PHY522; Quantum Mechanics II, Problem Set 7

Due Wed 4 Apr 2007 at the beginning of class.

1. Totally rad

We have considered the analytical and numerical values of the E1 decay rates of H atoms for a few special cases in class, but we do not yet have a more intuitive "feel" for the branching fractions of the higher excitations. Let's improve the situation by doing some simple numerical calculations, based using the analytic E1 decay rate formulas scanned into the class notes (output from my lost MAPLE program). Specifically:

a) (*3 pts*) Determine the E1 lifetimes in [nsec] of the series of "circular" hydrogen states 2P, 3D, 4F, 5G, 6H. (We already evaluated 2P in class, it was 1.596 [nsec].)

b) (*3 pts*) Which final states does a highly radially excited state prefer to decay to under radiative transitions? Develop some intuition for this by evaluating the numerical branching fractions of 6S hydrogen into 5P, 4P, 3P and 2P final states.

c) (*4 pts*) Visit the H gas discharge tube exhibit in the Chemistry department (near Rm.555). Note how many different lines you can see through the glasses provided, and identify them in terms of which radiative transition(s) they represent, such as 2P$\rightarrow$ 1S. Now evaluate the E1 radiative partial widths (in [sec^{-1}]) for each of the transitions you have seen.

After turning in the assignment, visit the display again and enjoy your understanding of the quantum mechanics of this problem.

2. E1 radiative transitions from another angle.

In class we found that the E1 decay rate ("partial width") of the 2P→ 1S transition in H is given by

$$\Gamma_{2P\to 1S} = \frac{2^8}{3^8}\,\frac{mc^2\alpha^5}{\hbar}. \tag{1}$$

We found this result by starting from the general E1 rate formula

$$\Gamma^{E1} = \frac{4}{3}|\,\vec{x}_{fi}|^2 k_\gamma^3 \tag{2}$$

and specializing to H levels and wavefunctions, with the arbitrary choice of $L_z = 0$ for the initial 2P state.

(*10 pts*) Show that the choice of the initial orbital polarization L_z does not affect the decay rate by again deriving $\Gamma_{2P\to 1S}$ from the E1 formula of Eq.(2), but assuming $L_z = +1$.

3. Other multipoles.

We have briefly discussed the existence of two infinite series of radiative transitions, the electric multipole series E1,E2,E3... and the magnetic multipole series M1,M2,M3...; in both series the integer gives the total angular momentum j carried by the photon (so there is a selection rule that $j \otimes j_i$ must contain j_f), and there is a parity selection rule that the parity of the radiating system must change by $(-)^j$ for E-pole radiation and $(-)^{j+1}$ for M-pole radiation.

List all the allowed multipole(s) for radiation in each of the following systems:

a) (*2 pts*) 3P → 1S hydrogen (ignore spin).

b) (*2 pts*) 4D → 3P hydrogen (ignore spin).

c) (*2 pts*) $\omega \to \pi^0\gamma$, where the ω is a $J^P = 1^-$ meson and the π^0 is a $J^P = 0^-$ meson.

d) (*2 pts*) $\Delta^+ \to p\gamma$, where the Δ^+ is a $J^P = 3/2^+$ baryon and p is a proton ($J^P = 1/2^+$).

e) (*2 pts*) You get 2 free points if you can remember that $j_i = 0$ to $j_f = 0$ radiative transitions are forbidden. ($j = 0$ is a special case for vector spherical harmonics.)