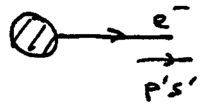
Remaining Feynman rules for QED

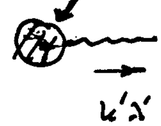
external lines

f

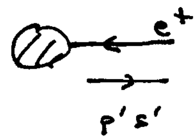


$\bar{u}_{p's'}$

connects to $-ie\gamma^\mu$ inside

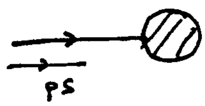


$\epsilon^{*\mu}_{k'\lambda'}$

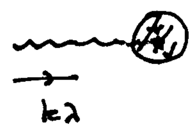


$v_{p's'}$

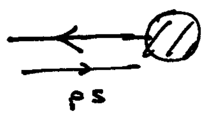
i



u_{ps}

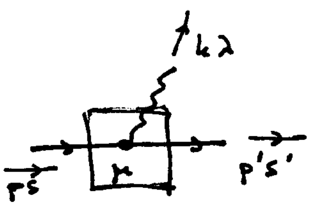


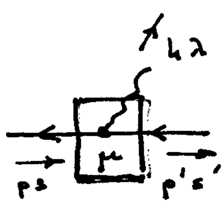
$\epsilon^{\mu}_{k\lambda}$



$\bar{v}_{ps}$

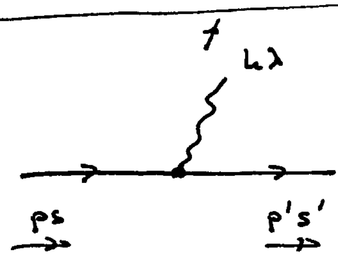
vertex





$-ie\gamma^\mu$ both

e.g.



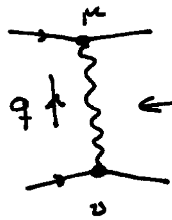
$$\mathcal{M} = \bar{u}_{p's'}(-ie\gamma^\mu)\, u_{ps}\; \epsilon^{*\mu}_{k\lambda}$$

$$= -ie\,\bar{u}_{p's'}\,\not{\epsilon}^{*}_{k\lambda}\, u_{ps}$$

Propagators

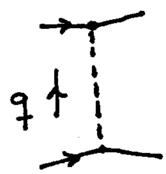
gauge-dependent, can drop

photon



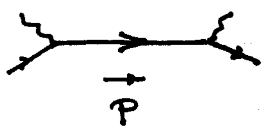
$$\frac{-ig^{\mu\nu} + (\ldots)^{\mu\nu}}{q^2 + i\epsilon}$$

scalar, mass μ



$$\frac{i}{q^2 - \mu^2}$$

Dirac fermion, mass m



$$\frac{i}{\not p - m} = \frac{i(\not p + m)}{p^2 - m^2}$$ (Dirac matrix)

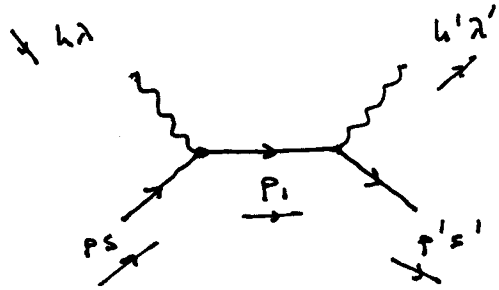
(note direction of momentum flow w.r.t. arrow matters)

$$\left[S_F^{\alpha\beta}(p) = \int d^4x \, e^{ip\cdot x} \langle 0 | T\{ \psi^\alpha(x) \, \bar\psi(0)^\beta \} | 0 \rangle = i \, \frac{(\not p + m)^{\alpha\beta}}{(p^2 - m^2)} \right.$$

if we spell out the explicit Dirac indices]

Another $\mathcal{M}$ example

A Feynman diagram for $e^-\gamma$ (Compton) scattering

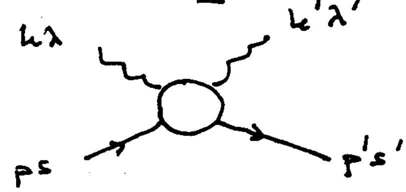


$$\mathcal{M} = \bar{u}_{p's'}\left(-ie\,\not{\epsilon}^{*}_{k'\lambda'}\right)\frac{i}{\not{p}_1 - m}\left(-ie\,\not{\epsilon}_{k\lambda}\right)u_{ps}$$

$$= -\frac{ie^2}{s-m^2}\left[\bar{u}_{p's'}\,\not{\epsilon}^{*}_{k'\lambda'}\,(\not{p}_1+m)\,\not{\epsilon}_{k\lambda}\,u_{ps}\right]$$

How to generate all $\mathcal{O}(e^n)$ Feynman diagrams for a QED proc

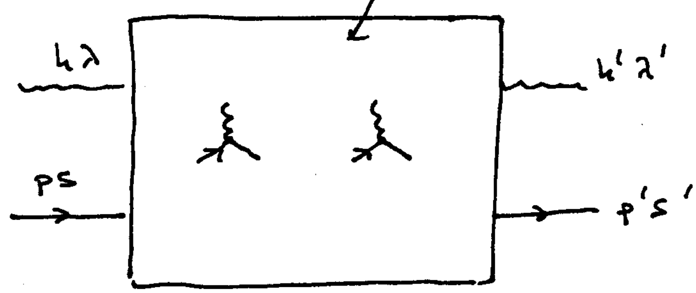
e.g. Compton scattering

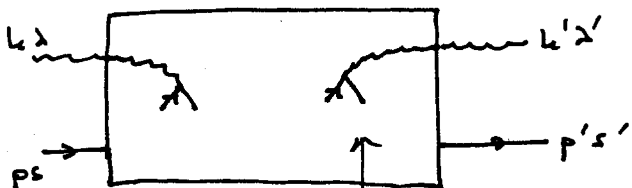


to second order, $n=2$. Two vertices $\frac{1}{n!}T\{\underbrace{H_I \ldots H_I}_{n \text{ factors}}\}$

n factors,
∴ n vertices

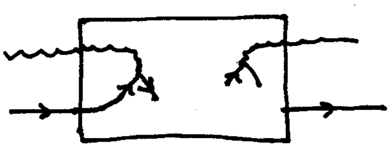
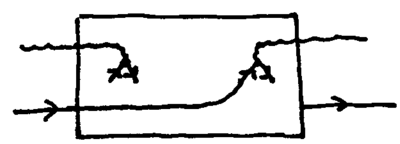
n vertices (this e.g. $n=2$)

$\frac{1}{n!}$ 

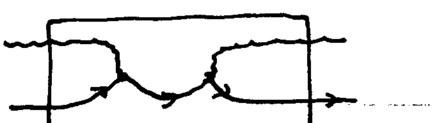
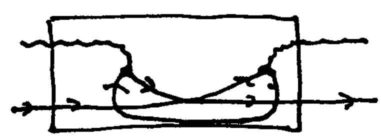
$= \frac{1}{2!} \cdot \underbrace{2}$ 

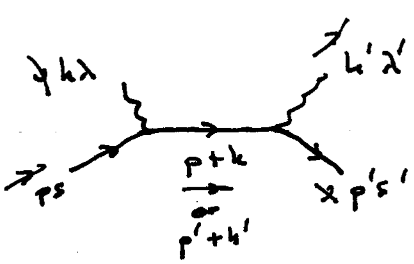
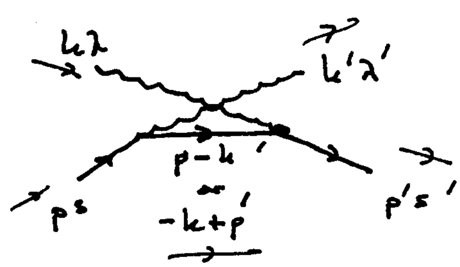
2 choices which vertex to connect kλ to

this one is then unique.

$=$  $+$ 

choices for p's' connection then forced,

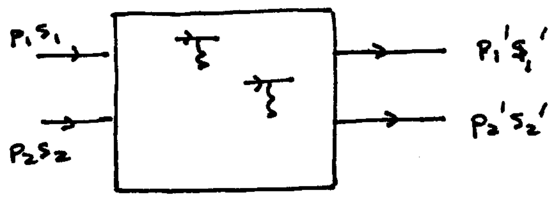
$=$  $+$ 

$=$  $+$ 

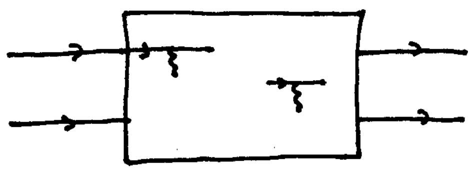
$\mathcal{M} = \mathcal{M}_1 + \mathcal{M}_2$

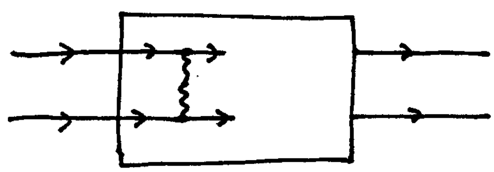
two indep. Feynman diagrams contribute to $\mathcal{M}$

Identical particles, e.g. e^-e^- scat, $\mathcal{O}(e^2)$

$\frac{1}{2!}$ 

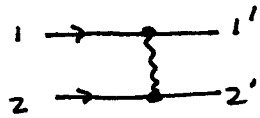
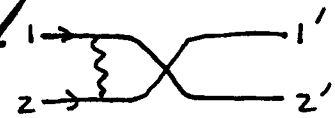
again factor of 2 from 1^{st} choice

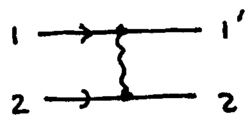
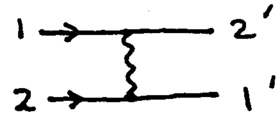
$\frac{1}{2!} \cdot 2 \cdot$ 

$=$ 

(−) for each fermion line crossing

forced

$=$  $-$ 

$=$  $-$ 

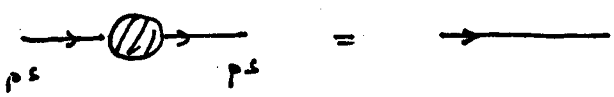
guarantees an antisymmetric scattering amplitude

actually we also get $\frac{1}{2!} \cdot 2 \cdot \Big\{$  $\Big\}$

but those don't contribute to scattering.
They only change the electron's properties from "bare" to

Feynman rules for loops

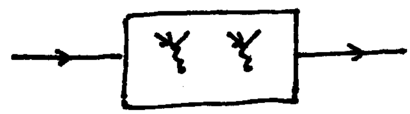
Consider as an example the higher order corrections to an e^- propagator



+ corrections with an even # of vertices

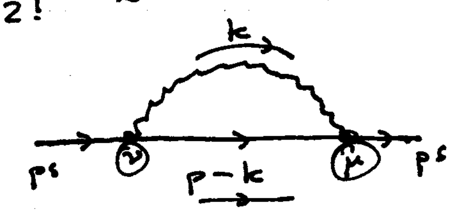
$$\mathcal{M} = \bar{u}_{ps} \frac{i}{\not{p}-m} u_{ps}$$

2^{nd} order $\quad \frac{1}{2!}$



$= \frac{1}{2!} \cdot 2 \cdot$

$=$



"rainbow diagram"

$$\left[\bar{u}_{ps} \frac{i}{\not{p}-m} (-ie\gamma^{\mu}) \frac{i}{\not{p}-\not{k}-m} (-ie\gamma^{\nu}) \frac{i}{\not{p}-m} u_{ps}\right]\left(\frac{-ig^{\mu\nu}}{k^2+i\epsilon}\right)$$

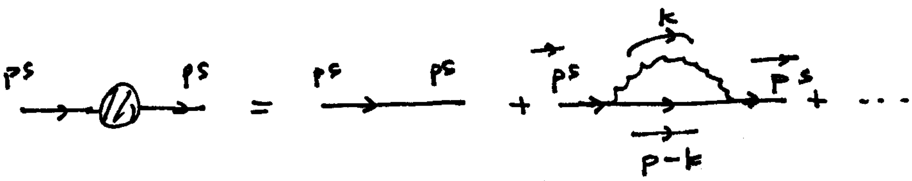
but what is k ? Not specified by the external momenta (p).

We have to return to $\quad S_{fi} = \langle ps | \frac{1}{2!} T\{H_I H_I\} | ps \rangle$,

working out this matrix element carefully we find an integral over the unspecified internal momentum,

$$\int \frac{d^4k}{(2\pi)^4}$$

So this can be viewed as a propagator & corrections



$$\mathcal{M} = \bar{u}_{ps}\left[\frac{i}{\not{p}-m_0} + \frac{i}{\not{p}-m_0}\int \frac{d^4k}{(2\pi)^4}(-ie\gamma^\mu)\frac{i}{\not{p}-\not{k}-m_0+i\epsilon}(-ie\gamma^\nu) \right.$$

↳ bare mass

$$\left. \left(\frac{-ig^{\mu\nu}}{k^2+i\epsilon}\right)\frac{i}{\not{p}-m_0}\right] u_{ps}$$

$$+ \dots$$

[ ] is the $\mathcal{O}(e^2)$ electron propagator $S_F(p)$

$$S_F^{(0)}(p) = \frac{i}{\not{p}-m_0}$$

with these corrections it can be written as

$$S_F(p) = \frac{i\,Z(p)}{\not{p}-\Sigma(p)}$$

1) pole shifts away from m_0 (change in e^- mass due to int. with γ).

2) residue of pole at $m_0+\delta m$ is no longer 1.

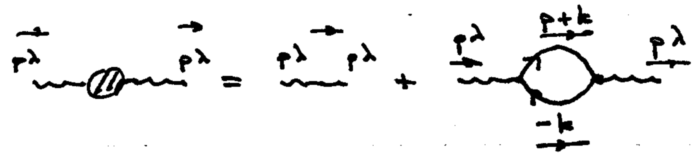
Unfortunately these corrections are ∞ :

$$\text{loop} \int \sim \int d^4k \frac{\not{p}-\not{k}+m_0}{[(p-k)^2-m_0^2]\,k^2} \sim \int \frac{d^4k}{k^4} \sim \int \frac{dk}{k}$$

[ln] diverges in the UV. Theory wrong?

Maybe, but only $m_{phys} = [m_0+\delta m]$ is observable, so $\delta m = \infty$ is not a pro

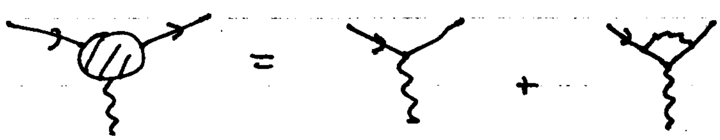
For a closed fermion loop,



there is in addition a $-\mathrm{Tr}\{\ \}$ over the Dirac propagators' indices.

This is also ln divergent, but doesn't shift the photon's mass, that's forced to be 0 by gauge invariance.
Instead this "vacuum polarization" graph modifies the $1/r$ Coulomb int. at short distances.

The last of those divergent graphs is the "vertex correction"



charge e_0 correction to charge, ln divergent

→ physical charge is e.

However this gives a finite correction to the magnetic moment of the electron, of $O(\alpha)$, which is observed & is in good agreement with expt.