de Broglie (PhD thesis 1924)

Since em waves came in quanta (hence were particle-like in some ways e.g. propagation through vacuum), maybe massive particles like the e^- & ... have waves associated with them.

For light we already know $E = \hbar\omega$ from Planck.

The momentum carried by an EM wave is

$$p = \frac{E}{c} = \hbar\frac{\omega}{c} = \hbar k = \frac{h}{\lambda}$$

e.g. from Poynting vector

So deBroglie assumed that the "pilot waves" associated with particles had frequencies and wave numbers of

$$\omega = \frac{E}{\hbar} \qquad \text{and} \qquad k = \frac{p}{\hbar}$$

$$\text{or} \qquad \nu = \frac{E}{h} \qquad\qquad \lambda = \frac{h}{p}$$

The wavelength associated with macroscopic objects is so small as to be $\approx$ unobservable

e.g. dust grain $\underline{r = 1\ \mu m}$, $\rho = 10.\ gm\ cm^{-3}$, $v = 1\ cm\ sec^{-1}$

$$p = 4 \cdot 10^{-11}\ gm\ cm\ sec^{-1} \rightarrow \underline{\lambda} = \frac{h}{p} = 1.6 \cdot 10^{-16}\ cm = \underbrace{\underline{1.6 \cdot 10^{-12}\ \mu m}}_{\lll r}$$

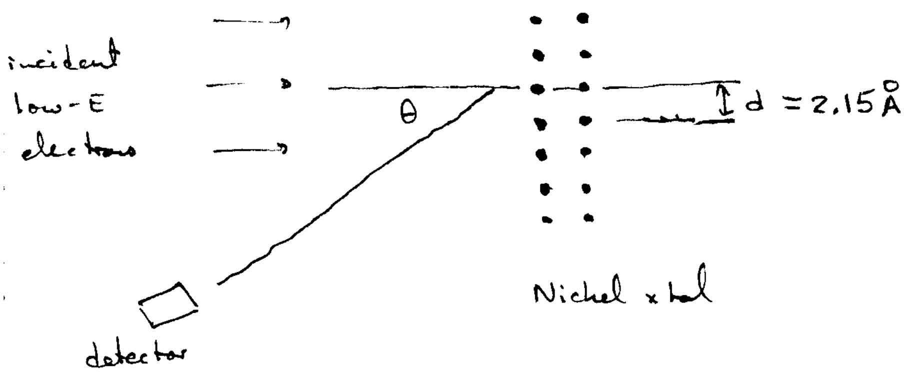
However at the atomic scale

e^- $\quad$ $KE = 10\,eV \rightarrow p = 1.7 \cdot 10^{-19}\ gm\ cm\ sec^{-1}$

$$\rightarrow \lambda = \frac{h}{p} = 3.9 \cdot 10^{-8}\ cm = 3.9\ \text{Å}$$

Somewhat larger than typical atomic size,
$\therefore$ wave nature of e^- should be important for atomic physics.

Expt test of de Broglie: Davisson & Germer (1927)



Condition for constructive interference, $\quad n\lambda = d \sin\theta$

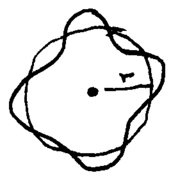
for $eV = KE = 54.\ eV$, a strong diffr. peak was observed at $50° = \theta$,

(if 1st diffr. peak) $n=1$ $\quad$ $\lambda = \underset{\substack{\uparrow \\ 2.15\text{Å} \\ Ni}}{d} \sin(50°) = 1.65\ \text{Å}$

for $V = 54.$ volts $\rightarrow \lambda = 1.67\ \text{Å}$

So de Broglie relation $\quad p = \hbar k \quad$ or $\quad \lambda = \frac{h}{p}$ $\quad$ appears confirmed.

Bohr's atom can now be motivated.
For stability we presumably want an integer number of wavelengths for each circular orbit.



e.g. $n=4$

$$2\pi r = n\lambda = n\frac{h}{p} \qquad \text{so} \qquad \underbrace{pr}_{mvr = L} = \frac{nh}{2\pi} = n\hbar$$

or $\quad L = n\hbar$

So we can "explain" the Bohr rule $L = n\hbar$ as requiring an integral number of waves of deBroglie's "pilot waves" around the orbit.

For free particles the associated waves are presumably plane waves,

$$\psi = e^{-i\omega t + ikx} \qquad \text{for motion along } x.$$

Then note that the E and p can be "measured" through the action of the operators $E_{op} = i\hbar\frac{\partial}{\partial t}$

and $\quad p_x{}_{op} = -i\hbar\frac{\partial}{\partial x} \qquad \vec{p}_{op} = -i\hbar\vec{\nabla}$

check:

$$E_{op}\psi = i\hbar\frac{\partial}{\partial t}\left(e^{-i\omega t + ikx}\right) = i\hbar\left(-i\omega e^{-i\omega t + ikx}\right) = \hbar\omega e^{-i\omega t + ikx}$$

$$= \hbar\omega\psi$$

similarly

$$P_{x\,op}\psi = -i\hbar\frac{\partial}{\partial x}\psi = \hbar k\psi$$

& for $\psi = e^{-i\omega t + i\vec{k}\cdot\vec{x}}$ $\qquad \vec{P}_{op}\psi = \hbar\vec{k}\,\psi$

n.b. If an operator $\mathcal{O}$ when operating on a function ψ returns a const time that function, then ψ is said to be an eigenfunction of $\mathcal{O}$, and the const. returned is the eigenvalue.

$$\underbrace{E_{op}}_{\text{operator}}\psi = \underbrace{\hbar\omega}_{\text{eigenvalue (= energy E in this case)}}\underbrace{\psi}_{\text{eigenfunction}}$$

Note for nonrelativistic particles $E = \frac{\vec{P}^2}{2m}$ so ψ satisfies the DE

$$E_{op}\psi = \frac{\vec{P}_{op}^{\,2}}{2m}\psi$$

$$\boxed{i\hbar\frac{\partial}{\partial t}\psi(\vec{x},t) = -\frac{\hbar^2}{2m}\nabla^2\psi(\vec{x},t)}$$

free Schrödinger equation

(whatever ψ is, physically!)

OK, but what if there is an interaction $V(\vec{x})$ present?
e.g. for our atomic physics problem the e^- "wavefunction" ψ has $V(\vec{x}) = -\frac{Ze^2}{r}$.
Generalization of this guessed by E. Schrödinger, 1925 (<u>central</u> to QM):