<u>differential cross section $2 \to n_f$</u>

$$d^{3n_f}\sigma = (2\pi)^4 \delta^{(4)}(p_f - p_i) \frac{1}{|v_1 - v_2|} \underbrace{\left(\frac{m_1}{E_1}\right)\cdots\left(\frac{m_n}{E_n}\right)}_{\text{from } \sqrt{\frac{m}{E}}\text{: for bosons these are } \left(\frac{1}{2E}\right)} \underbrace{|\mathcal{M}|^2}_{|\text{ampl}|^2} \underbrace{\frac{d^3p_{1f}}{(2\pi)^3} \cdots \frac{d^3p_{n_f}}{(2\pi)^3}}_{n_f\text{-particle phase space}}$$

<u>differential decay rate $1 \to n_f$</u>

$$d^{3n_f}\Gamma = \underbrace{\left(\frac{m_1}{E_1}\right)\cdots\left(\frac{m_n}{E_n}\right)}_{\text{again } \frac{1}{2E} \text{ for bosons}} |\mathcal{M}|^2 \frac{d^3p_{1f}}{(2\pi)^3} \cdots \frac{d^3p_f}{(2\pi)^3}$$

also, if ① is at rest, $\left\{ \begin{array}{l} \frac{m_1}{E_1} = 1, \text{ fermion} \\ \frac{1}{2M}, \text{ boson} \end{array} \right.$

$(2\pi)^4 \delta^{(4)}(p_f - p_i)$

$\mathcal{M}$ from diagram Feynman rules

These are obviously central results for decay rate + cross section calculations.

9Nov98

A heavy boson decay rate calc.

We now have the basic $W\ell\bar{\nu}_\ell$ and $Z\ell\bar{\ell}$ vertices,

$g\sin\theta_W = e$

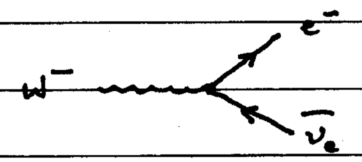
$g'/g = \tan\theta_W$

$$\mathcal{L}_I = -\frac{g}{2\sqrt{2}}\left(\bar{\nu}_{e^-}\gamma_\mu(1-\gamma_5)\psi_{e^-}W_\mu^+ + h.c.\right) - e\bar{\psi}_{e^-}\gamma_\mu\psi_{e^-}A_\mu$$

$$-\frac{g}{4\cos\theta_W}\left[\bar{\nu}_{e^-}\gamma_\mu(1-\gamma_5)\nu_{e^-} - \bar{\psi}_{e^-}\gamma_\mu\left((1-4\sin^2\theta_W)-\gamma_5\right)\psi_{e^-}\right]Z_\mu$$

$$+\mu^- + \tau^-$$

So W^- decay to lowest order proceeds as



$$\mathcal{M} = i\cdot\left(-\frac{g}{2\sqrt{2}}\right)\bar{u}_{e^-}\gamma_\mu(1-\gamma_5)v_{\nu_e}\underbrace{\epsilon_\mu(W)}_{\text{polarization 4-vector of initial W.}}$$

we can use this to calculate the decay rate as usual,

$$d^{\overset{=6}{3n_f}}\Gamma = (2\pi)^4\delta^{(4)}(p_f-p_i)\underbrace{\frac{1}{2M_W}}_{\substack{\text{init}\\ \text{boson}\\ \frac{1}{2E}}}\left(\frac{m}{E}\right)_e\left(\frac{m}{E}\right)_\nu|\mathcal{M}|^2\frac{d^3p_e}{(2\pi)^3}\frac{d^3p_\nu}{(2\pi)^3}$$

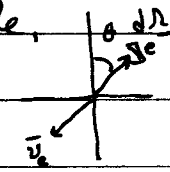
This is a familiar procedure. We can do one d^3p_f trivially (ν), $\vec{p}_\nu = -\vec{p}_e$ is the result. Assuming both m_{e^-} and m_ν are $\ll M$, these effectively set $E_{e^-} = E_{\nu_e}$, so

$$d^3\Gamma = (2\pi)\,\delta(M-2E_e)\,\frac{1}{2M}\,\frac{m_e m_\nu}{E_e^2}\,|\mathcal{M}|^2\,\frac{\mathbb{P}_e^2\,d\mathbb{P}_e\,d\Omega_e}{(2\pi)^3}$$

& with usual
$\mathbb{P}\,d\mathbb{P} = E\,dE$
& $\delta(ax) = \frac{1}{a}\delta(x)$,

$$\frac{d^2\Gamma}{d\Omega_{e^-}} = \frac{1}{8\pi^2}\,\frac{m_e m_\nu}{M^2}\,\mathbb{P}_e\,|\mathcal{M}|^2$$

rate to emit $W^- \to e^-\bar{\nu}_e$ into element $d\Omega_{e^-}$ of final solid angle.



$$\Gamma = \frac{1}{8\pi^2}\,\frac{m_e m_\nu}{M^2}\,\mathbb{P}_e\int|\mathcal{M}|^2\,d\Omega$$

The polarization 4-vector $\epsilon^\mu = (0,\hat{\epsilon})$ of the initial W^- can be any unit vector. For simplicity lets take $\hat{\epsilon} = \hat{z}$, a "sideways W^-".

$$\mathcal{M} = +\,\frac{ig}{2\sqrt{2}}\,\bar{u}_{e^-}\gamma_3(1-\gamma_5)v_\nu$$

$$|\mathcal{M}|^2 = \frac{g^2}{8}\,\bar{u}_{e^-}\gamma_3(1-\gamma_5)v_\nu\bar{v}_\nu(1+\gamma_5)\gamma_3 u_{e^-}$$

$$\langle|\mathcal{M}|^2\rangle = \sum_{\text{final pols}}|\mathcal{M}|^2 = \frac{g^2}{8}\,\mathrm{Tr}\left\{\frac{\not{p}_e + \overbrace{\cancel{m_{e^-}}}^{\text{neglect}}}{2m_e}\,\gamma_3(1-\gamma_5)\,\frac{\not{p}_\nu \mp \overbrace{\cancel{m_\nu}}^{\text{neglect}}}{2m_\nu}\,(1+\gamma_5)\gamma_3\right\}$$

$$= \frac{g^2}{16\,m_e m_\nu}\,\underbrace{\mathrm{Tr}\left\{\not{p}_e\gamma_3(1-\overbrace{\gamma_5}^{\text{drops out}})\not{p}_\nu\gamma_3\right\}}_{\underbrace{\mathrm{Tr}\{\not{p}_e\gamma_3\not{p}_\nu\gamma_3\}}_{\underbrace{4\left[2p_{e_3}p_{\nu_3} - g_{33}\,p_e\cdot p_\nu\right]}_{4\left[E_e E_\nu + p_{e_z}p_{e_\nu}\right]}}}$$

$$\doteq 4\mathbb{P}^2(1+\cos^2\theta)$$

$$\langle |\mathcal{M}|^2 \rangle \simeq \frac{g^2}{4 m_e m_\nu} \mathbb{P}^2 (1+\mu^2)$$

$$\Gamma = \frac{1}{8\pi^2} \frac{m_e m_\nu}{M^2} \mathbb{P} \int |\mathcal{M}|^2 d\Omega = \frac{g^2}{6\pi} \frac{\mathbb{P}^3}{M^2}$$

since $\mathbb{P} \approx E \approx M/2$, to this $m_e \approx m_\nu \approx 0$ accuracy

$g = \frac{e}{\sin\theta_W}$

$e^2 = 4\pi\alpha$

$$\underset{\sim\sim}{\Gamma_W} = \frac{g^2}{48\pi} M = \underset{\sim\sim\sim\sim}{\frac{\alpha}{12 \sin^2\theta_W} M_W}$$

Using $M_W = 80.3$ GeV, $\alpha = 137.^{-1}$, $\sin^2\theta_W = 0.232$, we find

$$\Gamma_{W^- \to e^- \bar{\nu}_e} = 0.21 \text{ GeV}$$

expt ('96 pdg) $= 0.224(11)$ GeV ✓

nb $\Gamma_W^{tot} = 2.1$ GeV approx, but this is a sum over

decays to

$$\left.\begin{array}{l} e^- \bar{\nu}_e \\ \mu^- \bar{\nu}_\mu \\ \tau^- \bar{\nu}_\tau \end{array}\right\} \text{modes all approx equal}$$

$$\left.\begin{array}{l} d\bar{u} \\ d\bar{c} \\ s\bar{c} \end{array}\right\} \text{quarks, as we shall see have diff. weak couplings}$$