Phy 512 review of topics (pre-midterm)

failures of classical physics

atomic stability $W = \frac{2}{3}\frac{e^2 a^2}{c^3}$

discrete spectral lines

H atom $\rightarrow E_n \propto \frac{1}{n^2}$

UV catastrophy in black body rad.

Black body prob. (Planck) modes, counting of d.o.f.s $\frac{dN}{dk}, \frac{dN}{d\omega}$

in 1D 2D 3D

$E = kT$ each SHO mode, ∞ # modes!

$P(\lambda) \propto \frac{1}{\lambda^4}$

$\frac{dN}{d\lambda}$ = Rayleigh–Jeans law

checking units!

Planck's assumption (1900) $E = \hbar\omega$ (steps in)

$$P(\lambda, T) \propto \frac{1}{\lambda^5\left(e^{\frac{hc}{kT\lambda}} - 1\right)}$$

✓ expt.

& how to get relevant $\int$s and sums by diff. w.r.t. params.

Bohr atom: classical except $L_n = n\hbar$. $\left(V = -\frac{e^2}{r},\ e^2 = \hbar c \alpha\right)$

$$r_n = n \cdot a_0$$

$$\frac{\hbar^2}{me^2} = \frac{\hbar c}{m\alpha} = \text{Bohr radius}$$

$$E_n = -mc^2\alpha^2 \cdot \frac{1}{2n^2}$$

deBroglie (1924) "pilot wave"

$p = \hbar k \qquad E = \hbar\omega$

Davisson & Germer ('27)

Schrödinger's wave eqn. (1925)

$$-\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi = i\hbar\frac{\partial}{\partial t}\Psi \qquad \Psi(\vec{x},t) \qquad \text{TDSE}$$

from $\qquad \frac{\vec{p}^{\,2}}{2m} + V(\vec{x}) = E$

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = E\psi \qquad \Psi(\vec{x},t) = e^{-iEt/\hbar}\,\psi(\vec{x})$$

TISE

"energy eigenstates"

$|\Psi|^2$ & the "Copenhagen interpretation"

energy eigenstate

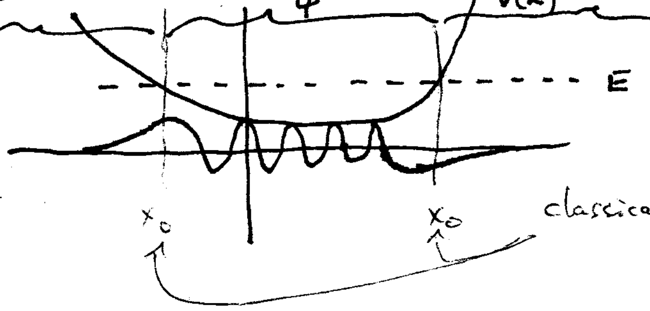
1D problems

Const. V_0

$$\begin{cases} E>V_0: \quad \lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2m(E-V_0)}}, \quad \hbar k = \sqrt{2m(E-V_0)} \\ \qquad\qquad \text{oscillatory region, classically allowed} \\ E<V_0 \quad \text{exp. damped}, \quad a = \dfrac{\hbar}{\sqrt{2m(V_0-E)}} \\ \qquad\qquad \text{quantum tunneling} \end{cases}$$

$\frac{\psi''}{\psi} > 0$ exp. $\qquad \frac{\psi''}{\psi} < 0$ osc. $\qquad V(x) \qquad \psi''/\psi > 0$ exp. damped

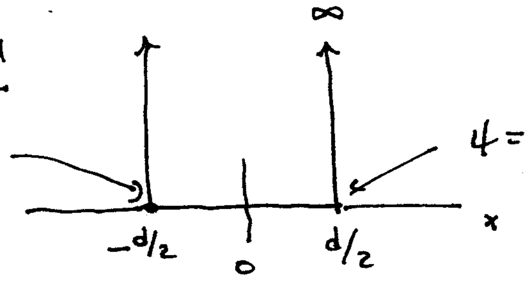
generally



classical turning pts $E = V(x_0)$

Specific probs

1D
∞ sq. well



$$E\psi = -\frac{\hbar^2}{2m}\psi''$$ even & odd (cos & sin) solns. $\cos(k_n x)$, $\sin(k_n x)$

$$\lambda_n = \frac{2d}{n} \qquad k_n = \frac{2\pi}{\lambda_n} = \frac{\pi n}{d}$$

$$\therefore E_n = n^2 \cdot \frac{\hbar^2 \pi^2}{2md^2}$$

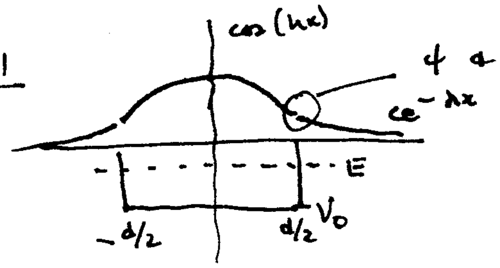
$(N+1)^2$ if $N=0$ is the g.s.

Normalization $$\int |\psi|^2 d^n x = 1$$

Heis. unc. princ. $\Delta x \cdot \Delta p_x \geq \hbar/2$ any wfn. (Gaussian gives min. uncertainty)

$$\Delta x = \left[\langle x^2 \rangle - \langle x \rangle^2 \right]^{1/2}$$

finite depth sq. well



ψ & ψ' continuous.

to get E_n only, matching ψ'/ψ suffices.

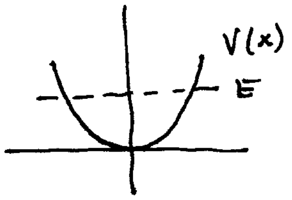
(even solns) $x \tan x = \sqrt{R^2 - x^2}$
(odd) $\rightarrow -x\cot x$

$$x = \frac{kd}{2}, \quad R^2 = \frac{m|V_0|d^2}{2\hbar^2}$$

always have at least 1 b.s.

1D SHO

$V(x) = \frac{1}{2}kx^2$



$$-\frac{\hbar^2}{2m}\psi_n'' + \frac{1}{2}kx^2\psi_n = E_n\psi_n$$

try $\psi = e^{-f}$

$$-\frac{\hbar^2}{2m}\left(-f'' + f'^2\right) + \frac{1}{2}kx^2 = E_0$$

f'^2 ← dom. large x

$$\psi_0 = \eta_0 e^{-\frac{\sqrt{km}}{2\hbar}x^2}$$ ✓ exact soln = Gaussian

excited states

$$\psi_n = p(x)\,e^{-\frac{\sqrt{km}}{2\hbar}x^2} = p(x)\,e^{-\frac{1}{2}cx^2} \qquad c \equiv \frac{\sqrt{km}}{\hbar}$$

$s = \sqrt{c}\,x$ dimensionless

$$\frac{d^2p}{ds^2} - 2s\frac{dp}{ds} + \underbrace{\left(\frac{E}{\frac{1}{2}\hbar\omega} - 1\right)}_{2n} p = 0$$ Hermite DE

solns are Hermite polynomials

$$\psi_n = \eta_n H_n(s)\,e^{-\frac{1}{2}s^2}$$

$$H_n(s) = (-)^n e^{s^2}\frac{d^n}{ds^n}e^{-s^2}$$

gen fn. $\quad F(r,s) = \sum_n \frac{r^n}{n!}H_n(s) = e^{2rs - r^2}$

orthog. $\quad \int_{-\infty}^{\infty} H_n(s)H_{n'}(s)e^{-s^2}\,ds = 2^n n!\sqrt{\pi}\,\delta_{nn'}$

why polynomials? (hence $\frac{E}{\frac{1}{2}\hbar\omega} - 1 = 2n$). normalizable solns, localized near $r=0$.

norm

$$\eta_0 = \left(\frac{c}{\pi}\right)^{1/4}$$

$$\eta_n = \frac{1}{\sqrt{2^n n!}} \left(\frac{c}{\pi}\right)^{1/4}$$

<u>expectation values</u>

$$\langle x \rangle = \int_{-\infty}^{\infty} x \overset{|\psi|^2}{P(x)}\, dx = \int_{-\infty}^{\infty} \psi^* x\, \psi \, dx$$ "sandwich formula"

↳ specify wfn like $\langle 0|x|0\rangle = \int_{-\infty}^{\infty} \psi_0^*(x)\, x\, \psi_0(x)\, dx$

$$\langle 1|x|0\rangle = \int_{-\infty}^{\infty} \psi_1^*(x)\, x\, \psi_0(x)\, dx$$

can also calc. expected values of operators, e.g. $p_x = -i\hbar \frac{d}{dx}$

$$p_x^2 = -\hbar^2 \frac{d^2}{dx^2}$$

for the SHO we derived

$$\langle n|x^2|n\rangle = \frac{\hbar}{\sqrt{km}}\left(n+\tfrac{1}{2}\right)$$ explicitly (checks $\langle\psi|V(x)|\psi\rangle = \frac{1}{2}E_n$).

SHO ± ops

$$a = \sqrt{\frac{\hbar}{2m\omega}}\left(+\frac{d}{dx} + \frac{m\omega}{\hbar}x\right)$$

$$a^\dagger = \sim \left(-\frac{d}{dx} \sim\right)$$

$$H = \hbar\omega\left(a^\dagger a + \tfrac{1}{2}\right)$$