Cheating and Enforcement in Asymmetric Rank-Order Tournaments

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Abstract

Imperfect monitoring of actions in rank-order tournaments makes it possible that undesirable but output-enhancing activities, such as cheating, may occur. Cheating may be especially tempting when one player has an advantage over another. We show that when audit probabilities are low, the leading player has more incentive to cheat; when audit probabilities are high, the incentive is reversed. Furthermore, we show that "correlated" audits are more effective at decreasing the frequency of cheating than independent audits. Finally, we show that differential monitoring schemes, where contestants are audited based on either their initial position or final ranking, more efficiently achieve full deterrence than schemes which monitor contestants with equal
probability.

1 Introduction

Among the many issues surrounding incentives to cheat in rank-order tournaments and contestants’ behavior when cheating is possible, one of the most important is the effect of asymmetry in the position of the contestants. If two contestants are competing for a prize but one has an advantage over the other (such as greater ability or talent, or simply being "ahead" due to the history of play up to that point in the contest), will the trailing player necessarily be more likely to cheat because he needs to close the gap and has less to lose from possible disqualification? Might there be circumstances under which the leader cheats in order to maintain her advantage, because if she does not, the trailing player can close the gap by cheating? This paper presents a model to provide insights into how positional concerns influence cheating in rank-order tournaments.¹

Rank-order tournaments present an environment in which the incentive to cheat² may be particularly strong for a few reasons. First, tournaments are generally used in environments in which actions are difficult to monitor. Second, a small increase in a contestant’s output can dramatically change his payoff if it increases his rank. Cheating in various forms has been observed in important competitive settings, such as corporate promotion tournaments and

¹We consider asymmetry because being ahead or behind may influence an individual’s incentive to cheat, just as asymmetry influences individuals’ choices to engage in risk-seeking behavior in tournaments. In particular, an individual who is ahead may want to "play it safe," while an individual who is behind may want to adopt a more risky strategy (Tsetlin, Gaba, and Winkler (2004), Bronars (1987), Kräkel and Sliwka (2004)).

²Cheating can take many forms, but here we refer to an agent taking some action which increases his output but which is prohibited by the organizer of the contest, or principal, who is seeking to elicit productive effort from the agent. We do not consider "influence activities" such as bribery and sabotage.
sports competitions; these settings can be broadly characterized as taking place in contests and can be modeled as rank-order tournaments of the sort first introduced by Lazear and Rosen (1981).

In this paper, we consider a game in which two heterogenous players simply choose whether or not to cheat. This can be thought of as the final stage of a tournament in which players have previously chosen effort and now find themselves having an opportunity to cheat while knowing their current position relative to each other. Or, it can be considered as the final round of a multi-stage tournament in which one player is ahead of the other entering the last phase of competition. Our focus is on how equilibrium cheating behavior is affected by varying the intensity of enforcement, represented by the probability that contestants are audited to detect cheating, and on the conditions required to achieve complete deterrence of cheating and thus a "clean" contest. Consistent with intuition we find that the trailing player has a strong incentive to cheat, but in some circumstances, it is the leading player who is more likely to cheat.

In addition, we explore how differences in the auditing regime affect cheating behavior. We find that employing "correlated" audits (both contestants are audited simultaneously with some probability and otherwise neither is) yields less frequent cheating in the mixed-strategy equilibrium of the game than if the two contestants are audited with an equal probability but as a result of independent random draws. Thus, using a correlated audit regime achieves more effective deterrence of cheating than independent audits for any given expenditure of resources when resources are not sufficient to achieve full deterrence. Then, relaxing the restriction that players must be monitored with equal probability, we find that full deterrence can be achieved at a lower cost by auditing the contestants with different
probabilities. Specifically, the "leading" contestant can be audited with a lower probability than the trailing contestant while maintaining no cheating in equilibrium. This suggests that conditioning enforcement on contestants’ positions (ex ante) and also perhaps final rank (ex post) is advantageous and that uniform enforcement is not the least cost method of fully deterring cheating.

Other papers have considered cheating in asymmetric tournaments or contests, and the most closely related paper to this is Berentsen (2002). This paper also considers a game with two heterogeneous players who simultaneously decide whether or not to cheat (take performance-enhancing drugs) before competing; it focuses on implementing the no-cheating outcome as a function of punishment. Berentsen finds that there are regions in which the underdog cheats, regions in which the favorite cheats, and regions in which both players follow a mixed-strategy. The author then proposes a ranking-based sanction scheme which can implement no cheating in equilibrium in a less-costly manner than one in which sanctions are identical regardless of ranking.

Kräkel (2007) analyzes a two-player asymmetric tournament in which contestants choose both effort and cheating, which are modeled as complements. Given this setup, the paper focuses on a no-cheating equilibrium and identifies three effects which determine whether an individual decides to cheat: the likelihood effect (cheating improves the probability of winning), the cost effect (how cheating affects effort costs), and the base-salary effect (if a

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3Haugen’s (2004) model is similar to Berensten (2002); the main difference is that Haugen’s presentation is less technical.

4In a markedly different approach, Berentsen and Lengwiler (2004) study the problem from an evolutionary perspective by analyzing the replicator dynamics of doping in games in which there are strong and weak players. They find that under certain parameter restrictions, there are cycles of doping; furthermore, there are some cases in which the stronger players are more likely to cheat than the weaker ones.
player cheats, he reduces his expected base salary since there is a chance he will get caught). Furthermore, Kräkel examines whether *ex ante* (before the tournament begins) or *ex post* (after rankings are determined) testing leads to higher levels of legal input and finds that greater effort is exerted in *ex ante* testing.

Gilpatric (2007) and Curry and Mongrain (2007) model the cheating decision in a symmetric contest. The former presents a model in which contestants simultaneously choose cheating and effort, and as a departure from previous models, cheating is a continuous choice. The latter also models cheating in symmetric contests in which players make a dichotomous choice whether to cheat or not; their model concentrates on the minimum audit probability required to deter all cheating. They focus on the important effect of "re-awarding" on cheating incentives, that is, whether the prize to the top-ranked contestant passes by default to the second-ranked contestant if the winner is found to have cheated.

The paper proceeds as follows. Section 2 presents a model of the decision to cheat or not in a two-player tournament when the players are heterogeneous. It explores pure and mixed-strategy equilibria, both of which depend on the probability of audit. In Section 3, we show that when both contestants are subject to an equal audit probability, correlated audits more effectively deter cheating than independent audits (each contestant is audited or not as a result of an independent random draw). In Section 4, we explore the role of differential audit rules, whereby one player may be audited with higher probability than the other conditional on their relative position or final rank, may be desirable if feasible. Section 5 concludes, and all proofs are collected in the appendix.
2 The Decision to Cheat

In a Lazear-Rosen tournament setting, consider risk-neutral players $j$ ("she") and $k$ ("he"). The winner of the tournament, determined by the player with the highest level of output $q$, receives $w_1$, while the loser receives $w_2$, where $w_1 > w_2$ and $w_1 > 0$. Denote the spread between the winning and losing prizes $S = w_1 - w_2$.

Consider a tournament in which players make a simple dichotomous choice to cheat or not; a player who cheats increases his average output by $x$. Moreover, one player may have an advantage over the other. Without loss of generality, suppose that player $j$ is weakly ahead of player $k$ by $\alpha \geq 0$ ($\alpha = 0$ corresponds to the case of a symmetric contest, and $\alpha > 0$ corresponds to an asymmetric contest). This difference in position is common knowledge to both contestants as well as the contest organizer. When asymmetry exists, it may be due to differences in talent, effort, luck, or may be a consequence of previous play (O’Keeffe, et al (1984)). We do not model the choice of effort or prior play, but simply analyze behavior when cheating is possible and prior choices or abilities may result in asymmetric positions.

As is standard in tournament models, output is also affected by $\varepsilon$, a random component symmetrically distributed around 0 which is not realized at the time of contestants’ decisions whether to cheat. So, if individual $j$ does not cheat, she produces $q_j = \alpha + \varepsilon_j$, whereas if she cheats, she produces $q_j = \alpha + x + \varepsilon_j$. Also, define $\zeta = \varepsilon_k - \varepsilon_j$, and let the density of $\zeta$ be $g(\zeta)$ with corresponding cdf $G(\zeta)$. Note that $\zeta$ is a symmetric, unimodal random variable with mean zero and that $G(0) = \frac{1}{2}$.

We assume initially that contestants are audited with probability $\eta$, and the contest

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5 Modeling asymmetry in this way permits comparative statics analysis of the results on the size of the lead and also allows the model to encompass both symmetric and asymmetric tournament settings.
organizer uses a random "correlated" audit. That is, with probability $\eta$ both players are audited, and with probability $1 - \eta$ neither is. We assume that the outcome of the audit is always accurate; a cheater is always detected cheating, and a non-cheater is never falsely accused.\(^6\) If a player is found to have cheated, he automatically loses the contest (receiving $w_2$) in addition to possibly being penalized by $r \geq 0$. This "outside penalty" can be thought of as capturing any punishment beyond mere disqualification, including any enforceable fine or criminal penalty, as well as the value of lost future revenues or opportunities as a result of being caught cheating (e.g. being banned from participating in future tournaments or losing lucrative contracts). In addition, the runner-up wins by default when the winner is caught cheating.

Players choose a strategy to maximize their expected payoffs. If neither player cheats, or if both players cheat, the probability that player $j$ will rank first\(^7\) is $p_j = \text{prob}(\alpha > \varepsilon_k - \varepsilon_j) = G(\alpha)$, while the probability $k$ will rank first is $p_k = \text{prob}(\alpha < \varepsilon_k - \varepsilon_j) = 1 - G(\alpha) = G(-\alpha)$. Similarly, if player $j$ cheats but player $k$ does not, the probability that player $j$ will rank first is $p_j = G(\alpha + x)$, whereas the probability that player $k$ will rank first is $p_k = 1 - G(\alpha + x) = G(-\alpha - x)$, and so on.

The expected payoffs to players $j$ and $k$ from cheating and not cheating are shown in the following payoff matrix, with the payoffs for player $j$ ($k$) on top (bottom) in each cell. Note that in the payoff matrix, setting $\alpha = 0$ corresponds to a symmetric contest while $\alpha > 0$ corresponds to an asymmetric contest.

\(^6\)It would be possible to incorporate another variable which captures the conditional probability of testing positive, as in Berentsen (2002).
\(^7\)We choose our terminology carefully because winning is not synonymous with ranking first; a player may rank first but be caught cheating and therefore not win.
We characterize the equilibrium as a function of the probability of audit, $\eta$. If $\eta$ is sufficiently large then not cheating is a dominant strategy, but the audit probability sufficient to render not cheating dominant differs for $j$ and $k$. For not cheating to be dominant it must yield a larger expected payoff than cheating regardless of the opponent’s strategy. It is generally the case that a contestant’s expected payoff from cheating is higher if his opponent does not cheat than if he does (this will be true unless $\alpha$ and $x$ are very large; we ignore this possibility), so the binding condition for not cheating to be dominant is that it yields a higher expected payoff than cheating if the opponent does not cheat. If $j$ believes $k$ will not cheat, player $j$’s expected payoff by not cheating will exceed that from cheating if

$$G(\alpha)S + w_2 \geq (1 - \eta)[G(\alpha + x)S] + w_2 - \eta r.$$ 

Therefore not cheating is a dominant strategy for $j$ if

$$\eta \geq \eta_j \equiv \frac{G(\alpha + x) - G(\alpha)}{G(\alpha + x) + \frac{x}{S}}. \quad (1)$$
Similarly, not cheating is a dominant strategy for $k$ if

$$\eta \geq \overline{\eta}_k \equiv \frac{G(-\alpha + x) - G(-\alpha)}{G(-\alpha + x) + \frac{r}{S}}.$$  

(2)

Of course if the contest is symmetric, i.e. $\alpha = 0$, then the critical value is identical for both contestants, or $\overline{\eta}_j = \overline{\eta}_k = \eta_s \equiv \frac{G(x) - G(0)}{G(x) + \frac{r}{S}},$  

(3)

where $\eta_s$ is the threshold above which not cheating is a dominant strategy for both players in the symmetric game.

For sufficiently low values of $\eta$ cheating is a dominant strategy. These critical values can be identified analogously to the above results. Cheating is a dominant strategy for $j$ if

$$\eta \leq \underline{\eta}_j \equiv \frac{G(\alpha) - G(\alpha - x)}{G(\alpha) + G(x - \alpha) + \frac{r}{S}},$$  

(4)

and cheating is a dominant strategy for $k$ if

$$\eta \leq \underline{\eta}_k \equiv \frac{G(-\alpha) - G(-\alpha - x)}{G(-\alpha) + G(\alpha + x) + \frac{r}{S}}.$$  

(5)

If the contest is symmetric ($\alpha = 0$) then again, $\underline{\eta}_j = \underline{\eta}_k = \eta_s \equiv \frac{G(x) - G(0)}{G(x) + G(0) + \frac{r}{S}}.$

We can rank these critical values for $\alpha > 0$ as follows: $\underline{\eta}_k < \underline{\eta}_j < \overline{\eta}_j < \overline{\eta}_k$. Equilibrium
Proposition 1 When $\eta \leq \eta_k$, both players cheat. When $\eta \in (\eta_k, \eta_j]$, it is a dominant strategy for player $j$ to cheat, and player $k$ best responds by not cheating. When $\eta \geq \eta_k$, neither player cheats. When $\eta \in [\eta_j, \eta_k)$, it is a dominant strategy for player $j$ to not cheat and player $k$ best responds by cheating. When $\eta \in (\eta_j, \eta_k)$ three equilibria exist: 1) $j$ cheats and $k$ does not cheat, 2) $k$ cheats and $j$ does not cheat, and 3) $j$ cheats with probability $\tilde{\rho}_j$ and $k$ cheats with probability $\tilde{\rho}_k$ as defined by the following expressions:

\[
\tilde{\rho}_j \equiv \frac{(1 - \eta)G(-\alpha + x) - G(-\alpha) - \eta r/S}{(1 - \eta)[G(-\alpha + x) + G(-\alpha - x)] - G(-\alpha)[2 - \eta] + \eta} \tag{6}
\]

\[
\tilde{\rho}_k \equiv \frac{(1 - \eta)G(\alpha + x) - G(\alpha) - \eta r/S}{(1 - \eta)[G(\alpha + x) + G(\alpha - x)] - G(\alpha)[2 - \eta] + \eta} \tag{7}
\]

[Figure 1 about here]

Figure 1 summarizes the players’ equilibrium cheating strategies for $\eta \in [0, 1]$. Regarding the central region where $\eta \in (\eta_j, \eta_k)$, note that pure strategy equilibria with only one player cheating presumes coordination among the players.\(^8\) Although such equilibria could occur if some basis for coordination exists, such as one contestant having a reputation for cheating while the other does not, we generally assume no coordination of this sort and focus on the mixed-strategy equilibrium in this range. In the mixed-strategy equilibrium, each player adopts a strategy that leaves his opponent indifferent between cheating and not cheating.

Given (6) and (7), we can further specify equilibrium cheating behavior when $\eta \in (\eta_j, \eta_k)$; in particular, we can determine which player cheats with a higher probability in the region

\(^8\)Figure 1 omits the two asymmetric pure-strategy equilibria in the interval $(\eta_j, \eta_k)$. 

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where players mix.

**Proposition 2** There exists a unique $\eta^* \in (\eta_j, \eta_j)$ where $\hat{\rho}_j = \hat{\rho}_k$. When $\eta \in (\eta_j, \eta^*)$, player $k$ cheats with higher probability, or $\hat{\rho}_k > \hat{\rho}_j$. When $\eta \in (\eta^*, \eta_j)$, player $j$ cheats with higher probability, or $\hat{\rho}_j > \hat{\rho}_k$.

Proposition 2 says that when the audit probability lies in the interval $(\eta_j, \eta^*)$, the trailing player cheats with a higher probability than the leading player, but when the audit probability lies in the interval $(\eta^*, \eta_j)$, the leading player cheats with a higher probability. Recall that mixed-strategy equilibrium probabilities are determined by rendering the opponent indifferent between his strategies. The trailing player cheats with a higher probability than the leading player when the audit probability is relatively low because this is required to make the leading player indifferent between cheating and not. The leading player has a stronger incentive to cheat when the audit probability is low, and therefore the trailing player must cheat with a higher probability to elicit indifference. When the audit probability lies in the interval $(\eta^*, \eta_j)$, the logic is reversed as the leading player cheats with a higher probability to elicit indifference on the part of the trailing player. Also, note that while the result that the leading player may have more incentive to cheat than the trailing player is somewhat unexpected, it is similar to results found in Berentsen (2002), Haugen (2004), and Kräkel (2007).

The equilibrium strategies in Figure 1 resemble those Berentsen’s (2002) Figure 2, which illustrates cheating behavior as a function of the penalty under IOC (International Olympic Committee) regulations, in which all contestants found to have cheated are equally penalized regardless of their rank. There is an important difference, however. We find that the proba-
bility of cheating on the part of the trailing player is *decreasing* in the audit probability, as reason suggests, whereas Berentsen finds the counterintuitive result that the probability of cheating on the part of the trailing player is *increasing* in the size of the penalty.

An important question our model allows us to address is how asymmetry in contestants’ positions affects their incentive to cheat and the corresponding difficulty (or enforcement cost) of deterring cheating. In other words, we can analyze whether cheating is more likely (and hence whether deterrence is more costly) in a lopsided contest or a tight contest, and in the limit, a symmetric contest. Recall that the audit probability required to render not cheating a dominant strategy is higher for the trailing player, $k$, than for the leading player, and therefore to completely deter cheating the probability of audit must be $\eta \geq \eta_k$. As $\alpha$ increases from zero (starting at a symmetric contest), $\eta_k$ increases until at least $\alpha = x/2$ and eventually begins decreasing. At some point, when the size of the lead becomes large enough, the audit probability required to deter all cheating actually falls below the threshold in the symmetric game (see equation (3)), implying that there is more incentive to cheat in the symmetric game. This leads to the following result, which indicates that asymmetry in contestants’ positions generally requires that a contest organizer increase enforcement resources to ensure a "clean" contest because the trailing contestant becomes more tempted to cheat as his disadvantage grows, unless the differential becomes sufficiently large relative to the potential gain from cheating.

**Proposition 3** For a given gain in output from cheating ($x$), as the difference in contestants’ positions ($\alpha$) increases from zero, the minimum audit probability required to completely deter cheating is increasing and hence is greater than the corresponding threshold in the symmetric
game. However, if a contest is sufficiently "lopsided" (α sufficiently greater than \( x \), or \( \alpha \gg x \)), then the audit probability required to completely deter cheating is lower than that in a symmetric contest.

While it is not our emphasis here, it is worth noting how the equilibrium of the game depends on parameters other than \( \alpha \). All the critical values, \( \eta_k, \bar{\eta}_j, \bar{\eta}, \bar{\eta}_k \), and thus also the cheating probabilities in mixed-strategy equilibrium, \( \hat{\rho}_j \) and \( \hat{\rho}_k \), are increasing in the gain in output due to cheating \( x \). The tournament organizer must monitor more intensely to deter cheating when the contestants stand to gain more from cheating, as one would expect. These values are decreasing in \( r/S \), the ratio of the outside penalty to the prize spread. When either the outside penalty is increasingly significant or prize spread shrinks, the organizer need not monitor as intensely to deter cheating, again as one would expect. This is consistent with the standard result first presented in Becker (1968), in which individuals can be deterred from criminal activities by either increasing the probability of getting caught or increasing the fine.

### 3 The Advantage of Correlated Audits

Throughout Section 2, we assume that the contest organizer uses random correlated audits such that either both players are audited (and disqualified if caught cheating) or neither player is audited. From a contest organizer’s perspective, the *ex ante* enforcement cost of a correlated audit with probability \( \eta \) is identical to that of auditing each player with an independent probability \( \eta \) (assuming no economies or diseconomies of scale regarding the number of players who are audited at a given time), and clearly each player faces the same
probability of being caught if he cheats under the two regimes. Nevertheless, the player’s expected payoffs are not quite identical in these two regimes, and we show in this section that random correlated audits are more effective at deterring cheating than random independent audits when enforcement resources are insufficient to achieve total deterrence.

When random independent audits are used, the payoff matrix is nearly identical to that in Section 2; the only payoffs that change are in the southeast cell which correspond to the case when both players cheat. Player $j$’s expected payoff to cheating when her opponent cheats is $G(\alpha)(1 - \eta)^2 + \eta(1 - \eta) S + w_2 - \eta r$, whereas $k$’s expected payoff to cheating when his opponent cheats is $G(-\alpha)(1 - \eta)^2 + \eta(1 - \eta) S + w_2 - \eta r$.

Under random independent audits, the expected payoffs to cheating when the opponent cheats increase relative to the correlated audits case because a cheater can win by default even when both players cheat. Under correlated audits, a cheater cannot win by default when both players cheat because if one player is audited, the other player is audited with probability one. Because the expected payoff to cheating increases when both players cheat under random independent audits, the following proposition states the advantage of using correlated audits.

**Proposition 4** The probability of audit necessary to fully deter cheating, $\eta_k$, is identical when audits are correlated or uncorrelated, but correlated audits reduce the mixed-strategy equilibrium probabilities of cheating for audit probabilities on the interval $(\eta_j, \eta_j)$.

The value of the upper thresholds are identical in both settings because this is determined by a contestant’s relative expected payoffs between cheating and not cheating when he believes his opponent will not cheat, and thus only his own probability of audit matters.
But if a contestant believes his opponent will cheat with some probability, correlated audits reduce the expected payoff from cheating and thus reduce probabilities of cheating in the mixed-strategy equilibrium.

4 Differential Auditing

In many contexts, a contest organizer may be constrained to using enforcement rules which monitor contestants equally, as we assume in the model as developed in Sections 2 and 3. However, we now relax that assumption and show that if it is feasible to audit the players with different probabilities, full deterrence can be achieved at a lower cost.

For simplicity, we assume here that $r = 0$. This avoids some considerable added complexity arising from differential probabilities between the two contestants of incurring $r$ when audit probabilities are conditional on final rank or initial position, but we anticipate that these results should also hold as long as $r$ is not too large. We first assume that auditing must occur prior to the conclusion of the contest (after cheating takes place but before the uncertainty is realized), which we will describe as ex ante auditing. In this case audit probabilities can be conditional only on contestants’ initial positions (trailing or leading).

The condition for full deterrence of cheating is that both players find it a best response to not cheat if they believe their opponent is not cheating. When audits are conducted ex ante but audit probabilities may differ between the players, the condition to fully deter cheating is simply that each player’s audit probability meets the critical value identified in equations (1) and (2) in Section 2 with $r = 0$: $\eta_j \geq \bar{\eta}_j,A \equiv \frac{G(a+x)-G(a)}{G(a+x)}$ and $\eta_k \geq \bar{\eta}_k,A \equiv \frac{G(-a+x)-G(-a)}{G(-a+x)}$.

The use of differential audit probabilities reduces the enforcement resources required to fully
deter cheating simply because $\bar{\eta}_{j,A} < \bar{\eta}_{k,A}$. If it is feasible to audit with different probabilities, the organizer can save on expected costs by reducing the probability that player $j$ is audited to $\bar{\eta}_{j,A}$ rather than $\bar{\eta}_{k,A}$.

Now, consider the case that auditing takes place ex post, or conditional on the contestants’ final rankings. The idea here is that it is less likely that the trailing player will win; if he does, it may raise suspicions that he cheated, and hence he should be audited with a higher probability than the leading player conditional on winning. In this case the organizer can audit the contestant who ranks first; the second-ranked contestant is audited only if the winner is disqualified (this is similar in spirit to testing policies in the Olympic Games, where all three medalists are tested, and testing continues through the rankings until three clean medalists are found). In this context, let $\eta_j$ be the probability that $j$ is audited if she wins, and similarly let $\eta_k$ be the probability $k$ is audited if he wins. Once again the condition for full deterrence is that $\eta_j > \eta_{j,P} \equiv \frac{G(\alpha+x) - G(\alpha)}{G(-\alpha+x)}$ and $\eta_k > \eta_{k,P} \equiv \frac{G(-\alpha+x) - G(-\alpha)}{G(\alpha+x)}$. However, there is an additional advantage to differential auditing. Not only is $\bar{\eta}_{j,P} < \bar{\eta}_{k,P}$, but $j$ will win more than half of the time in equilibrium (since neither player will cheat and $j$ has an advantage in the contest, $j$ will be audited with probability $G(\alpha)\bar{\eta}_{j,P}$).

These discussions prove the following proposition.

**Proposition 5** Suppose the contest organizer’s objective is to fully deter cheating and that $r = 0$. Ex post differential monitoring yields the smallest enforcement cost while ensuring full deterrence, followed by ex ante differential monitoring. Ex ante identical monitoring requires the greatest enforcement cost to achieve full deterrence.

This result bears similarities to Berentsen’s (2002) result on ranking-based sanctions. He
shows that rank-based punishments are both incentive compatible and individually rational, and that they achieve full deterrence at a lower cost than IOC regulations, which mandate equal fines for cheaters regardless of their rank.

5 Conclusion

There are several possible extensions for this model. They include considering tournaments with more than two players as well as multi-stage tournaments. When there are more than two contestants who each may have a different ability or "starting point" this greatly increases the complexity of the model. Another interesting extension may be the consideration of multi-stage tournaments. In this environment, players take actions to improve their chances of winning the tournament, but there are many periods in which they can cheat. This type of tournament is considered in Tsetlin, Gaba, and Winkler (2004).

In a different vein, one might consider a model which incorporates a variable to allow for the possibility that cheating may entail psychological costs even when one is not caught. Some individuals attribute cheating to "just being part of the game," and that in order to win, one must cheat, whereas others experience remorse for cheating and do not subscribe to the "win at all costs" mindset. The former individuals experience more utility than the latter conditional on having cheated, and hence integrity influences their optimal decisions. At its extreme, some individuals will sacrifice the chance to win in order to maintain their integrity.

Finally, one can investigate the optimal design of the tournament given the parameters of the model. This problem is demanding, because as others have pointed out, it is not entirely
clear what the contest organizer’s objective should be, and more specifically, it is not clear that the contest organizer wants to fully deter cheating. In the corporate world, violating regulations might make the firm more profitable unless cheating is detected. In a sports contest, fans prefer high-quality competition, and most fans also prefer a close contest over a blowout. To the extent that cheating increases the quality of competition or increases the likelihood of a "photo finish," the contest organizer may find it optimal to allow for a higher level of cheating, especially if cheating is costly only when it becomes public.
6 Appendix

6.1 Proof of Proposition 1

We first show the ranking that $\bar{\eta}_k < \bar{\eta}_j < \bar{\eta}_j < \bar{\eta}_k$. Since $G$ is the cdf of a distribution that is symmetric around its mean of zero then $G(\alpha) - G(-\alpha) > G(\alpha - x) - G(-\alpha - x)$; this can be rearranged to show $G(\alpha) - G(\alpha - x) > G(-\alpha) - G(-\alpha - x)$. Both (4) and (5) are of the form $y/(y+z)$, where $y > 0$ and $z = \frac{x}{S} + 1 > 0$. Then $y/(y+z)$ is increasing in $y$ proves that $\bar{\eta}_k < \bar{\eta}_j$. Similarly, again relying on the symmetry of the distribution represented by $G$, we have $G(\alpha) - G(-\alpha) > G(\alpha + x) - G(-\alpha + x)$. Rearrange this to show $G(-\alpha + x) - G(-\alpha) > G(\alpha + x) - G(\alpha)$. The numerator of (2) is greater than the numerator of (1), and since $G(\alpha + x) > G(-\alpha + x)$, the denominator of (2) is less than the denominator of (1). Hence, $\bar{\eta}_k > \bar{\eta}_j$. The relation $\bar{\eta}_j < \bar{\eta}_j$ is true when

$$\frac{G(\alpha) + \frac{x}{S}}{1 + \frac{x}{S}} < \frac{G(\alpha + x) - G(\alpha)}{G(\alpha) - G(\alpha - x)}.$$

This condition may be violated when $\alpha$ and $x$ are both sufficiently large; we ignore this possibility.

The expression for $\hat{\rho}_j$ determined by letting $\rho_j$ be the probability that player $k$ believes player $j$ will cheat. Player $k$’s expected payoffs from cheating and not cheating are:

$$\pi_k(\text{cheat}) = (1 - \rho_j)[(1 - \eta)G(-\alpha + x)S] + \rho_j[G(-\alpha)(1 - \eta)S] + w_2 - \eta r$$

$$\pi_k(\text{not cheat}) = (1 - \rho_j)[G(-\alpha)S] + \rho_j[\eta((1 - \eta)G(-\alpha - x) + \eta)S] + w_2$$
Setting $\pi_k(\text{cheat}) = \pi_k(\text{not cheat})$ and solving for $\rho_j$ yields the expression found in (6). One can similarly determine $\hat{\rho}_k$.

### 6.2 Proof of Proposition 2

**Proof.** First, we need to show that $\hat{\rho}_j$ and $\hat{\rho}_k$ are monotonically decreasing in $\eta$. We will show the former: $\hat{\rho}_j$ is decreasing and convex in $\eta$ (showing that $\hat{\rho}_k$ is decreasing and convex in $\eta$ is similar and hence omitted). We have

$$\hat{\rho}_j = \frac{(1 - \eta) G(-\alpha + x) - G(-\alpha) - \eta r/S}{(1 - \eta) [G(-\alpha + x) + G(-\alpha)] - G(-\alpha) [2 - \eta] + \eta}.$$

Denote the denominator of this expression $D = (1 - \eta) [G(-\alpha + x) + G(-\alpha - x)] - G(-\alpha) [2 - \eta] + \eta$ and the numerator $N = (1 - \eta) G(-\alpha + x) - G(-\alpha) - \eta r/S$. Note $D, N > 0$ when $\eta \in (\eta_j, \bar{\eta}_j)$. The partial derivative of $\hat{\rho}_j$ with respect to $\eta$ is

$$\frac{\partial \hat{\rho}_j}{\partial \eta} = \frac{D \cdot \frac{\partial N}{\partial \eta} - N \cdot \frac{\partial D}{\partial \eta}}{D^2}.$$  \hspace{1cm} (8)

where $\frac{\partial N}{\partial \eta} = -G(-\alpha + x) - \frac{r}{S} < 0$ and $\frac{\partial D}{\partial \eta} = [(1 - G(-\alpha + x)) + (G(-\alpha) - G(-\alpha - x))] > 0$. Hence, the numerator of (8) is negative, proving that $\hat{\rho}_j$ is decreasing in $\eta$.

The second derivative of $\hat{\rho}_j$ with respect to $\eta$ is

$$\frac{\partial^2 \hat{\rho}_j}{\partial \eta^2} = \frac{D^2 \left[ D \cdot \frac{\partial^2 N}{\partial \eta^2} - N \cdot \frac{\partial^2 D}{\partial \eta^2} \right] - 2 \left[ D \cdot \frac{\partial N}{\partial \eta} - N \cdot \frac{\partial D}{\partial \eta} \right] \cdot D \cdot \frac{\partial D}{\partial \eta} \cdot D}{D^4}. \hspace{1cm} (9)$$

Both $\frac{\partial^2 N}{\partial \eta^2}, \frac{\partial^2 D}{\partial \eta^2} = 0$, and $D \cdot \frac{\partial N}{\partial \eta} - N \cdot \frac{\partial D}{\partial \eta} < 0$ from (8). Furthermore, $D, \frac{\partial D}{\partial \eta} > 0$, so the
nominator of (9) is positive. This proves $\frac{\partial^2 \hat{\rho}_j}{\partial \eta^2} > 0$.

We cannot unambiguously determine the rank of $\hat{\rho}_j$ and $\hat{\rho}_k$. However, we have endpoint conditions: $\hat{\rho}_j = 0$ at $\bar{\eta}_k$, $\hat{\rho}_j = 1$ at $\bar{\eta}_k$, $\hat{\rho}_k = 0$ at $\bar{\eta}_j$, and $\hat{\rho}_k = 1$ at $\bar{\eta}_j$. We also know $\bar{\eta}_j < \bar{\eta}_k$, and $\bar{\eta}_k < \bar{\eta}_j$. Since both $\hat{\rho}_j$ and $\hat{\rho}_k$ are monotonically decreasing and convex in $\eta$, $\hat{\rho}_j$ and $\hat{\rho}_k$ can cross only once, so there must exist a unique $\eta^* \in (\bar{\eta}_j, \bar{\eta}_j)$ where $\hat{\rho}_j = \hat{\rho}_k$. The endpoint conditions also establish that $\hat{\rho}_k > \hat{\rho}_j$ when $\bar{\eta}_j < \eta < \eta^*$ and $\hat{\rho}_j > \hat{\rho}_k$ when $\eta^* < \eta < \bar{\eta}_j$.

### 6.3 Proof of Proposition 3

**Proof.** Suppose a given value of $x$. Starting at $\alpha = 0$, note that $\bar{\eta}_k = \bar{\eta}_s$. The threshold $\bar{\eta}_k$ is increasing in $\alpha$ as long as $\alpha < x/2$:

$$\frac{\partial \bar{\eta}_k}{\partial \alpha} = \frac{D \cdot \frac{\partial N}{\partial \alpha} - N \cdot \frac{\partial D}{\partial \alpha}}{D^2},$$

where $D = G(-\alpha + x) + \frac{r}{S} > 0$, $N = G(-\alpha + x) - G(-\alpha) > 0$, $\frac{\partial D}{\partial \alpha} = -g(-\alpha + x) < 0$, and $\frac{\partial N}{\partial \alpha} = -g(-\alpha + x) + -g(-\alpha)$. Since $g$ is symmetric around its mean of 0, $\frac{\partial N}{\partial \alpha} > (>)0$ when $\alpha < (>)\frac{x}{2}$. With certainty, $\frac{\partial \eta_k}{\partial \alpha} > 0$ when $\alpha < \frac{x}{2}$. As $\alpha$ increases beyond $\frac{x}{2}$, $\bar{\eta}_k$ reaches its maximum at $\alpha^*$, which is the value that solves

$$\left[G(-\alpha + x) + \frac{r}{S}\right] \left[-g(-\alpha + x) + -g(-\alpha)\right] = \left[G(-\alpha + x) - G(-\alpha)\right] \left[-g(-\alpha + x)\right],$$
and then begins decreasing. Even when \( \alpha = x \), however, the threshold for full deterrence in the asymmetric game is still greater than the threshold in the symmetric game:

\[
\bar{\eta}_k = \frac{G(0) - G(-x)}{G(0) + \frac{r}{S}} = \frac{G(x) - G(0)}{G(0) + \frac{r}{S}} > \bar{\eta}_s.
\]

There exists an \( \tilde{\alpha} \), however, such that when \( \alpha > \tilde{\alpha} \), \( \bar{\eta}_s > \bar{\eta}_k \). To show this, take the limit of \( \pi_k \) as \( \alpha \) approaches infinity:

\[
\lim_{\alpha \to \infty} \bar{\eta}_k = 0 < \bar{\eta}_s.
\]

So, if the contest is sufficiently lopsided (\( \alpha \) is large enough given \( x \)), more enforcement is needed to fully deter cheating in a symmetric contest than in an asymmetric contest. ■

### 6.4 Proof of Proposition 4

**Proof.** When audits are uncorrelated, player \( j \)'s expected payoffs from cheating and not cheating respectively are:

\[
\begin{align*}
\pi_j(\text{cheat}) &= (1 - \rho_k)[(1 - \eta)G(\alpha + x)S] + \rho_k[(G(\alpha)(1 - \eta)^2 + (1 - \eta)\eta)S] + w_2 - \eta r \\
\pi_j(\text{not cheat}) &= (1 - \rho_k)[G(\alpha)S] + \rho_k[((1 - \eta)G(\alpha - x) + \eta)S] + w_2.
\end{align*}
\]

The mixed-strategy equilibrium probability of cheating for \( k \) is found by equating \( \pi_j(\text{cheat}) \) and \( \pi_j(\text{not cheat}) \), which yields the following (the \( u \) superscript denotes the uncorrelated audit equilibrium)

\[
\hat{\rho}_k^u \equiv \frac{(1 - \eta)G(\alpha + x) - G(\alpha) - \eta r / S}{(1 - \eta)[G(\alpha + x) + G(\alpha - x)] - G(\alpha)[2 - 2\eta + \eta^2] + \eta^2}.
\]

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Recall that the equilibrium cheating probability with correlated audits is

\[
\hat{\rho}_k \equiv \frac{(1 - \eta)G(\alpha + x) - G(\alpha) - \eta r / S}{(1 - \eta)[G(\alpha + x) + G(\alpha - x)] - G(\alpha)[2 - \eta] + \eta}.
\]

Note that the numerator of these two expressions is identical. We show that the denominator of \(\hat{\rho}_k\) is greater than the denominator of \(\hat{\rho}_u\) and therefore that \(\hat{\rho}_k < \hat{\rho}_u\). To establish this we subtract the denominator of \(\hat{\rho}_k\) from the denominator of \(\hat{\rho}_u\):

\[
\{ (1 - \eta)[G(\alpha + x) + G(\alpha - x)] - G(\alpha)[2 - \eta] + \eta \} - \{ (1 - \eta)[G(\alpha + x) + G(\alpha - x)] - G(\alpha)[2 - 2\eta + \eta^2] + \eta^2 \} = \eta(1 - \eta)(1 - G(\alpha)) > 0.
\]

A directly analogous result holds for \(\hat{\rho}_j\). This establishes that ceteris paribus correlated audits result in lower mixed-strategy equilibrium cheating probabilities.
References


Figure 1 – equilibrium behavior for player $j$ and player $k$ as a function of $\eta$