

PHY522; Quantum Mechanics II, Problem Set 8

Due Wednesday 11 Apr 2007 at the beginning of class.

1. One last radiation problem.

Now that we have some intuition for the (E1) transition rates to states with different values of radial excitation in hydrogen, let's consider a similar numerical problem in the orbital excitations.

In the previous problem set we found that the lifetime of the $N = 6, L = 5$ 6H (maximum L) state of hydrogen was 608.0 [nsec], unless I have made another mistake. One might wonder how the lifetime of these highly excited radial levels varies with the orbital angular momentum L .

(10 pts) By summing the partial widths to the various E1 allowed transitions, determine the numerical values in [nsec] of the lifetimes of the six different $N = 6$ levels in hydrogen, 6S, 6P, 6D, 6F, 6G and 6H (you have already done the last one).

2. Path integral

In the evaluation of the path integral for a free particle in 1D QM, we found that the integral over x_k at time slice k required a generic coupled Gaussian integral of the form

$$\mathcal{I}_k = a_k \eta \int_{-\infty}^{\infty} dx_k e^{F_k} \quad (1)$$

where

$$F_k = \frac{im}{2\epsilon\hbar} \left\{ (x_2 - x_1)^2 + (x_3 - x_2)^2 + \dots + (x_k - x_{k-1})^2 + b_k (x_k - x)^2 \right\} \quad (2)$$

and $\eta = \sqrt{m/2\pi i\epsilon\hbar}$.

a) (5 pts) Show that doing this integral produces a similar Gaussian with one less variable, with a different overall constant a_{k-1} and final term $b_{k-1}(x_{k-1} - x)^2$;

$$\mathcal{I}_k = a_{k-1} e^{F_{k-1}} \quad (3)$$

where

$$F_{k-1} = \frac{im}{2\epsilon\hbar} \left\{ (x_2 - x_1)^2 + (x_3 - x_2)^2 + \dots + (x_{k-1} - x_{k-2})^2 + b_{k-1} (x_{k-1} - x)^2 \right\} \quad (4)$$

In particular, evaluate the ratios a_{k-1}/a_k and b_{k-1}/b_k .

b) (5 pts) A plausible guess for the Green function of the time-dependent free Schrödinger equation in three dimensions is

$$G(0, 0; r, t) \propto t^{-p} e^{icr^2/t} . \quad (5)$$

Test this Ansatz by substitution in the Schrödinger equation. Does it work, and if so what are the required values of the power p and constant c ?