PHY522; Quantum Mechanics II, Problem Set 9

Due Wednesday 25 Apr 2007 at the beginning of class.

1. Properties of S_3.

The $3! = 6$ elements of S_3 may be written as the set of operations {e, (12), (13), (23), (123), (132)}.

a) (*5 pts*)

Using this form for the group elements, construct the full 6×6 multiplication table for S_3.

b) (*5 pts*)

Now take each element g_i in sequence and determine $g g_i g^{-1}\ \forall\ g \in S_3$. This lets you work out the three conjugacy classes in S_3. What are they in terms of their explicit membership? ($\mathcal{C}_1 = \{e\}$, etc.) In this case one may use the subscript n $\mathcal{C}_n$ to refer to the conjugacy class *and* to give the number of members in the each class.

2. A factor group of S_3

As noted in class, the subset of S_3 elements {e, (123), (132)} forms a subgroup of S_3, which is the (Abelian) cyclic group C_3.

a) (*2 pts*)

From the results of problem 1, is this C_3 an *invariant* subgroup of S_3?

b) (*4 pts*)

Assuming that C_3 **is** an invariant subgroup of S_3 ;), form the elements of the factor group S_3/C_3 by identifying each of the independent left cosets $g_i H$ (where $H = C_3$ and either $g_i = e$ or $g_i \in S_3$ but $\notin H$).

c) (*4 pts*)

Work out the multiplication table for this factor group, and identify it as one of the finite groups discussed in class.

3. Irreducible representations (IRs) of S_3

The dimensions $\{d_i\}$ of the IRs of a finite group satisfy

$$\sum_{i=1}^{k} d_i^{\,2} = N \tag{1}$$

where N is the order of the group and k is the number of classes.

a) (*3 pts*)

Given this fact, what are the dimensions of all the IRs of S_3?

b) (*3 pts*)

Two of the IRs of S_3 are already known to you (they are the trivial and alternating representations). Construct a character table with unknown values $\alpha = \chi(C_2)$ and $\beta = \chi(C_3)$ for the values of the characters of the two non-identity classes.

c) (*4 pts*)

Now use the orthogonality relations between characters to determine the values of α and β. Those who have access to a copy of Wigner's "Group Theory" may wish to compare with the explicit S_3 representation matrices in Eq.(7.E.1).