PHY612; QFT Problem Set 6: *Quark Model Baryons*

Due Mon. 3 Apr 2006 at the beginning of class.
n.b. Use natural units $\hbar = c = 1$ in all problems unless explicitly specified otherwise.

1. Nucleon states in the quark model.

In class the quark model state of a spin-up proton was given as

$$|P(+1/2)\rangle = \frac{1}{\sqrt{18}}\Big(2|u_+u_+d_-\rangle+2|u_+d_-u_+\rangle+2|d_-u_+u_+\rangle$$

$$-|u_+u_-d_+\rangle-|u_-u_+d_+\rangle-|u_+d_+u_-\rangle-|u_-d_+u_+\rangle-|d_+u_+u_-\rangle-|d_+u_-u_+\rangle\Big) \quad (1)$$

and the spin-up neutron state can be obtained by the substitution $u \to d$ and a change of overall sign.

a) (*5 pts*) Operate on the spin-up proton state with the isospin lowering operator I_- and confirm that the neutron state suggested above does indeed follow.

b) (*5 pts*) Using I_z, I_+ and I_-, confirm that the proton state above is an eigenvector of total isospin I^2, with eigenvalue $I(I+1) = 3/4$.

2. A step into strangeness.

a) (*5 pts*) Starting from the nucleon states above, one may construct the remaining members of the light baryon octet using SU(3) raising and lowering operators. Do this explicitly for the singly strange baryon states $|\Sigma^o(+1/2)\rangle$ and $|\Lambda(+1/2)\rangle$. Take care to confirm that your states have the correct isospin; since they occupy the same point in the group weight diagram, one typically produces a linear combination of these states when using $U_\pm$ and $V_\pm$.

b) (*5 pts*) Assuming as usual in the quark model that the magnetic moment operator is of the additive form

$$\vec{\mu} = \sum_q (e_q/e)\mu\,\vec{\sigma}_q \ , \quad (2)$$

derive quark model predictions for the ratios μ_N/μ_P and μ_Λ/μ_P. Compare these to experiment (in the PDG); how might you account for the relative accuracy in each case?

3. Heavy quarks.

a) (*5 pts*) B factories are machines that produce B mesons from the decay of the 4S Υ $b\bar{b}$ resonance, typically at an e^+e^- collider. Given the PDG mass of the 4S Υ and an electron energy of 12 GeV, approximately what positron energy is required to operate this collider?

b) (*5 pts*) It has been suggested that one could study J/ψ interactions with nucleons by making a J/ψ in flight from an antiproton beam incident on a nuclear target ($p\bar{p} \rightarrow J/\psi$; only the p is at rest). Given the J/ψ mass of 3.097 GeV and the proton mass of 0.939 GeV, approximately what momentum in GeV (frequently written as GeV/c) must the antiproton beam have for this application?