The CKM matrix (charged weak decays)

19apr99

Arranging fermions as mass eigenstates we have 3 "generations":

$Q=+\frac{2}{3}$	$Q=-\frac{1}{3}$	$Q=-1$	$Q=0$	
quarks		leptons		
t	b	τ^-	ν_τ	3rd gen
c	s	μ^-	ν_μ	2nd gen
u	d	e^-	ν_e	1st gen

The mass term is diagonal in quark flavor $\mathcal{L}_0 = \sum_q \bar{\psi}_q (\not{p} - m_q)\psi_q$

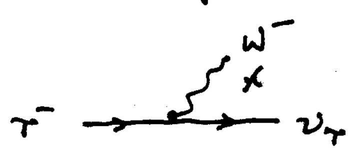
$$= \bar{\psi}_t(\not{p} - m_t)\psi_t + \ldots$$

and the neutral weak & EM couplings are also diagonal, e.g.

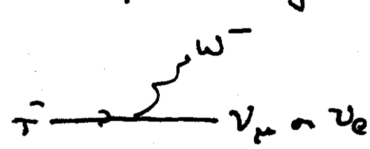
$$\mathcal{L}_{em} = -\sum_q e_q \bar{\psi}_q \not{A} \psi_q$$

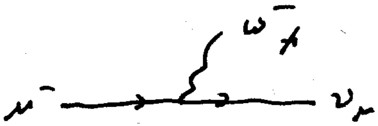
e_q: $\frac{2}{3}$ for $t\ c\ u$, $-\frac{1}{3}$ for $b\ s\ d$

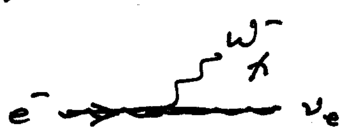
However there is a complication with charged weak currents, which change fermion charge. In the leptons they are diagonal



but not







all with $\mathcal{L}_I = -\frac{g}{2\sqrt{2}} \bar{\psi} \gamma_\mu (1-\gamma_5) \psi W_\mu$

but in the quarks the charged weak decays don't just couple $t \leftrightarrow b$ $c \leftrightarrow s$ $u \leftrightarrow d$ within each generation. This is roughly true but there is some mixing, parametrized by the CKM matrix,

$$\mathcal{L}_I = -\frac{g}{2\sqrt{2}} \begin{array}{c} \begin{array}{ccc} 1^{st} & 2^{nd} & 3^{rd} \\ \downarrow & \downarrow & \downarrow \end{array} \\ \left[\bar{\psi}_u \quad \bar{\psi}_c \quad \bar{\psi}_t\right] \end{array} \underset{\gamma_\mu(1-\gamma_5)}{\cdot} \underbrace{\begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix}}_{\substack{\text{unitary matrix} \\ \text{"CKM matrix"}}} \begin{array}{cc} \begin{bmatrix} \psi_d \\ \psi_s \\ \psi_b \end{bmatrix} & \begin{array}{l} \leftarrow 1^{st}\text{ gen} \\ \leftarrow 2^{nd}\text{ gen} \\ \leftarrow 3^{rd}\text{ gen} \end{array} \end{array}$$

(need not be if weak strength are not universal!)

The coupling of quark flavors here is approx diagonal. The magnitudes $|V_{ij}|$ of the CKM matrix entries are (90% c.l.)

$$\begin{bmatrix} 0.9745-9760 & 0.217-224 & 0.0018-0045 \\ 0.217-224 & \underset{\text{Apr. 99}\ \ 0.9745(5)}{\underline{0.9737-9753}} & 0.036-042 \\ 0.004-013 & 0.035-042 & 0.9991-9994 \end{bmatrix}.$$

Note the really important off-diag entry is V_{us} & V_{cd}.
If we specialize to only the 1^{st} 2 generations & ignore their weak couplings to t & b we have a 2×2 matrix

$$\begin{bmatrix} V_{ud} & V_{us} \\ V_{cd} & V_{cs} \end{bmatrix} = \begin{bmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{bmatrix}$$

where θ_c is the "Cabibbo angle".

$$\text{"}\mathcal{L}_I \propto \underbrace{\overset{-\frac{g}{2\sqrt{2}}}{}\ \bar{u}\,\overset{\gamma_\mu(1-\gamma_5)}{}(d\cos\theta_c + s\sin\theta_c)}_{u,d,s\ \text{parts.}}\overset{W_\mu}{}+\dots$$

still a very useful parametrization of u,d,s weak decays.

There is an official PDG parametrization of the 3×3 CKM matrix.

After choosing free phases of fermi fields, with an $n\times n$ matrix (n quarks) we still have

$$\underbrace{\underbrace{\underbrace{\tfrac{1}{2}n(n-1)}_{}}_{3 \text{ for } n=3}}_{\theta_{12},\,\theta_{23},\,\theta_{13}} \text{ angles} \quad + \quad \underbrace{\tfrac{1}{2}(n-1)(n-2)}_{}\text{ phases}$$

$\theta_{12}, \theta_{23}, \theta_{13}$ can be chosen to lie in 1^{st} quadrant

1 for $n=3$

δ_{13}

$0 \to 2\pi$.

If $\neq 0$ it implies CP violation.

The standard parametrization is

$$\begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & \underline{s_{13}}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}\underline{\underline{s_{23}s_{13}}}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}\underline{\underline{s_{23}s_{13}}}e^{i\delta_{13}} & s_{\underline{23}}c_{13} \\ s_{12}\underline{s_{23}} - c_{12}c_{23}\underline{s_{13}}e^{i\delta_{13}} & -c_{12}\underline{s_{23}} - s_{12}c_{23}\underline{s_{13}}e^{i\delta_{13}} & c_{23}c_{13} \end{bmatrix} \quad \begin{array}{ccc} ud & us & \underline{ub} \\ cd & cs & cb \\ td & ts & tb \end{array}$$

— small = very small

~ = CP viol. phase

but since $c_{13} = 1 - \mathcal{O}(10^{-6})$, to very good accuracy we can write this as

$$\begin{bmatrix} c_{12} & s_{12} & s_{13}e^{-i\delta_{13}} \\ & & s_{23} \\ & & c_{23} \end{bmatrix}$$

those ~~as the case~~ we can measure directly in decay processes, so can easily constrain the angles

$$\begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ & & V_{cb} \\ & & V_{tb} \end{bmatrix}$$

$b \to u$ transitions will show any CP violation.

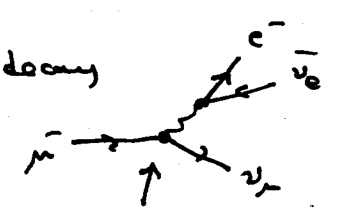
There is another commonly used set, "Wolfenstein params", which are only approximate

but recall the Cabibbo off-diag as most important,

$$\begin{bmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{bmatrix}.$$

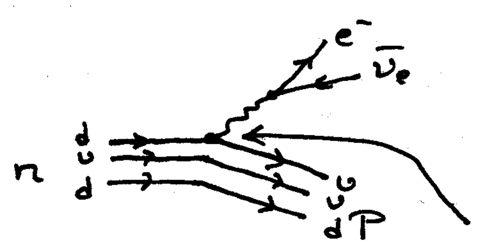
Experimental determination of the CKM matrix

1) $|V_{ud}|$ nuclear β decay, compared to μ^- decay



strength $1 \cdot \left(-\frac{g}{2\sqrt{2}}\right)$

e.g.

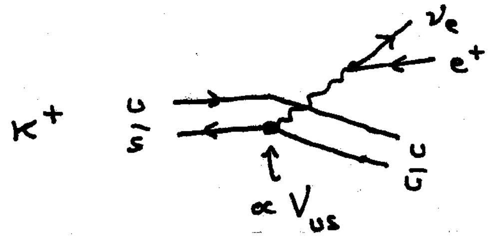


$V_{ud} \cdot \left(-\frac{g}{2\sqrt{2}}\right)$

$|V_{ud}| = 0.9740(10)$

$K^+ = u\bar{s}$ (K^0 agrees)

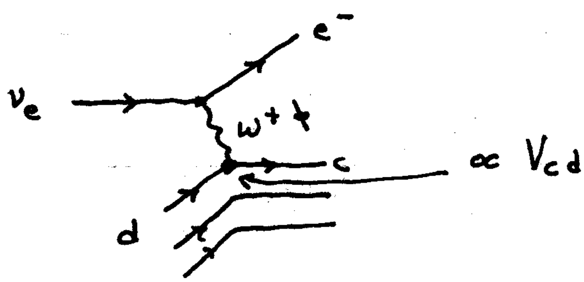
2) $|V_{us}|$ best data is kaon weak decays $K^+ \to \pi^0 e^+ \nu_e$ "K^+_{e3}"



$\left(\pi^0 = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})\right)$

$|V_{us}| = 0.2196(23)$

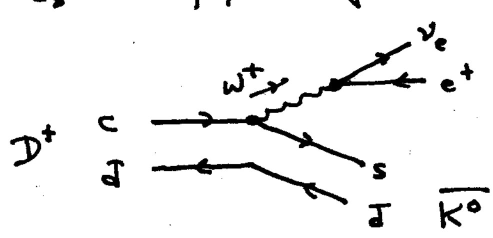
3) $|V_{cd}|$ determined from ν + $\bar{\nu}$ production of c from valence d quarks



$|V_{cd}| = 0.224(16)$

4) $|V_{cs}|$ D_{e3} decays, analogous to K_{e3}. $D \to \bar{K} e^+ \nu_e$

e.g.



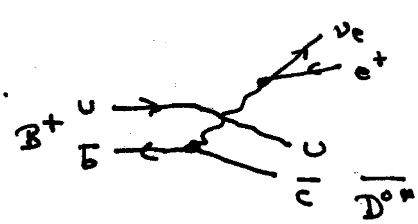
$|V_{cs}| = 1.04(16)$ (tighter bound with 3 gens & unitarity assumed)

5) $|V_{cb}|$ (uses HQET)

$B \to \overline{D^*} e^+ \nu_e$

($\bar{D} e^+ \nu_e$ less accurate)

e.g.



$|V_{cb}| = 0.0395(17)$

6) $|V_{ub}|$ semileptonic decays of B mesons from $\Upsilon(4S)$

study lepton energy spectrum
above energy endpoint for
$b \to c \ell \bar{\nu}_\ell$

these have
$b \to u \ell \bar{\nu}_\ell$ events

(don't study exclusive meson modes, just ℓ
E spectrum)

result $\dfrac{|V_{ub}|}{|V_{cb}|} = 0.08(2)$

include recent exclusive CLEO decay
$B \to \pi \ell \bar{\nu}_\ell$ & $\rho \ell \bar{\nu}_\ell$

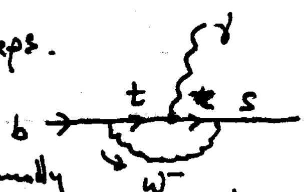
$|V_{ub}| = 3.3(4)(7) \cdot 10^{-3}$

7) $|V_{tb}|$ rough estm from CDF & DØ at FNAL

$$\frac{|V_{tb}|^2}{|V_{td}|^2 + |V_{ts}|^2 + |V_{tb}|^2} = 0.99\ (29).$$

Can infer rather than directly observe constraints esp on t-quark couplings in diagrams involving loops.

e.g. CLEO obs $b \to s\gamma$,



, constrains V_{ts}.

presumably

+ . . .

even light processes like $K^+ \to \pi^+ \nu\bar{\nu}$ are expected to be driven by c & t quark loops, so rare K decay searches are very interesting.