Performance optimisation of laminar fully developed flow through square ducts with rounded corners

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Abstract

A study on combined first and second law based optimisation of thermal-hydraulic performance of laminar fully developed flow through square ducts with rounded corners has been presented in this paper. The objective functions have been considered according to suggestions of Webb and Bergles \cite{7}. Four specific geometric constraints have been imposed on the shape of the ducts and these ducts have also been subjected to three different thermal and (or) hydraulic constraints. Two different thermal boundary conditions have been considered and the correlations for friction factor and Nusselt numbers have been adopted from the study of Ray and Misra \cite{21}. The results obtained from the present study clearly show that the optimal duct geometry strongly depends on geometric and thermal-hydraulic constraints, as well as, the objective functions and hence, no general comment can be made with respect to the superiority of a particular geometry of the ducts. Nevertheless, the present study also shows that although entropy generation minimisation may be considered to be an important tool, one requires being careful in using it for thermal-hydraulic optimisation since it may lead to contradictory results for some of the performance evaluation criteria.

1. Introduction

Much effort in the past has been aimed at providing economical methods for improving the thermal performance of heat exchangers. Improvement is generally identified in terms of augmented heat transfer, which would require less surface area and consequently a smaller heat exchanger. At the same time, reduction in pumping power for a specified heat duty or approach temperature difference is also desirable. Various augmentation techniques, both active and passive, have been developed to achieve these objectives, and they are also well documented (see for example, Ray \cite{1}, for details). In general, heat transfer augmentation is brought about by disrupting the development of thermal boundary layers and therefore is associated with enhanced pumping power. Usually, the pressure drop and heat transfer characteristics of flow in enhanced ducts are highly nonlinear. As a result, it is almost impossible to make a general statement regarding superiority of any specific augmentation technique with respect to the overall thermal-hydraulic performance. Webb and Eckert \cite{2}, Bergles et al. \cite{3,4}, Bergles \cite{5}, Webb \cite{6} and Webb and Bergles \cite{7} have therefore introduced several performance evaluation criteria (PEC) for the assessment of overall improvement in thermal-hydraulic behaviour. However, since all of the criteria prescribed in these works are mainly based on first law of thermodynamics, none of these analyses takes into account the amount of entropy generated due to the overall process, although enhanced entropy generation is a natural consequence of increased temperature difference and pressure drop.

In order to overcome this drawback, Bejan \cite{8,9} has proposed an optimisation procedure, based on second law of thermodynamics, in which minimisation of irreversibility or entropy generation (Entropy Generation Minimisation method or EGM method), associated with an augmentation technique, is used as the objective function. Subsequently, Nelson and Bergles \cite{10} have further extended the optimisation analysis by treating the entropy generation as a variable, as suggested by Bejan \cite{8}. However, in many of their cases, Nelson and Bergles \cite{10} have evaluated only one objective function while treating the entropy generation as a constraint. PEC equations, based on both first and second law analyses have been developed by Zimparov and Vulchanov \cite{11}. These equations have been further extended by Zimparov \cite{12} in order to include the effect of fluid temperature variation along the...
length of a tubular heat exchanger. Similar studies have also been reported by Prasad and Shen [13] and Hesselgears [14].

In recent years, the EGM method, mentioned above, has been widely utilised in order to evaluate the relative merits of various augmentation techniques. Prasad and Shen [15] have carried out an experimental investigation to assess the effect of wire-coil inserts for forced convection heat transfer, using exergy analysis. This method has also been used by Lin and Lee [16] for optimisation of pin-fin array under cross flow and by Zimparov [17] in order to study the effectiveness of different inserts in tubes. In view of the brief literature review presented here, it is obvious that the EGM method combined with the brief literature review presented here, it is obvious that the EGM method, mentioned above, has been widely utilised in order to evaluate the relative merits of various augmentation techniques. Prasad and Shen [15] have carried out an experimental investigation to assess the effect of wire-coil inserts for forced convection heat transfer, using exergy analysis. This method has also been used by Lin and Lee [16] for optimisation of pin-fin array under cross flow and by Zimparov [17] in order to study the effectiveness of different inserts in tubes. In view of the brief literature review presented here, it is obvious that the EGM method combined with the

Compared to a circular tube, however, a polygonal duct (with number of sides = n) has a definite advantage as it offers more surface area than its conventional counterpart occupying the same volume. Furthermore, the ratio of surface area to volume (or, perimeter to cross-sectional area) increases with the decrease in n. Therefore, logically the triangular duct should be the most preferred geometry, followed by the square duct. Inspite of this advantage, however, the polygonal ducts are not in common use, probably because of the associated manufacturing difficulties and more importantly, because the sharp corners of these ducts eventually act as hot spots and offer poor heat transfer surfaces. As a result, rounding of corners appears to be a logical alternative. The geometry of such a rounded corner square duct is shown in Fig. 1. It may be observed from the figure that for \( r_c = 0 \), one obtains the geometry of a perfectly square duct, whereas, for \( r_c = a \), a perfectly circular duct is obtained. Therefore, in the range \( 0 \leq r_c \leq a \), with the increase in radius of curvature, one obtains a family of rounded corner ducts, whose shape changes from a perfectly square duct to that of a perfectly circular duct.

In this paper, an analysis has been carried out for the performance of a square duct with rounded corners, for single-phase, fully developed, laminar flow. The PEC criteria identified by Webb and Bergles [7] have been taken as objective functions and for each case, a second law analysis has been made in an attempt to find out an optimal operating point, i.e., a particular radius of curvature for the corners, which is advantageous from both first and second law point of view. In the present paper, only H1 (i.e., constant heat input per unit axial length and uniform temperature along the periphery of the duct at a given axial location) and H2 (i.e., constant axial, as well as, peripheral heat flux) boundary conditions have been considered. The correlations for friction factor and Nusselt number as functions of dimensionless radius of curvature \( R_c = r_c/a \) for such ducts have been taken from Laha [20] and Ray and Misra [21] and...
have been used for the evaluation of thermal-hydraulic performance. The friction factor correlation is given as follows,

\[ f\text{Re} = 14.226 \left[ 1 + \sum_{k=1}^{5} C_{\text{Re},k} \frac{R_k}{d} \right] \]

where, the constants are given as, \( C_{\text{Re},1} = 0.4316 \), \( C_{\text{Re},2} = -0.5549 \), \( C_{\text{Re},3} = 0.2067 \), \( C_{\text{Re},4} = 0.1451 \), and \( C_{\text{Re},5} = -0.1040 \). The maximum error associated with the above correlation is reported to be around 0.007%. For H1 boundary condition, the corresponding Nusselt number is correlated as,

\[ Nu_{H1} = 3.608 \left[ 1 + \sum_{k=1}^{3} C_{H1,k} \frac{R_k}{d} \right] \]

The constants, appearing in Eq. (2), are given as \( C_{H1,1} = 0.4258 \), \( C_{H1,2} = -0.00093 \), \( C_{H1,3} = -0.7139 \), \( C_{H1,4} = 0.7976 \), and \( C_{H1,5} = 0.2909 \). The maximum error associated with Eq. (2) is around 0.007%. For H2 boundary condition, the corresponding Nusselt number correlation is reported as,

\[ Nu_{H2} = 3.102 \left[ 1 + \sum_{k=1}^{6} C_{H2,k} \frac{R_k}{d} \right] \]

where the constants are, \( C_{H2,1} = 0.5552 \), \( C_{H2,2} = 0.8649 \), \( C_{H2,3} = -3.1650 \), \( C_{H2,4} = 4.3328 \), \( C_{H2,5} = -3.0036 \), and \( C_{H2,6} = 0.8184 \) and the maximum error is around 0.006%. The exact numerical data points, obtained by Laha [20] and Ray and Misra [21], and the performance of these correlations are shown in Fig. 2(a) and (b) for friction factor and Nusselt numbers, respectively. From these figures, it may be observed that both \( f\text{Re} \) and \( Nu_{H1} \) (or, \( Nu_{H2} \)) are low for perfectly square duct and their values increase with the increase in radius of curvature of the corners. Subsequently, with further increase in radius of curvature, these quantities asymptotically tend to their corresponding values for perfectly circular ducts.

2. Mathematical formulation

2.1. Expression for entropy generation

The second law of thermodynamics for the elemental control volume, shown in Fig. 3, under steady state flow (SSSF) condition through ducts of arbitrary cross section may be written as,

\[ ds_{\text{gen}} = \dot{m} ds - \dot{q} \frac{dx}{T + \Delta T} \]

(4)

Since the cross-sectional area for the flow does not change in the axial direction, under SSSF condition, the mass flow rate through the duct (\( \dot{m} \)) remains constant. For H1 boundary condition (i.e., constant heat input per unit axial length and uniform temperature along the periphery of the duct at a given axial location), \( \dot{q} \) is considered to be known and for H2 boundary condition (i.e., constant axial, as well as, peripheral heat flux), \( \dot{q} = \dot{q} \phi \), that can be calculated from the knowledge of wall heat flux and heated perimeter. The \( Tds \) relationship may now be considered, which is given as,

\[ Tds = \dot{h} \frac{dx}{\rho} - \dot{P} = c_{p}dT - \frac{1}{\rho} dp \]

(5)

The simplification of the above equation, using \( d\dot{h} = c_{p}dT \), is valid either for an ideal gas or for an incompressible fluid, for which, \( \dot{h} = \dot{h}(T) \). The first law of thermodynamics for the same elemental control volume is given as,

\[ \dot{q} dx = mc_{p}dT = h_{P}dx\Delta T \]

(6)

Substituting the value of \( ds \) from Eq. (5) and using Eq. (6), the expression for entropy generation in Eq. (4) may be written as;

\[ ds_{\text{gen}} = \dot{m} c_{p}dT \left[ \frac{1}{T} - \frac{1}{T + \Delta T} \right] - \frac{\dot{m} c_{p}dT}{\rho} dp \]

(7)

\[ ds_{\text{gen}} = \dot{m} c_{p}dT \left[ \frac{1}{T} \Delta T + \frac{\dot{m} c_{p}dT}{\rho} \right] = \dot{m} c_{p}dT \left[ \frac{1}{T} \Delta T + \frac{\dot{m} c_{p}dT}{\rho} \right] \]

(8)

Few comments are now in order. For a given flow and heat transfer situation, \( \rho \), \( \dot{m} \), \( c_{p} \), \( \Delta T \) and \( \Delta p \) (the last two, under the assumption of hydrodynamically and thermally fully developed flow with constant wall heat flux) remain constant. It may be recognised here that while integration of the first part on the right hand side of Eq. (8) is relatively straightforward, integration of the second part requires an expression for the bulk temperature as a function of axial location. This expression may be obtained by integrating Eq. (6) as follows:

\[ Tds_{\text{gen}} = \dot{m} c_{p} \frac{dT}{T_{2}} - \frac{\dot{m} \Delta p}{\rho L} dx \]

Fig. 1. Geometry of a square duct with rounded corners.

1 Eq. (9) may also be written in terms of Stanton Number, if one allows the heated and the wetted perimeters to be same, i.e., \( \rho_{h} = \rho_{w} \), Under this situation, Eq. (9) may be written as, \( T = T_{1} + 4S(\Delta T/\rho_{h}L) \).
\[ T = T_i + \frac{h_p \Delta T}{m c_p} x \]  

The total rate heat transfer, \( \dot{Q}_i \), on the other hand, is obtained by integrating Eq. (8) over the entire length of the duct as;

\[ \dot{Q} = \dot{m} c_p (T_e - T_i) = h_p \Delta T \]

Substituting \( T \) from Eq. (9), Eq. (8) may be integrated and after carrying out some algebraic manipulations, the total entropy generation for the duct is obtained as;

\[ \dot{S}_{gen} = \dot{Q} \frac{\Delta T}{T_i} + \left( \frac{\dot{m} c_p}{\rho T_i} \right) \ln \left[ 1 + \left( \frac{\Delta T}{T_i} \right) \right] \]

In the above expression, \( \Delta T = T_e - T_i \) denotes the temperature rise (or drop, as the case may be) over the length of the heat exchanger. It may be recognised here that the above expression is the most general one and it does not assume any particular form for \( \dot{Q}, \rho, c_p \) or, \( T_i \). \n
2.2. Further simplification

The pressure drop for fully developed forced convective flow through horizontal ducts can be expressed in terms of the friction factor and since the correlation for \( fRe \) is already available for the present configuration, one obtains;

\[ \frac{\Delta p}{\rho} = \frac{2 f L u_o^2}{D_h} = \frac{2 \mu m fRe L}{\rho^2 A_i D_h^2} \]

Further, neglecting higher order terms in the expansion of \( \ln[1 + (\Delta T)] \), such that \( \ln[1 + (\Delta T)]/(\Delta T) \approx 1 \), and using Eq. (12), one may rewrite Eq. (11) as;

\[ \dot{S}_{gen} = \frac{\dot{Q} \Delta T}{T_i (T_e/T_i)} + \frac{2 \mu m fRe L}{\rho^2 A_i D_h^2 T_i} \]

From Eq. (13) it may be easily recognised that the total entropy generation consists of two distinct components. The first term on the right hand side denotes the entropy generation arising out of heat transfer through finite temperature difference and the second term signifies the entropy generation due to frictional losses, i.e., the dissipative work done in the system. Therefore, Eq. (13) may also be written as,

\[ \dot{S}_{gen} = \dot{S}_{gen, \Delta T} + \dot{S}_{gen, \Delta p} \]

where, \( \dot{S}_{gen, \Delta T} \) and \( \dot{S}_{gen, \Delta p} \) are defined as follows,

\[ \dot{S}_{gen, \Delta T} = \frac{\dot{Q} \Delta T}{T_i (T_e/T_i)} \]

\[ \dot{S}_{gen, \Delta p} = \frac{2 \mu m^2 fRe L}{\rho^2 A_i D_h^2 T_i} \]

The augmentation entropy generation number, \( N_a \), due to Bejan [8,9], may now be defined as follows;

\[ N_a = \frac{(\dot{S}_{gen})_A}{(\dot{S}_{gen})_0} = \frac{N_T + \phi_D N_P}{1 + \phi_D} \]

where, the suffix ‘A’ and ‘0’ stand for the ‘augmented’ and ‘original’ configurations, respectively. Further, \( \phi_D \) is the irreversibility distribution ratio of the original duct whereas, \( N_T \) and \( N_P \) are the entropy generation ratios due to heat transfer and friction, respectively. These quantities are defined as,

\[ \phi_D = \frac{(\dot{S}_{gen, \Delta p})_A}{(\dot{S}_{gen, \Delta T})_0} N_T = \frac{(\dot{S}_{gen, \Delta T})_A}{(\dot{S}_{gen, \Delta T})_0} \] and \( N_P = \frac{(\dot{S}_{gen, \Delta p})_A}{(\dot{S}_{gen, \Delta p})_0} \)

2 According to this form, the definitions of these numbers could be expressed either on the basis of the hydraulic diameter, or, on the basis of the length scale of the original (unaugmented) tube, which is quite common in the enhanced heat transfer literature.
Since the same working fluid, with same thermo-physical properties and inlet temperature, is considered for both original (circular) and augmented (square duct with rounded corners) configurations, \( N_I \) and \( N_P \) may be further expressed as,

\[
N_I = \frac{Q^* \Delta T^*}{T_{e0}} \quad \text{(18a)}
\]

\[
N_P = \frac{m^*^2 (\Delta fR_e^*)^2}{\lambda^* \Lambda_D^2} \quad \text{(18b)}
\]

where, \( T_{e0}^* \) is the ratio of exit temperatures for the augmented and original configurations and may be obtained from the energy balance as follows;

\[
T_{e0}^* = \frac{T_i}{T_{e0}} + \left(1 - \frac{T_i}{T_{e0}}\right) \frac{Q^*}{m^*} \quad \text{(19)}
\]

Obviously, evaluation of \( T_{e0}^* \) requires the value of \( T_i/T_{e0} \), i.e., the ratio of inlet to outlet temperatures for the original duct, other than \( Q^* \) and \( m^* \). It is also evident that for \( Q^* = 1 \) and \( m^* = 1 \), \( T_{e0}^* \) is also equal to unity. The total rate of heat transfer, \( Q_t \), is already given by Eq. (10). This can also be expressed in terms of Nusselt number, and hence, the ratio of heat transfer for the rounded corner square duct and the circular duct, \( Q^* \), may be expressed as;

\[
Q^* = \frac{Nu \lambda^* \Lambda_D \Delta T^*}{Dh} \quad \text{(20)}
\]

It is evident that \( Q^* \) is directly related to \( \Delta T^* \). Wherever required, the same equation may be used to calculate \( \Delta T^* \). Another quantity of interest, while dealing with performance evaluation, is the pumping power, given as, \( P = \Delta \rho \text{Pe} \text{ul}_{\text{dns}} \). Using the expression for \( \Delta \rho \) from Eq. (12), the ratio of pumping powers may be written as;

\[
P^* = \frac{m^* \alpha (\Delta fR_e^*)^2}{\Lambda_D^2} \quad \text{(21)}
\]

For a given pumping power, however, \( m^* \) can be calculated from Eq. (21). Comparing Eqs. (18b) and (21), and with the help of Eq. (18a), one may now rewrite the expression for augmentation entropy generation number in Eq. (16) as;

\[
N_S = \frac{(\frac{Q^* \Delta T^*}{T_{e0}^*}) + \phi_0 P^*}{1 + \phi_0} \quad \text{(22)}
\]

Quite obviously, as \( \phi_0 \to 0 \), \( N_S \to N_I = \frac{Q^* \Delta T^*}{T_{e0}^*} \). It may be recognised here that for same pumping power, \( N_S \) is only a strong function of \( N_I \) and a relatively weak function of \( \phi_0 \) (since the values of \( \phi_0 \) are quite less). Further, for same \( \Delta T^* \), \( N_S \) increases with the increase in \( Q^* \), whereas, for same \( Q^* \), \( N_S \) decreases with the decrease in \( \Delta T^* \).

2.3. Geometric relations

The geometry of the rounded corner square duct is already shown in Fig. 1. The cross-sectional area, the heated perimeter and the hydraulic diameter can be completely expressed in terms of \( R_c \) and \( \alpha \) as follows;

\[
A_c = \alpha^2 [4 - (4 - \pi)R_c^2] \quad \text{(23a)}
\]

\[
P_h = a[8 - (8 - 2\pi)R_c] \quad \text{(23b)}
\]

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Types of ducts considered for the present study.</th>
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<tbody>
<tr>
<td>Duct type</td>
<td>Constraint</td>
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<tr>
<td>Duct-I</td>
<td>Same physical (linear) space</td>
</tr>
<tr>
<td>Duct-II</td>
<td>Same hydraulic diameter</td>
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<tr>
<td>Duct-III</td>
<td>Same heat transfer area</td>
</tr>
<tr>
<td>Duct-IV</td>
<td>Same cross-sectional area</td>
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\[
D_h = a \left[ \frac{4 - (4 - \pi)R_c^2}{2 - (2 - \pi/2)R_c} \right] \quad \text{(23c)}
\]

All these quantities may be normalised with respect to those of the circular counterpart (\( A_c = \pi/4 \alpha^2 \), \( R_{ch} = \pi d \) and \( D_{ho} = d \)) and may also be expressed as,

\[
A_c^* = \frac{4[4 - (4 - \pi)R_c^2]}{\pi} \left( \frac{a}{d} \right)^2 \quad \text{(24a)}
\]

\[
P_h^* = \frac{8 - (8 - 2\pi)R_c}{\pi} \left( \frac{a}{d} \right) \quad \text{(24b)}
\]

\[
D_h^* = \frac{4 - (4 - \pi)R_c^2}{2 - (2 - \pi/2)R_c} \left( \frac{a}{d} \right) \quad \text{(24c)}
\]

These equations clearly show that the normalised cross-sectional area, heated perimeter and hydraulic diameter depend on \( R_c \) and the \( a/d \) ratio. Since the main purpose of the present study is to ascertain the effect of \( R_c \) on the performance of square ducts with rounded corners, one requires information regarding ratio, \( a/d \). This may be obtained from the imposed geometrical constraints. These constraints are identified in Table 1. It is evident from Table 1 that in each case, \( a/d \) appears as a particular function of \( R_c \), or, as a constant (as for Duct-I) and hence, the results are presented for these ducts with \( R_c \) as the variable parameter.

2.4. Performance evaluation criteria

Various performance evaluation criteria, as suggested by Webb and Bergles [7], are considered for the present study and they are listed in Table 2. It may be noted that these criteria are based on first law analysis and for each of them there are some fixed parameters and one objective function. In the present analysis, in addition to the objective functions listed in Table 2, the entropy generation number is also considered as another objective function, based on second law analysis.

3. Results and discussions

In the present paper, performances of rounded corner square ducts, based on first and second law analyses, are evaluated for both
H1 and H2 boundary conditions. For each of these boundary conditions, four different geometric constraints are imposed as listed in Table 1. While obtaining the entropy generation number, the irreversibility distribution ratio for the circular configuration, $f_O$, is varied from $10^{-1}/C_0^1$ to $10^{-3}/C_0^3$, whereas, $T_i/T_{eo}$ is kept constant at 0.8. In this section, results are first presented for H1 boundary condition followed by those for H2 boundary condition.

3.1. Results for H1 boundary condition

3.1.1. Evaluation criterion: FG-1a

The objective function for the performance evaluation criterion FG-1a is the enhancement of heat transfer, which is presented in Fig. 4(a). The entropy generation numbers for different ducts are also presented in Fig. 4(b)–(d) for $f_O = 10^{-1}$, $f_O = 10^{-2}$ and $f_O = 10^{-3}$ respectively. The figure shows $Q'$ is maximum for a perfectly square duct, indicating the best performance, whereas, it is minimum for the circular duct. For any value of $R_c$, $Q'$ is always greater than unity which implies that the rounded corner square ducts always perform better than the circular one and with the increase in $R_c$, $Q'$ gradually decreases. It may be noted from Eq. (24) that since both $P_h'$ and $D_h'$ are linear functions of $a/d$ ratio, $P_h'/D_h'$ and hence, $Q'$ for same $\Delta T'$ (see Eq. (20)), are independent of $a/d$ ratio and are functions of only $R_c$. As a result, the variations of $Q'$ for all the four ducts converge into a single curve. The second law based criterion, $N_s$, on the other hand, shows some interesting trends. In general, $N_s$ is always greater than unity for perfectly square duct, irrespective of the geometric constraints and the value of $f_O$. This clearly indicates that the performance of the square duct is poor from the viewpoint of second law. For $f_O = 10^{-1}$, however, a range of $R_c$ is observed ($R_c \geq 0.46$), only for Duct-I, where the entropy generation number is less than unity. This observation may be attributed to the reduction in pressure drop irreversibility for Duct-I in the given range of $R_c$. The figure also shows that for lower

Fig. 4. Variations of objective function and $N_s$ with $R_c$ for FG-1a criterion and H1 boundary condition. (a) $Q'$, (b) $N_s$ for $f_O = 10^{-1}$, (c) $N_s$ for $f_O = 10^{-2}$, (d) $N_s$ for $f_O = 10^{-3}$.
values of $\phi_0$, the variations in $N_S$ for all the four ducts become negligible, which is also evident from the discussion presented in the mathematical formulation section. It may be noted here that for FG-1a criterion, since $m^*$ is fixed, $T_r^*$ is a weak function of $\dot{Q}$ (i.e., $R_c$, see Eq. (19)) and hence, according to Eq. (18a), $N_T$ (as well as $N_S$) increases with the increase in $\dot{Q}$ for constant $D_T^*$. As a result, the augmentation entropy generation number is generally more than unity for almost all the cases under consideration. In view of this, it appears that since the present analysis deals only with the internal irreversibilities, the second law based criterion is probably inappropriate for evaluation of performances.

3.1.2. Evaluation criterion: FG-1b

Results for PEC–FG-1b are shown in Fig. 5, which clearly show the most encouraging result on the basis of both first and second law. The objective of the criterion FG-1b is to obtain reduced approach temperature difference, which according to Eq. (20) and for the given constraints, is inverse of the objective function for FG-1a criterion. The variation in the objective function is shown in Fig. 5(a). It may be observed from the figure that the reduction in approach temperature difference is obtained over the entire range of $R_c$, the reduction being most effective for a perfectly square duct. The square duct also turns out to be the most advantageous from the principles of entropy generation minimisation, particularly for Ducts-I, II and IV. On the other hand, for higher values of $f_0$, the entropy generation number for Duct-III shows a distinct minimum around $R_c \approx 0.48$ and a range of $R_c$ is observed within which $N_S$ is less than unity. Once again, this is clearly the effect of pressure drop irreversibility which is prominent for higher values of $\phi_0$.

3.1.3. Evaluation criterion: FG-2a

The objective function for criterion FG-2a is to achieve increased heat duty, $\dot{Q}$, and the curve has exactly the same characteristics as observed for the case FG-1a since $\dot{Q}$ for $L = 1$ does not depend upon $m^*$ or $P^*$ and is already shown in Fig. 4(a). The variations of augmentation entropy generation number $N_S$ with $R_c$ for $\phi_0 = 10^{-1}$,
on the other hand, show marked differences, which may be attributed to the variations in \( T_e \) arising out of changes in mass flow rates other than the variations in the objective function itself. As Fig. 6 shows, use of all the ducts, except Duct-III, results in an entropy generation number that is greater than unity for any value of \( R_c \), indicating that the thermal-hydraulic performance of these ducts are poorer in comparison to a duct of circular cross section from a second law point of view. However, for Duct-III, \( N_S \) is minimum for a perfectly square duct, which increases with increase in \( R_c \). It is also observed that below \( R_c = 0.34 \), the performance of the rounded corner ducts is better than their circular counterpart. Since in this case \( P^* \) is fixed and \( N_T \) is a weak function of \( \phi_0 \), the results are not shown for other values of \( \phi_0 \).

3.1.4. Evaluation criteria: FG-2b and FN-1

The objective function for FG-2b criterion is to achieve reduced approach temperature difference for a constant heat duty, pumping power and length of the heat exchanger and as hence is identical to that observed for FG-1b criterion (see Fig. 5(a)). The objective for FN-1 criterion, on the other hand, is to achieve reduction in the length of the tube keeping the approach temperature difference, pumping power, as well as, the heat duty constant. As evident from Eq. (20), the expression for \( L^* \) with \( \Delta T^* = 1 \) reduces to that for \( \Delta T^* \) with \( L^* = 1 \). Hence the objective function has exactly the same characteristics as that for FG-1b and FG-2b criteria as may be observed from Fig. 6. The variations in \( N_S \) for PEC-FG-2b and FN-1 are presented in Fig. 7(a) and (b), respectively, for \( \phi_0 = 10^{-1} \). Since the variations for other lower values of \( \phi_0 \) are similar to each other (as explained before), they are not shown here.

It is evident from the figure that all the four ducts do not offer the same level of advantage, while considering entropy generation minimisation as the objective. Ducts-III and IV, which show better performance based on second law for any value of \( R_c \), have the least value of \( N_S \) for the square duct. For FG-2b criterion, Duct-II, however, shows that a definite radius of curvature exists (\( R_c \approx 0.26 \)), where the entropy generation number is minimum and hence, the best thermal-hydraulic performance is obtained from the viewpoint of second law. It is also observed that the performance of Duct-II is always better than that of the circular duct, irrespective of \( R_c \). Duct-I, on the other hand, shows a rapid increase in the value of \( N_S \), which eventually becomes greater than unity and thereby indicating that there exists a threshold value of the radius of curvature of the corners, \( R_c \approx 0.50 \), after which the duct provides a poorer thermal-hydraulic performance in comparison to a reference circular duct.

For FN-1 criterion, on the other hand, Ducts-I and II perform poorly in comparison with the circular duct, \( N_S \) being greater than unity for all values of \( R_c \). Once again, these observations could be attributed to the variations in \( T_e^* \) for different cases under consideration.

3.1.5. Evaluation criteria: FN-2 and FN-3

The objective functions for FN-2 and FN-3 criteria are to obtain reduced tube length and pumping power, respectively, for same set
of constraints. Since \( L^* \) for these cases is calculated from Eq. (20), it remains identical to the one obtained for FN-1 criterion and hence is not shown here. The pumping power \( P^* \), on the other hand, shows some interesting variations, as presented in Fig. 8(a). It may be observed from Eq. (21) that \( P^* \) is a function of as well as the \( a/d \) ratio. Hence four distinct curves are obtained for four different geometric constraints in contrast to a single curve obtained for the earlier cases. A careful look into Eq. (18a) reveals that the value of \( N_T \) for these cases is equal to unity since other than \( _m^* \) and \( _e^* \), \( T^* \) also turns out to be unity for \( m^* = 1 \). For this reason, it may be noted that the general pattern for the objective functions and the entropy generation curves are similar to each other, particularly for higher values of \( \phi_0 \) (see Fig. 8(b)). It may be further noted that \( N_s \) converges to unity for progressively lower values of \( \phi_0 \), and hence, these results are not shown here. As far as performances of four ducts are concerned, under the present criterion, Ducts-I and II perform best in terms of minimisation of \( P^* \) as well as \( N_s \). Best results are, however, obtained for the perfectly square duct.

### 3.1.6. Evaluation criterion: VG-2a

The objective for the performance evaluation criterion VG-2a is again to achieve reduction in the length of the tubing, i.e., to

![Fig. 8. Variations of objective function and \( N_s \) with \( R_c \) for FN-2 and FN-3 criteria and H1 boundary condition. (a) \( P^* \), (b) \( N_s \) for \( \phi_0 = 10^{-1} \).](image)

![Fig. 9. Variations of objective function and \( N_s \) with \( R_c \) for VG-2a criterion and H1 boundary condition. (a) \( L^* \), (b) \( N_s \) for \( \phi_0 = 10^{-1} \).](image)
achieve more compact heat exchangers. It is clear from Eq. (21) that for $P^* = 1$ and $m^* = 1$, $L^*$ depends upon the $a/d$ ratio, other than the radius of curvature and hence, different values are obtained for different duct geometries. From Fig. 9(a), it may be observed that use of Ducts-III and IV can effectively reduce the size of the heat exchanger, a perfectly square duct of type III being the most advantageous. It may be further noted that the extent of reduction that may be achieved is much superior in this case than any of the earlier cases with almost 50% reduction in length being possible. The general trend of the variation of $N_S$ with $R_c$, as presented in Fig. 9(b), is similar to that observed for the PEC–FN-1 (see Fig. 7(b)), but quantitatively the extent of variation in $N_S$ among the four ducts is much more pronounced in this case. Since $Q^*$ is a linear function of $L^*$ and $N_S$ is a strong function of $Q^*$, Ducts-III and IV shows better performance even from the viewpoint of second law, while Ducts-I and II exhibit the opposite.

3.1.7. Evaluation criterion: VG-2b

The results for objective function for the VG-2b criterion is shown in Fig. 10. It may be observed from the figure, that Ducts-I and II are effective in reducing both $\Delta T^*$ as well as $N_S$, with the best performance being offered by the perfectly square ducts. In this case, $\Delta T^*$ is inversely proportional to $L^*$, the latter remaining same as that obtained for the VG-2a criterion due to same imposed

![Fig. 10. Variations of objective function and $N_S$ with $R_c$ for VG-2b criterion and H1 boundary condition. (a) $\Delta T^*$, (b) $N_S$ for $\phi_0 = 10^{-1}$.](image)

![Fig. 11. Variations of objective function and $N_S$ with $R_c$ for FG-1a criterion and H2 boundary condition. (a) $Q^*$, (b) $N_S$ for $\phi_0 = 10^{-1}$.](image)
constraint on mass flow rate and pumping power. Further, since $T_e^* = 1$ (see discussion on FN-2 and FN-3 criteria), the numerical values of $N_f$ equal those of $\Delta T^*$. Due to this reason, the variations in $\Delta T^*$ and $N_f$ appear to be quite similar to each other.

3.2. Results for H2 boundary condition

It has already been shown in Fig. 2(b) that the average Nusselt number for H2 boundary condition is lower than that for H1 boundary condition. As a result, the ducts with H2 boundary conditions are expected to behave poorly from the viewpoint of first law of thermodynamics. The general nature of variations and their reasons have already been discussed earlier in this section. In the subsequent part, only the results for H2 boundary condition are presented for the sake of completeness, without going into details.

3.2.1. Evaluation criterion: FG-1a

The results for FG-1a criterion are presented in Fig. 11. It is evident from the figure that the objective function shows poorer performance for rounded corner ducts. On the other hand, $N_f$ shows that the performance of the rounded corner ducts is always better than their circular counterpart, except for Duct-III, particularly for $\phi_0 = 0.1$. For further lower values of $\phi_0$, since all the curves tend towards the one for Duct-II, they are not shown here.

3.2.2. Evaluation criterion: FG-1b

The results for the criterion FG-1b, presented in Fig. 12, suggest that the rounded corner square ducts offer quite poor performance with respect to the reference circular duct, as far as the objective function $\Delta T^*$ is concerned. From the viewpoint of second law, however, Duct-I offers a region, $R_C \geq 0.38$, for $\phi_0 = 0.1$, where $N_f$ is less than unity and hence in this region, the performance of the rounded corner duct is better than its circular counterpart from the second law viewpoint. For all the other ducts, $N_f$ is always greater than unity over the entire range of $R_C$.

3.2.3. Evaluation criterion: FG-2a

The augmentation entropy generation number for FG-2a criterion is presented in Fig. 13. It may be noted, that $Q_e^*$ being less than unity for all values of $R_C$, as shown in Fig. 11(a), the objective function does not show any advantage. However, all the ducts offer a certain amount of advantage in terms of entropy generation, with the perfectly square duct being the most efficient. It may also be observed that $N_f$ for all the ducts is less than unity, irrespective of $R_C$, except for Duct-I, for which a region may be identified ($R_C \geq 0.28$), where $N_f$ is greater than unity.

3.2.4. Evaluation criteria: FG-2b and FN-1

The results for FG-2b criterion, presented in Fig. 14(a), show that only Duct-III has a certain range of $R_C$ ($R_C \geq 0.32$) for which $N_f$ is less than unity. Similar results for the FN-1 criterion are presented in Fig. 14(b).
Fig. 14(b). They indicate that use of Ducts-I and II results in values of $N_s$, those are consistently greater than unity, while $N_s$ for Ducts-III and IV is always less than unity, the minimum being obtained for the perfectly square ducts.

3.2.5. Evaluation criteria: FN-2 and FN-3

As mentioned before, the objective function for FN-2 criterion remains the same as that obtained for FN-1 criterion. However, it may be observed from Fig. 15 that for FN-3 criterion, where reduced pumping power is the objective, the curves show quite different trend. It may also be noted that particularly for this case, two ducts (namely Ducts-I and II) are obtained which provide a better performance than the reference circular duct in terms of first law objective, which has not been observed for any other case with H2 boundary condition. Interestingly, Duct-I also offers a definite radius of curvature ($R_c = 0.36$), where least pumping power is required. For $\phi_0 = 0.1$, variations in $N_s$ also show a similar trend and with further decrease in $\phi_0$, $N_s$ tends to unity due to the reasons described before.

3.2.6. Evaluation criterion: VG-2a

A unique feature of the VG-2a criterion is that for this set of constraints, the variation of $L^*$ with $R_c$ is the same for both H1 and H2 boundary conditions (see Fig. 16). The reason for this invariance may

![Fig. 14. Variations of $N_s$ with $R_c$ for $\phi_0 = 10^{-1}$ and H2 boundary condition. (a) FG-2b criterion (b) FN-1 criterion.](image1)

![Fig. 15. Variations of objective function and $N_s$ with $R_c$ for FN-2 and FN-3 criteria and H2 boundary condition. (a) $P^*$, (b) $N_s$ for $\phi_0 = 10^{-1}$.](image2)
be attributed to the fact that for this case \( L^* \) is only functions of \((fRe)^*\) and the ratio of geometric properties. Since \( Nu \) is the only parameter that is affected by the boundary conditions, \( L^* \) remains the same for both H1 and H2 type of applications. From the viewpoint of second law, Ducts-III and IV always provide better performance, with \( NS \) reaching a minimum for the perfectly square duct. Ducts-I and II, on the other hand, show poorer performance since \( NS \) is always greater than unity, irrespective of the value of \( Rc \). These observations are consistent with the variations in the objective function.

### 3.2.7. Evaluation criterion: VG-2b

The objective function for the VG-2b criterion is reduced approach temperature difference, which is shown in Fig. 17(a). It can be observed that, in terms of both first law as well as second law objectives, Ducts-I and II, in contrast to Ducts-III and IV, offer better performance than the reference circular duct, over the entire range of \( Rc \). It may also be noted that for Duct-I, an optimal radius of curvature exists (\( Rc = 0.36 \)), for which the required approach temperature difference, as well as, the entropy generation rate are simultaneously minimum. For the case of Duct-II, however, a perfectly square duct appears to be most advantageous.

### 4. Conclusions

In the present paper, an analysis has been presented for combined heat transfer and pressure drop characteristics based
performance optimisation of a square duct with rounded corners, for single-phase laminar flow. The PEC criteria, identified by Webb and Bergles [7], have been taken as objective functions, however, importance has been laid on a second law analysis of the performances. Both H1 and H2 boundary conditions have been considered and the correlations for friction factor and Nusselt numbers have been taken from the study of Ray and Misra [21]. An attempt has also been made to find out optimal operating points, i.e., particular radii of curvature for the corners, those are advantageous from both first and second law point of view.

In general, it has been observed that, when first law objectives are considered, the rounded corner ducts produce better performance under H1 boundary conditions, rather than H2. For all the FG cases, the perfectly square ducts offer the best performance in terms of the first law objective function. However, when entropy generation is considered, no such generality applies. Four different geometric constraints have been considered in the present study and for these ducts, a variety of entropy generation curves have been obtained, with values of NS varying widely. For few cases (namely FG-1a, 1b and 2b for H1, and FG-1b and 2b for H2), optimum radii of curvature of the corners have also been obtained, for which the entropy generation number is minimum. For the rest of the cases, such as FN and VG, the results are even more varied, with both the objective function, as well as, NS ranging from about 0.6 to 1.8 in some cases. Optimum radii of curvature have been obtained for a limited number of cases, namely for FN-2, FN-3 and VG-2b criteria under H2 boundary conditions.

A careful analysis of the performances thus reveals that no general comment can be made regarding the superiority of one particular type of duct over the others. Rather, under different conditions (geometric constraints, objective functions, fixed operating parameters, etc.), different variety of ducts show optimum performance. Moreover, a number of situations have been encountered where a particular geometry could be identified to be the optimal from a first law analysis, which are not at all advantageous when entropy generation is considered as the performance evaluation criteria. Therefore, the results obtained from the present analysis also clearly highlight the fact that one requires to be careful in using the entropy generation minimisation as a tool for thermal-hydraulic optimisation since it may lead to contradictory results (for example, in FG-1a, FG-2a, FN-1 and FN-2 criteria).

References