If $\Delta_F \succeq_B \phi$ given E, then $\psi \succeq_B \phi$ given E.

These axioms are the standard Savage axioms modified so that they govern preferences over horse lotteries instead of preferences over acts. The main difference between these axioms and Savage's, then, is that here probability distributions in D replace outcomes in X. A second modification is that in Savage's axioms, the preference ordering over outcomes in X is induced by the preference ordering over constant acts. Here, the preference ordering over roulette lotteries in D is kept separate, and axiom (B3) states that the preference ordering over constant horse lotteries is consistent with the preference ordering over D.

The axioms on preferences over Φ do not impose any particular behavior on preferences over D. For example, consider the sure-thing principle (B2). To apply this axiom to preference over D, suppose that the horse lotteries $\phi, \phi', \psi, \psi' \in \Phi$ yield the following probability distributions in states $E, E^c \in \Sigma$, where $F, G, H, I \in D$:

	E	E^c
ϕ	F	H
ϕ'	F	I
ψ	G	H
ψ'	G	I

The sure-thing principle states that $\phi \succeq_B \psi$ if and only if $\phi' \succeq_B \psi'$, that is, preferences only depend on states in which the two horse lotteries being considered have different outcomes. This has the same spirit as the independence axiom (A3), but with one key difference. Here the mixture ϕ of F and

H is *not* a probability mixture, and thus it is not in D. Consequently, axiom (B2) does not imply axiom (A3), and thus (B1) - (B7) do not imply linearity of preferences over D, as axioms (A1) - (A3) do. The only restriction is that (B3) guarantees that preferences on constant horse lotteries are identical to preferences over the corresponding roulette lotteries.

The axioms governing $\succeq_A$ and $\succeq_B$ restrict the form of functions representing preferences. This can be seen in the next theorem and its corollary. Proofs are contained in the Appendix.

Theorem 1 *Assume that an individual has preferences $\succeq_A$ over D and preferences $\succeq_B$ over Φ. If axioms (A1) - (A2) on $\succeq_A$ and axioms (B1) - (B7) on $\succeq_B$hold then there exists a function $V : D \to \mathbb{R}$, a probability measure $\mu : \Sigma \to [0,1]$, and a function $w : \mathbb{R} \to \mathbb{R}$ such that, for all $F, G \in D$, $F \succeq_A G$ if and only if $V(F) \geq V(G)$, and for all $\phi, \psi \in \Phi$, $\phi \succeq_B \psi$ if and only if*

$$\int w(V(\phi(s)))d\mu(s) \geq \int w(V(\psi(s)))d\mu(s). \tag{3}$$

Moreover, the function V is unique up to increasing transformations, and for a given specification of V the function w is unique up to increasing affine transformations.

Corollary 2 *Assume that an individual has preferences $\succeq_A$ over D and preferences $\succeq_B$ over Φ. If axioms (A1) - (A3) on $\succeq_A$ and axioms (B1) - (B7) on $\succeq_B$hold then there exists a function $u : X \to \mathbb{R}$, a probability measure $\mu : \Sigma \to [0,1]$, and a function $w : \mathbb{R} \to \mathbb{R}$ such that, for all $F, G \in D$,*

$F \succeq_A G$ if and only if

$$\int u(x)dF(x) \geq \int u(x)dG(x), \tag{4}$$

and for all $\phi, \psi \in \Phi$, $\phi \succeq_B \psi$ if and only if

$$\int w\left(\int u(x)d(\phi(s)(x))\right)d\mu(s) \geq \int w\left(\int u(x)d(\psi(s)(x))\right)d\mu(s). \tag{5}$$

Moreover, the function u is unique up to increasing affine transformations, and for a given specification of u the function w is unique up to increasing affine transformations.

To keep these functions straight, the function u is referred to as an *A-utility* function, V is referred to as an *A-preference* function, and w is referred to as a *B-utility* function. The A-utility function u can be used to describe attitudes toward risk, while the B-utility function w can be used to discuss attitudes toward ambiguity.

The proofs of Theorem 1 and its corollary are fairly straightforward. Axioms (A1) and (A2) together are equivalent to the existence of a continuous A-preference function V representing $\succeq_A$ (Debreu, 1954). Substituting the A-preference values $V(F)$ for F and the "acts" $V(\phi)$ for ϕ into axioms (B1) - (B7) yields the standard Savage axioms governing "acts" which are functions mapping states into A-preference outcomes. Letting $Y \equiv V(D)$, these acts are mappings from S into $Y \subseteq \mathbb{R}$. Savage's theorem then states that

(B1) - (B7) are equivalent to the existence of a probability measure μ and a B-utility function w satisfying (3). The addition of axiom (A3) makes the A-preference function linear in the probabilities, and so there exists an A-utility function u such that $\succeq_A$ is represented by (4).

Note that both von Neumann-Morgenstern's expected utility and Savage's subjective expected utility are special cases of (5). Expected utility holds when there is no ambiguity, that is, when μ places probability one on a single state. Subjective expected utility holds when all horse lotteries assign degenerate distributions in D to every state. More specifically, when $\phi(s)$ is a degenerate distribution for every $s \in S$ and every $\phi \in \Phi$, let $x(\phi, s)$ denote the outcome assigned probability one by $\phi(s)$, and $V(\phi(s))$ can be written as $x(\phi, s)$. Preferences are then represented by $\int w(x(\phi, s) d\mu(s)$. In the terminology used here, this is the case of ambiguity but no risk.

The system of assumptions in Klibanoff, Marinacci, and Mukerji (2005) holds strong similarities to the systm of axioms used here. Their first assumption is that preferences over risk have an expected utility representation, the counterpart of axioms (A1) - (A3) here. Their second assumption is that preferences over states have a subjective expected utility representation, corresponding to axioms (B1) - (B7) here. Their third assumption requires that the subjective expected utility preferences from Assumption 2 are consistent with the expected utility preferences from Assumption 1, and here that assumption is covered by axiom (B3). The contribution of this axiomatization in light of their result, then, is to show that the Savage axioms can be used both to generate the subjective probability distribution over objective probability distributions and to provide the consistency required by their Assumption 3. Furthermore, the paper shows that an expected utility representation for preferences over objective probability distributions is stronger than what is needed to derive the subjective probability distribution over states, and that it is possible to have second-order preferences where ambiguity preferences have a (subjective) expected utility form but risk preferences do not.

3 Conclusion

By treating horse lotteries and roulette lotteries separately, it is possible to capture ambiguity attitudes with a second-order expected utility representation. If an individual satisfies the usual expected utility axioms over roulette lotteries, then she behaves as an expected utility maximizer in situations of pure risk with risk attitudes governed by the shape of her von Neumann-Morgenstern utility function, which is termed an A-utility function in this paper. If, in addition, she satisfies a modified version of the Savage axioms over horse lotteries, then she forms a subjective probability distribution over objective probability distributions, and possesses another utility function, termed a B-utility function, which governs ambiguity attitudes. The modified Savage axioms guarantee consistency of the risk attitudes caputred by the A-utility function and the more general attitudes embodied by the preference ordering over horse lotteries. The existence of the B-utility function, and the fact that it enters the decision process in the same way as the von Neumann-Morgenstern utility function, makes it possible to extend many of the results from the study of situations of pure risk to situations of ambiguity, and this is true whether or not preferences toward risk have an expected utiltiy representation.

A Appendix

Proof of Theorem 1. Axiom (B3) guarantees that A-preferences over probability distributions coincide with B-preferences over constant lotteries, and therefore the A-preference relation is unique and well-defined. The existence of a function V, unique up to increasing transformations, which represents $\succeq_A$ is proved by Debreu (1954). Recall that a horse lottery ϕ assigns probability distributions in D to states in S. Take one function V representing $\succsim_A$ as given and define $h_\phi(s) \equiv V(\phi(s))$. Let H denote the set of all such h_ϕ, and let $\succeq'_B$ be the preference ordering imposed on H by the preference ordering $\succeq_B$ over Φ. Axioms (B1) - (B7) imply the following axioms on $\succeq'_B$, where v is a real number, Δ_v is a constant lottery in H yielding preference value v, $Y \equiv V(D)$, and we use the notation that $h_\phi = v$ on E if $h_\phi(s) = v$ for all $s \in E$:

C1: $\succeq'_B$ is complete, reflexive, and transitive.

C2: If $h_\phi = h_{\phi'}$ and $h_\psi = h_{\psi'}$ on E, and $h_\phi = h_\psi$ and $h_{\phi'} = h_{\psi'}$ on E^c, then $h_\phi \succeq'_B h_\psi$ if and only if $h_{\phi'} \succeq'_B h_{\psi'}$.

C3: If E is not null and if $h_\phi = v$ and $h_\psi = v'$ on E, then $h_\phi \succeq'_B h_\psi$ given E if and only if $v \geq v'$.

C4: Suppose that $v_1 \geq v_2$, $h_\phi = v_1$ on E, $h_\phi = v_2$ on E^c, $h_\psi = v_1$ on E', and $h_\psi = v_2$ on E'^c, and suppose that $v_3 \geq v_4$, $h_{\phi'} = v_3$ on E, $h_{\phi'} = v_4$ on E^c, $h_{\psi'} = v_3$ on E', and $h_{\psi'} = v_4$ on E'^c. Then $h_\phi \succeq'_B h_\psi$ if and only if $h_{\phi'} \succeq'_B h_{\psi'}$.

C5: $v > v'$ for some $v, v' \in Y$.

C6: If $h_\phi \succ'_B h_\psi$, for every $v \in Y$ there is a finite partition of S such that for every E_i in the partition, if $h_{\phi'} = v$ on E_i and $h_{\phi'} = h_\phi$ on E_i^c then $h_{\phi'} \succ'_B h_\psi$, and if $h_{\psi'} = v$ on E_i and $h_{\psi'} = h_\psi$ on E_i^c then $h_\phi \succ'_B h_{\psi'}$.

C7: For all $E \in \Sigma$ and for all $v \in h_\psi(E)$, if $h_\phi \succeq'_B \Delta_v$ given E, then $h_\phi \succeq'_B h_\psi$ given E. If $\Delta_v \succeq'_B h_\phi$ given E, then $h_\psi \succeq'_B h_\phi$ given E.

Axioms (C1) - (C7) are the usual Savage axioms for preferences on H, except with the implicit assumption that preferences are monotone over outcomes. By Savage's theorem (see, for example, Fishburn, 1970, Theorem 14.1), axioms (C1) - (C7) hold if and only if there exists a probability measure μ on Σ and a function $w : \mathbb{R} \to \mathbb{R}$, unique up to increasing affine transformations, such that $h_\phi \succeq'_B h_\psi$ if and only if

$$\int w(h_\phi(s))d\mu(s) \geq \int w(h_\psi(s))d\mu(s)$$

where $h_\phi(s) \equiv V(\phi(s))$, which completes the proof.

Proof of Corollary 2. By Theorem 1 all that is left to prove is the linearity of V, which follows from the usual expected utility theorem (see, for example, Fishburn, 1970).

References

[1] Ahn, D.S. (2008). "Ambiguity without a state space," *Review of Economic Studies* 75, 3-28.

[2] Anscombe, F.J. and R.J. Aumann (1963). "A definition of subjective probability," *Annals of Mathematical Statistics* 34, 199-205.

[3] Chew, S.H. (1983). "A generalization of the quasilinear mean with applications to the measurement of income inequality and decision theory resolving the Allais paradox," *Econometrica* 51, 1065-1092.

[4] Chew, S.H. and J.S. Sagi (2008). "Small worlds: Modeling attitudes toward sources of uncertainty," *Journal of Economic Theory*, forthcoming.

[5] Debreu, G. (1954). "Representation of a preference ordering by a numerical function," in R. Thrall, C. Coombs, and R. Davis, eds., *Decision Processes*, New York: Wiley.

[6] Ellsberg, D. (1961). "Risk, ambiguity, and the Savage axioms," *Quarterly Journal of Economics* 75, 643-669.

[7] Ergin, H. and F. Gul (2004). "A subjective theory of compound lotteries," working paper.

[8] Fishburn, P.C. (1970). *Utility Theory for Decision Making*, New York: Wiley.

[9] Gilboa, I. (1987). "Subjective utility with purely subjective non-additive probabilities," *Journal of Mathematical Economics* 16, 65-88.

[10] Gilboa, I. and D. Schmeidler (1989). "Maxmin expected utility with a non-unique prior," *Journal of Mathematical Economics* 18, 141-153.

[11] Hansen, L.P. and T.J. Sargent (2001). "Robust control and model uncertainty," *American Economic Review Papers and Proceedings* 91, 60-66.

[12] Hazen, G.B. (1987). "Subjectively weighted linear utility," *Theory and Decision* 23, 261-282.

[13] Hazen, G.B. (1989). "Ambiguity aversion and ambiguity content in decision making under ambiguity," *Annals of Operations Research* 19, 415-434.

[14] Hazen, G. and J.S. Lee (1991). "Ambiguity aversion in the small and in the large for weighted linear utility," *Journal of Risk and Uncertainty* 4, 177-212.

[15] Klibanoff, P., M. Marinacci, and S. Mukerji (2005). "A smooth model of decision making under ambiguity," *Econometrica* 73, 1849-1892.

[16] Kreps, D.M. and E.L. Porteus (1978). "Temporal resolution of uncertainty and dynamic choice theory," *Econometrica* 46, 185-200.

[17] Maccheroni, F., M. Marinacci, and A. Rustichini (2006). "Ambiguity aversion, robustness, and the variational representation of preferences," *Econometrica* 74, 1447-1498.

[18] Nau, R.F. (2006). "Uncertainty aversion with second-order utilities and probabilities," *Management Science* 52, 136-145.

[19] Sarin, R.K. and P. Wakker (1992). "A simple axiomatization of nonadditive expected utility," *Econometrica* 60, 1255-1272.

[20] Savage, L.J. (1954). *Foundations of Statistics*, New York: Wiley.

[21] Schmeidler, D. (1989). "Subjective probability and expected utility without additivity," *Econometrica* 57, 571-587.

[22] Segal, U. (1987). "The Ellsberg paradox and risk aversion: An anticipated utility approach," *International Economic Review* 28, 175-202.

[23] Segal, U. (1990). "Two-stage lotteries without the reduction axiom," *Econometrica* 58, 349-377.

[24] Strzalecki, T. (2007). "Axiomatic foundations of multiplier preferences," working paper.

[25] Yaari, M.E. (1987). "The dual theory of choice under risk," *Econometrica* 55, 95-115.