Quiz # 4
Physics 222, Section 009
Feb. 21, 2008

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1. When measuring resistance, it is advisable not to let the experiment run for too long because the resistance would heat up, and change with temperature. Recalling that as a function of temperature, an ohmic resistance would change as follows:

$$R = R_0\,[1 + \alpha\,\Delta\,T] = R_0\,[1 + \alpha\,(T - T_0)\,],$$

   a. Derive an expression for the error (fluctuation) in resistance as a result of the temperature fluctuation.
   (*Hint: Treat R_0, α and T_0 as constants known with zero uncertainty*)

   b. What would the error (fluctuation) in resistance be if temperature fluctuated during some measurement by 1°C, and that the resistor has a temperature coefficient of α = 0.004°C^{-1}, and at 20°C, the resistance is 2Ω?

   c. Would the fluctuation be greater or smaller for a smaller α ?

2. <u>Name</u> the main physical principle that today's experiment will demonstrate?

Solution

1. **(2 points per part)** This is a "propagation of error" problem, where we will start with error (fluctuation) in temperature, and propagate that to resistance.

   a. Starting with the equation that relates the two, we can proceed in general by looking at the error in R as a function of errors in the variables it depends on. Let's first write the given relation in a more practical way:

   $$R(T)=a+bT\,,\ a=R_0(1-\alpha T_0)\,;\ b=R_0\alpha$$

   Using the formulas of error propagation, we see that ΔR is given by:

   $$\Delta R=\sqrt{(\Delta a)^2+(\Delta(bT))^2}$$

   also:

   $$\Delta(bT)=bT\sqrt{\left(\frac{\Delta b}{b}\right)^2+\left(\frac{\Delta T}{T}\right)^2}$$

   It would be quite a bit of work to go through this if errors in all quantities are known. Fortunately, we are told that R_0, α, and T_0 have zero errors, i.e., ΔR_0, $\Delta\alpha$, and ΔT_0 are all zeros, which implies that Δa and Δb are also zeros because a and b depend on quantities that are known with certainty. This means that the above equations become:

   $$\Delta R=\sqrt{(\Delta a)^2+(\Delta(bT))^2}=\sqrt{(\Delta(bT))^2}=\Delta(bT)=b\,\Delta T\Rightarrow\Delta R=R_0\alpha\,\Delta T$$

   So, that's the long way of doing it. But looking at this result, you notice that that it's actually just a rearrangement of the original equation:

   $$R=R_0+\alpha(T-T_0)\Leftrightarrow\Delta R=R_0\alpha\,\Delta T$$

   where $\Delta R = R - R_0$ and $\Delta T = T - T_0$, and this should make physical sense to you because the equation that relates changes in temperature to those in resistance is just an approximation, when either changes in temperature are small, or R is a linear function of temperature or both.

   b. Now we use our result by plugging in numbers:

   $\Delta T = 1°\text{C}$, $\alpha = 0.004°\text{C}^{-1}$, $R_0 = 2\Omega \Rightarrow\ \Delta R = R_0\,\alpha\,\Delta T = 2 \times 0.004 \times 1 = 0.008\Omega$

   c. It is clear from the relation we've found that the fluctuation in R is directly proportional with α, which means that for a smaller α, ΔR would be less, and physically, this means that the material in question responds less to changes in temperature.

2. **(4 points for a satisfactory explanation)** The main physical principle in today's experiment is conservation of energy: we move current through a wire, which means that we move charge through a potential difference, and this implies will result in a transformation of electrical energy into heat. Both are different types of energy, but the amount of electrical energy (assuming good insulation) will (almost) completely be converted into heat. The experiment will measure both and show that they're (almost) equal.