PHY612; QFT Problem Set 4:
Electroweak vector boson decays

Due Mon. 27 Feb 2006 at the beginning of class.
n.b. Use natural units $\hbar = c = 1$ in all problems unless explicitly specified otherwise.

1. W boson leptonic decay, including the lepton mass

In the previous HW set you evaluated the partial width of the W boson to lepton and neutrino (e.g. $W^- \to e^- \bar{\nu}_e$) due to the charged current part of the Weinberg-Salam lagrangian,

$$\mathcal{L}_I = -\frac{g}{2\sqrt{2}} \bar{\psi}_\ell \gamma_\mu (1 - \gamma_5) \psi_{\nu_\ell} W_\mu^- \ , \tag{1}$$

neglecting the lepton mass m_ℓ. The result was

$$\Gamma(W^- \to \ell^- \bar{\nu}_\ell) = \frac{\alpha}{12 \sin(\theta_W)^2} M_W = \frac{1}{6\sqrt{2}\,\pi} G_F M_W^3 \ , \tag{2}$$

which is numerically 227.6(3) MeV, in good agreement with the experimental 228.(6) MeV.

a) (*5 pts*) Repeat this calculation without neglecting the lepton mass m_ℓ, and express your result in the form

$$\Gamma(W^- \to \ell^- \bar{\nu}_\ell) = \Gamma_0 \cdot f(\xi = m_\ell / M_W) \tag{3}$$

where Γ_0 is the $m_\ell = 0$ result given above.

b) (*5 pts*) Plot this W partial width suppression factor $f(\xi)$ over the range $\xi = [0, 1]$, and evaluate it numerically for the three leptons e^-, μ^- and τ^- using 2006 PDG masses. What do you predict for the ratio $\Gamma(W^- \to \tau^- \bar{\nu}_\tau)/\Gamma(W^- \to e^- \bar{\nu}_e)$? By what factor would the current experimental error on this ratio have to be decreased to be sensitive to this lepton mass effect?

2. Z boson leptonic decay

The neutral Z boson decays to $\ell^+\ell^-$ lepton pairs through the term

$$\mathcal{L}_I = +g\,\bar{\psi}_\ell \gamma_\mu (a - b\gamma_5)\psi_\ell\, Z_\mu \tag{4}$$

in the Weinberg-Salam lagrangian, where $a = (1 - 4\sin(\theta_W)^2)/4\cos(\theta_W)$ and $b = 1/4\cos(\theta_W)$.

a) (*3 pts*) Write the invariant amplitude $\mathcal{M}$ for this decay in terms of lepton and antilepton spinors, the weak coupling parameters g, a, b, and the initial Z polarization vector ϵ_μ.

b) (*5 pts*) Use this invariant amplitude to calculate the differential decay rate $d\Gamma/d\Omega$ for the emission of the ℓ^- along $\Omega = (\theta, \phi)$ in this decay process ($Z \to \ell^-\ell^+$), assuming an initial spin-up Z boson, $\epsilon_\mu = (0, -(\hat{x} + i\hat{y})/\sqrt{2})$. You may set the lepton masses equal to zero. For simplicity you should write this using a, b rather than trig functions of θ_W.

c) (*2 pts*) Integrate this $d\Gamma/d\Omega$ over angles to find the partial width $\Gamma(Z \to \ell^-\ell^+)$. Evaluate this numerically, and compare with the result you get by multiplying the 2006 PDG's total width and $B(Z \to e^-e^+)$ branching fraction.