Chapter 5
Income and Substitution Effects

Effects of Changes in Income and Prices on Optimum Consumer Choices

- As shown earlier for utility maximization, x^* (optimal x) is a function of prices and income:

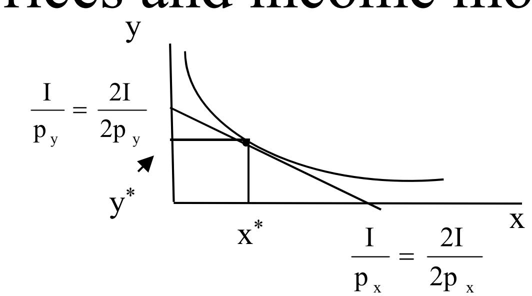
  $$x_i^* = x_i(p_{x_1}, p_{x_2}, ..., p_{x_n}, I); i = 1...n$$

  For two goods, it becomes

  $$x^* = x(p_x, p_y, I); \quad y^* = y(p_x, p_y, I)$$

- This chapter deals with this relationship.
- These functions can be derived from the constrained utility maximization problem from the last chapter.
- These are **<u>demand functions</u>** for goods for <u>an individual consumer</u>! If we know p_x, p_y, and I, we can calculate x^*, the optimal quantity of x demanded, for a given individual's utility function.

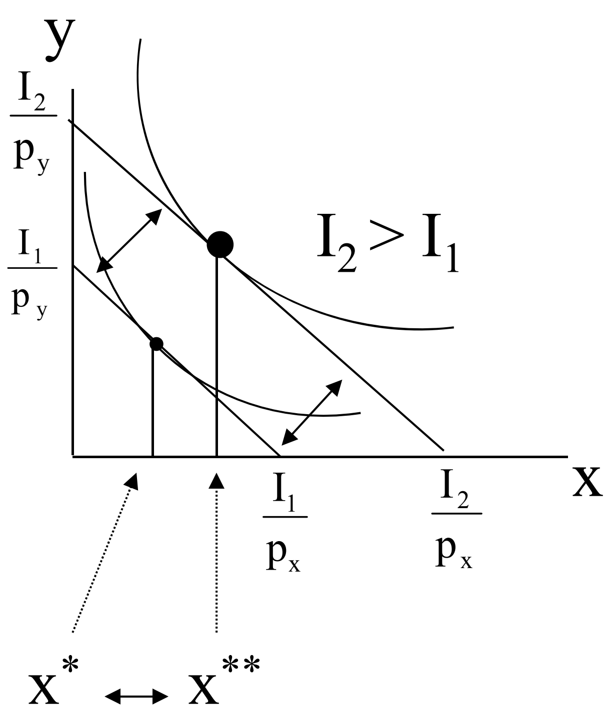
- Individual demand functions are ***homogeneous of degree zero*** in **all** prices and I. That is, if all prices and income double, x^* will not be affected. For example, if $I = p_x x + p_y y$, then $2I = 2p_x x + 2p_y y$.
  - On the graph of the budget constraint $\frac{I}{p_x}$ and $\frac{I}{p_y}$ will not change if both numerator and denominator are multiplied by 2 or any other constant (t).
  - Preferences do not change, ie., indifference curves do not change with changes in prices or income.
  - So if all prices and income move together, no change in x^* and y^*.



Pure inflation does not affect choices among goods.

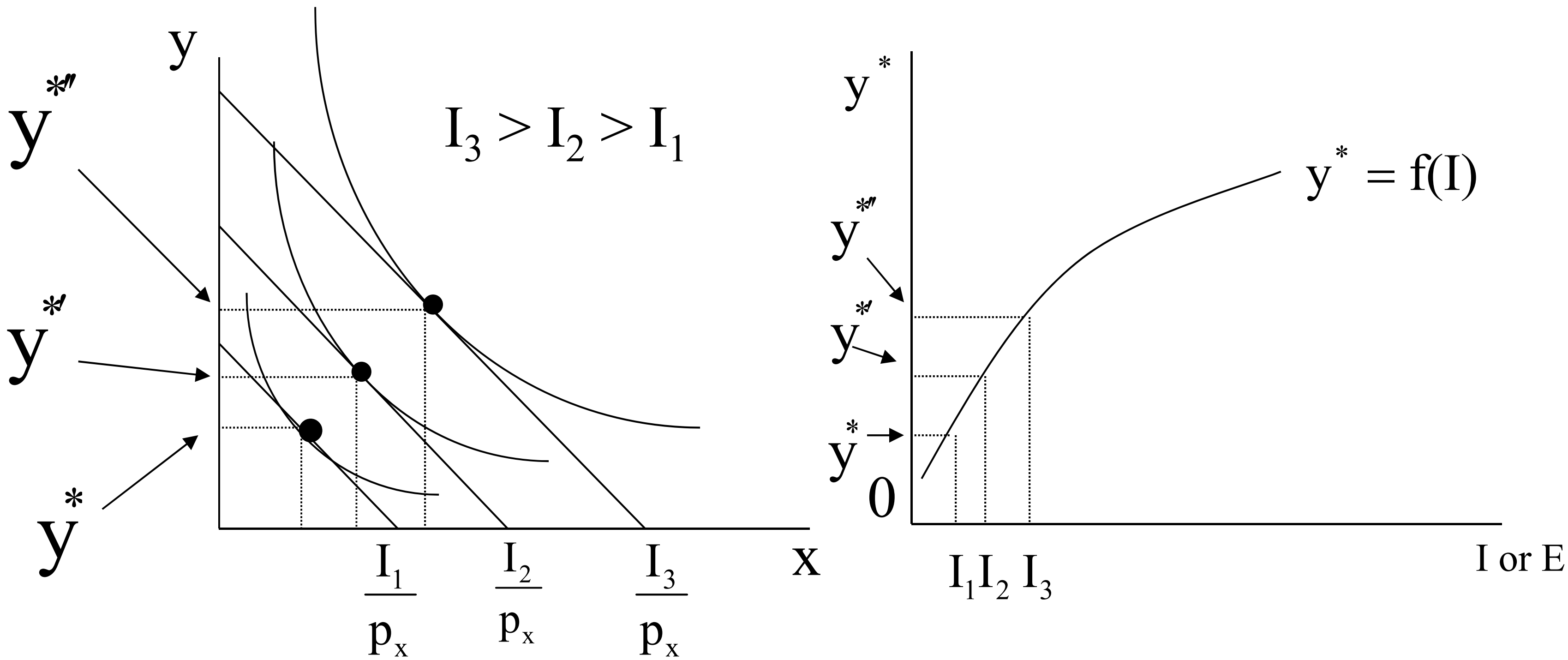
- <u>Changes in Income</u> – *Ceteris Paribus* Prices and preferences (utility function)

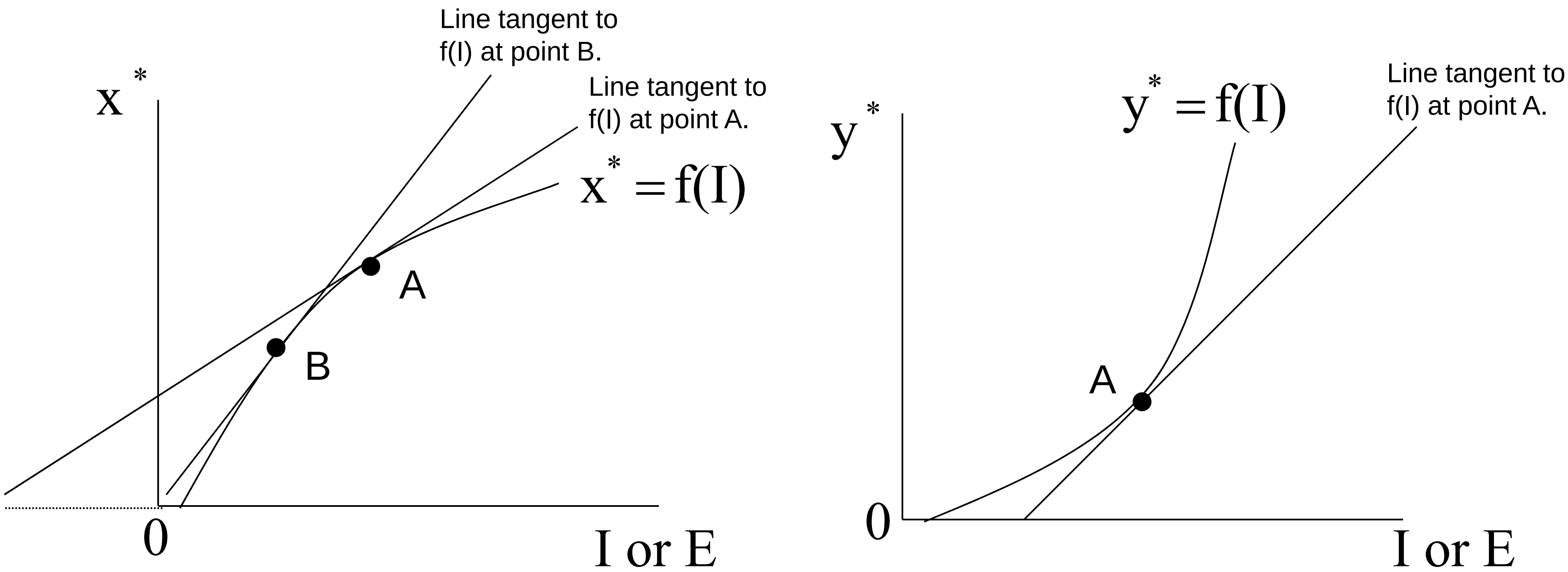
Changes in income shift the budget constraint parallel because prices are assumed not to change.



Successive optima when I changes will have same MRS because the slope of the budget constraint remains the same.

Engel Curves show how x^* or y^* changes as I changes, *Ceteris Paribus*.





If a line tangent to f(I) at point A has a negative intercept, expenditures on the good ($p_x x$) **decrease** as a percentage of total expenditures (E) as I increases around point A.

If a line tangent to f(I) at point A has a positive intercept, expenditures on the good ($p_y y$) **increase** as a percentage of total expenditures (E) as I increases around point A.

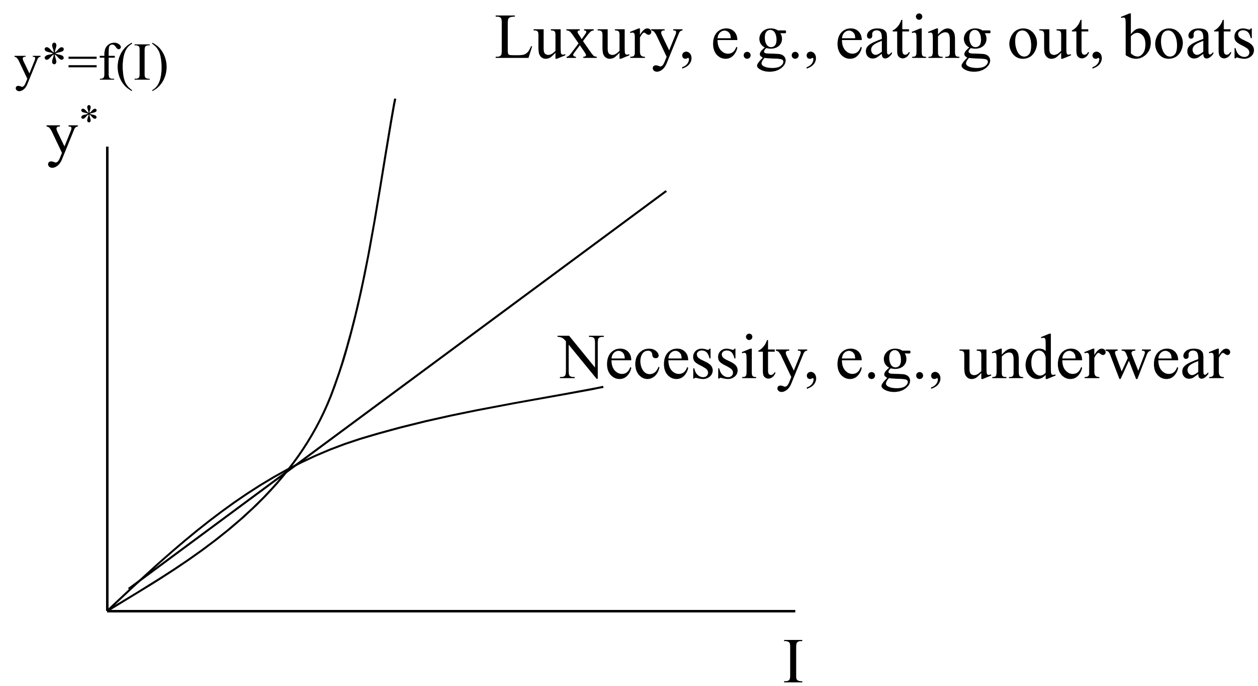
If a line tangent to f(I) at point B has a zero intercept, expenditures on the good ($p_x x$) **remain constant** as a percentage of total expenditures (E) as I increases around point B.

Normal and Inferior Goods

Normal good $\dfrac{\partial y^*}{\partial I} \geq 0$ Mostly we talk about $\dfrac{\partial y^*}{\partial I} > 0$.

Curves may bend up for **luxury** goods ($f_{II} > 0$) and down for **necessities**

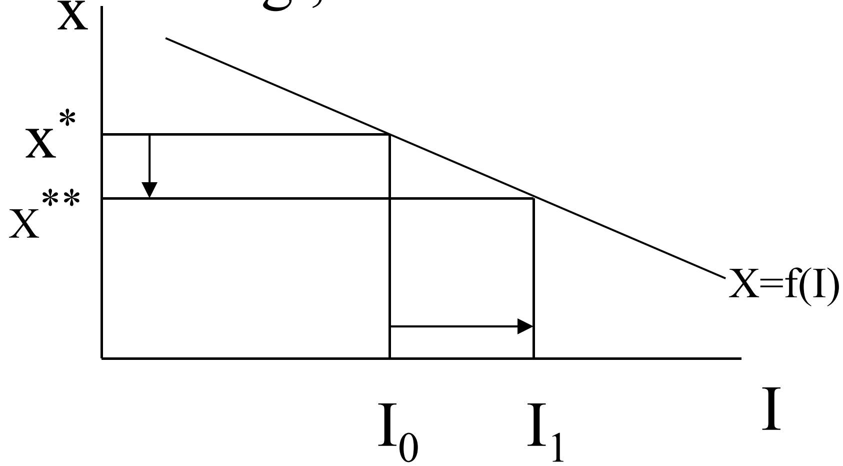
($f_{II} < 0$).

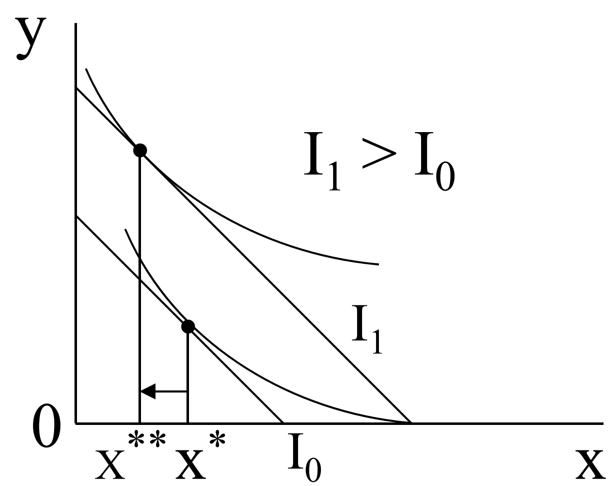


Both luxuries and necessities are Normal Goods.

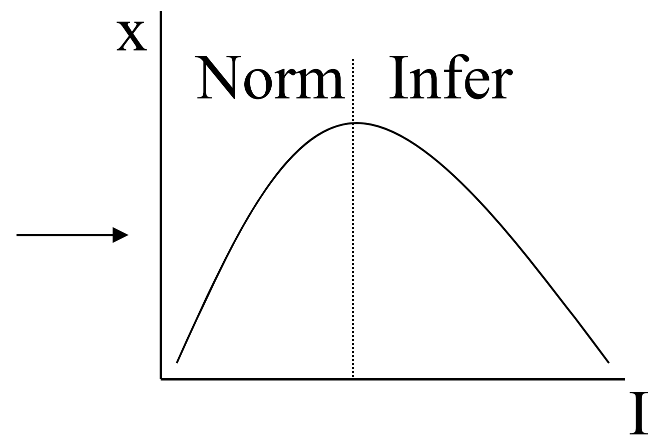
Inferior good $\frac{\partial x^{*}}{\partial I} < 0$

e.g., macaroni & cheese

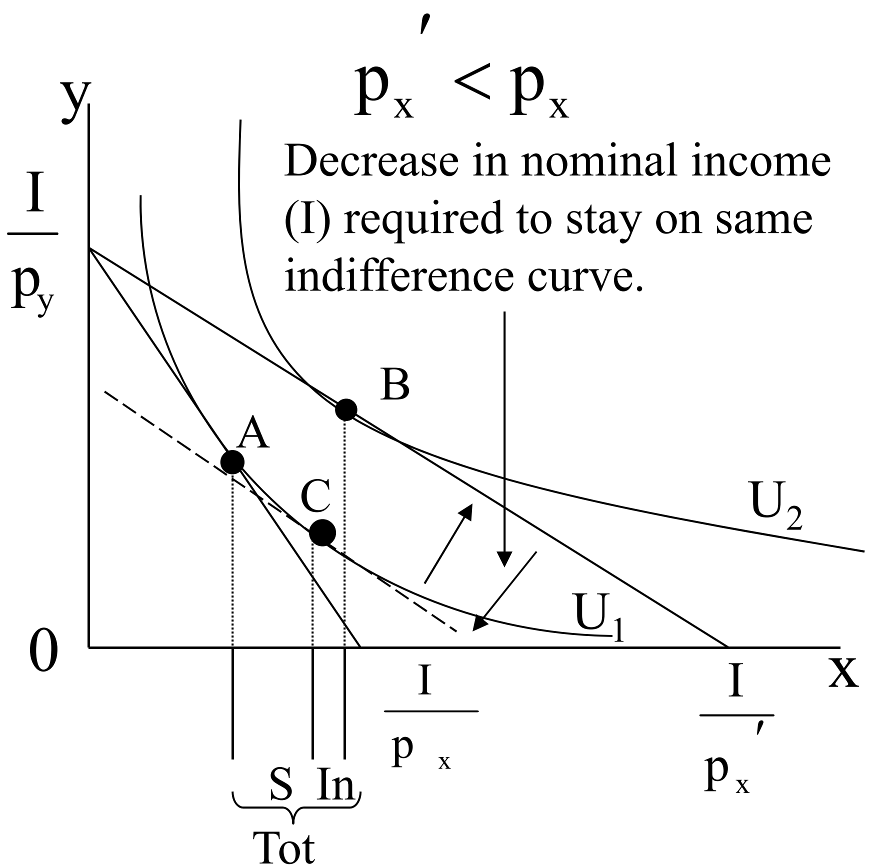




Whether a good is normal or inferior may vary along the Engle Curve. The good may change from Normal to Inferior.



- <u>Changes in Prices</u> – *Ceteris Paribus*. Involves a change in the <u>price ratio</u> (p_x/p_y) and thus a change in the <u>slope</u> of the budget constraint and a <u>change in the MRS at the new optimal point</u>, because at optimal, $MRS_{xy} = p_x/p_y$.



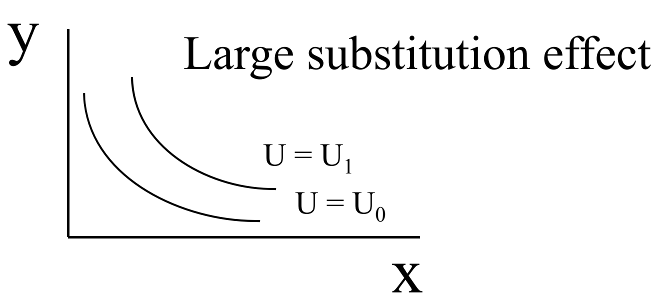
p_x decreases to p_x' causing optimum to move from A to B (Δx^* is total effect).

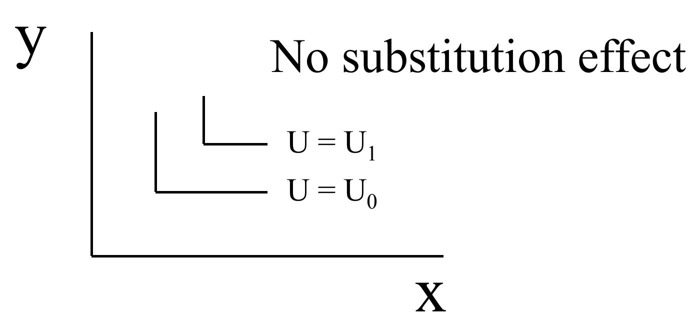
The total effect is divided into two effects, the substitution effect and the income effect. The substitution effect is the change in x^* in going from A to C, while the income effect is the change in x^* in going from C to B.

To find C, use the original indifference curve and find the point of tangency with a fictitious budget constraint that has the new price ratio.

What kind of good is x with regard to income? It is a normal good because $\frac{\partial x^*}{\partial I} > 0$.

- <u>Summary of substitution and income effects</u> – The movement from A to B is composed of <u>two effects:</u>
  - **Substitution effect -** Caused by change in $p_x/p_y|_{U = U1}$.
    - Because p_x is lower, the **<u>price ratio is smaller</u>** and the new tangency point must be at a smaller MRS (smaller –dy /dx).
    - Can measure the substitution effect by **<u>holding real income constant (hold U constant)</u>**, remaining on the same Indifference Curve, but using the new price ratio to find point C. The change in x^* in going from A to C measures the substitution effect.
    - For a given price change (Δp_x), the magnitude of the substitution effect depends on the availability of substitute goods.





– **Income Effect** – Caused by an increase in real income represented by an increase in utility, or movement to a higher indifference curve. A price decrease brings about an increase in real income; purchasing power. The income effect is the change in x^* in going from C to B. The magnitude of the income effect depends on the portion of income spent on x.

- The sum of the Income and Substitution Effects is the total effect of a price change (total change in x^*).
- Could show a similar analysis for a price increase (text p. 127).
- In most situations, the two effects are complementary, in that they move in the same direction and reinforce each other as in the case of **Normal Goods.**

$$\left.\frac{\partial x^*}{\partial p_x}\right|_{U^*} < 0$$

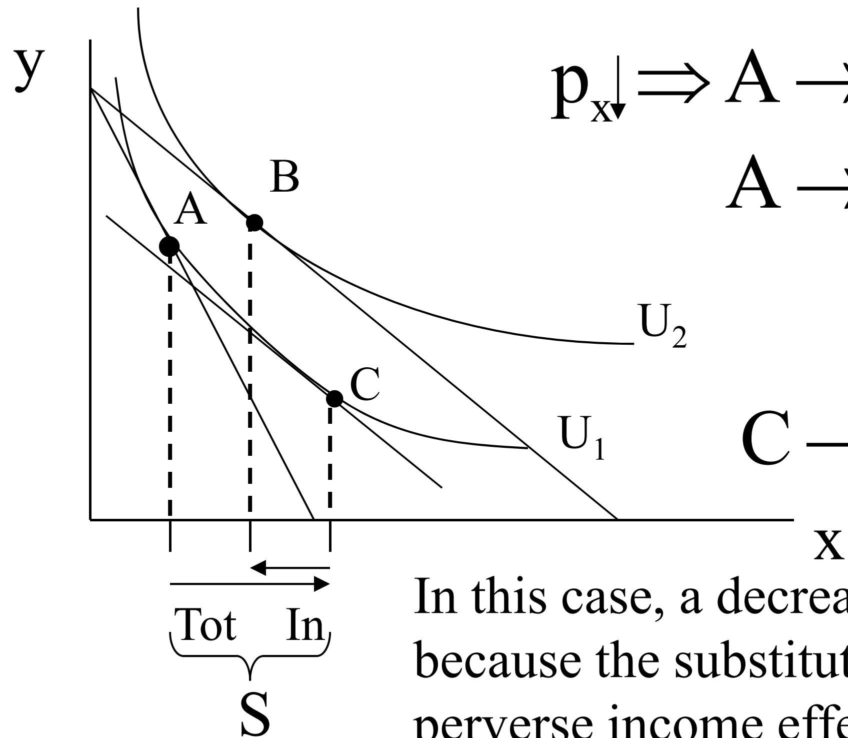
For a decrease in p_x , the substitution effect causes x^* to increase holding real income constant (have to reduce nominal income to keep real income constant).

$$\frac{\partial x^*}{\partial I} > 0$$

The increase in nominal income required to reach the higher indifference curve (higher real income resulting from the lower price), causes x^* to increase further.

Income and Substitution Effects for Inferior Goods

- Increasing income causes a decline in purchases of x, $\frac{\partial x^*}{\partial I} < 0.$ The income effect is perverse for an inferior good.



$p_x \downarrow \Rightarrow A \rightarrow B$ Gives **Total** effect on x^*.

$A \rightarrow C$ Gives **Substitution** effect $\left.\frac{\partial x^*}{\partial p_x}\right|_{U=U_1} < 0.$

$C \rightarrow B$ Gives **Income** effect $\frac{\partial x^*}{\partial I} < 0.$

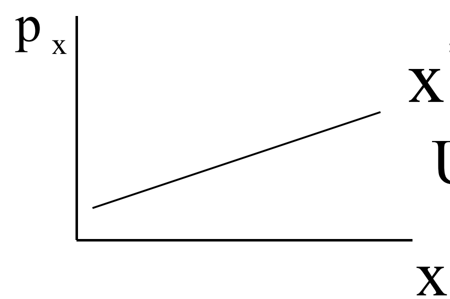
In this case, a decrease in p_x causes an increase in x^*, but only because the substitution effect is large enough to offset the perverse income effect.

<u>The substitution effect</u> is <u>**always**</u> (for typical Indifference Curves) opposite the price movement!

- The income effect generally cannot be classified. The income effect is opposite the price movement for a normal good and in the same direction as the price movement for an inferior good.

<u>Giffen Goods</u> – When the <u>perverse income effect</u> for an <u>inferior</u> good is large enough to <u>overwhelm</u> the substitution effect (very unusual). Probably requires the inferior good to make up a very large portion of total expenditures (see text p.128) and have no close substitutes.

$$\frac{\partial x^*}{\partial p_x} > 0!\,!$$



$$x^* = x(p_x \mid p_y, I)$$

Upward sloping demand curve

<u>Individual Demand Curve</u> (single consumer)

Demand functions $\longrightarrow$ $x_1^* = x_1(p_{x_1}, p_{x_2}, \ldots, p_{x_n}, I)$

$x_n^* = x_n(p_{x_1}, p_{x_2}, \ldots, p_{x_n}, I)$ from earlier

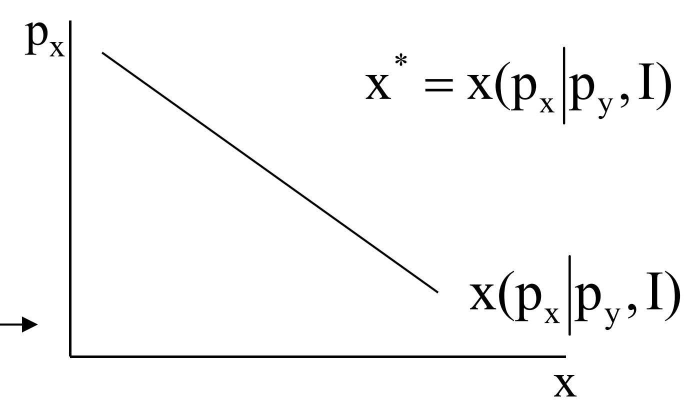
In a two-good world:

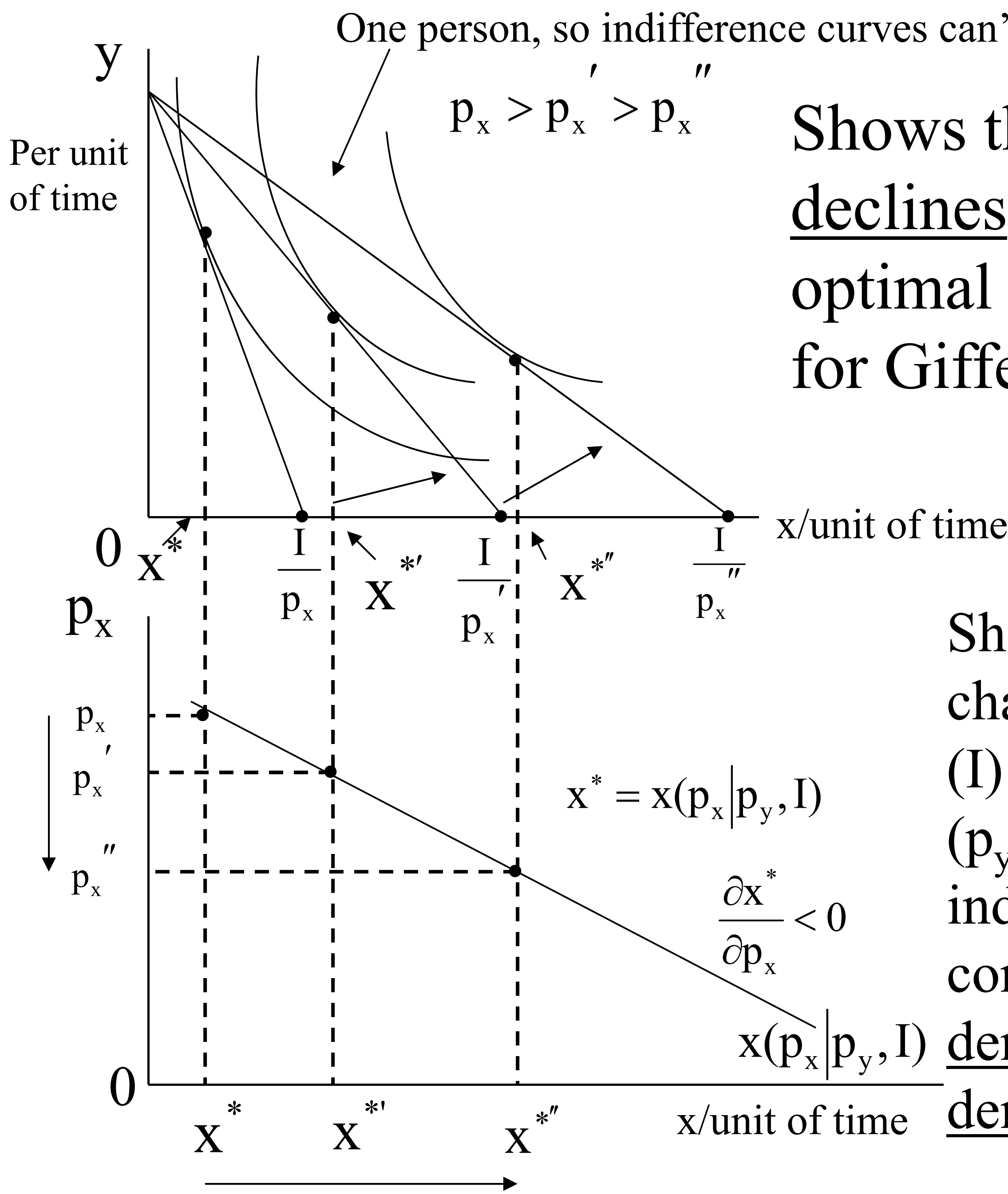
Demand functions $\longrightarrow$ $x^* = x(p_x, p_y, I)$

$y^* = y(p_x, p_y, I)$

Traditional or Marshallian demand functions

Could hold p_y, preferences, and I (nominal) constant and vary p_x to get typical <u>demand curve</u> in p_x and x space. $\longrightarrow$





Shows that successive price declines result in larger optimal quantities of x (except for Giffen goods).

Shows how x^* changes as p_x changes when nominal income (I) and prices of all other goods (p_y) are constant and when the individual's preference system constant. This is a Marshallian demand curve (uncompensated demand curve).

•Shifts in the demand curve are caused by shifts in the individual's preferences or utility function (shifts in the indifference curves implying changes in the MRS), the prices of other goods, and nominal income.

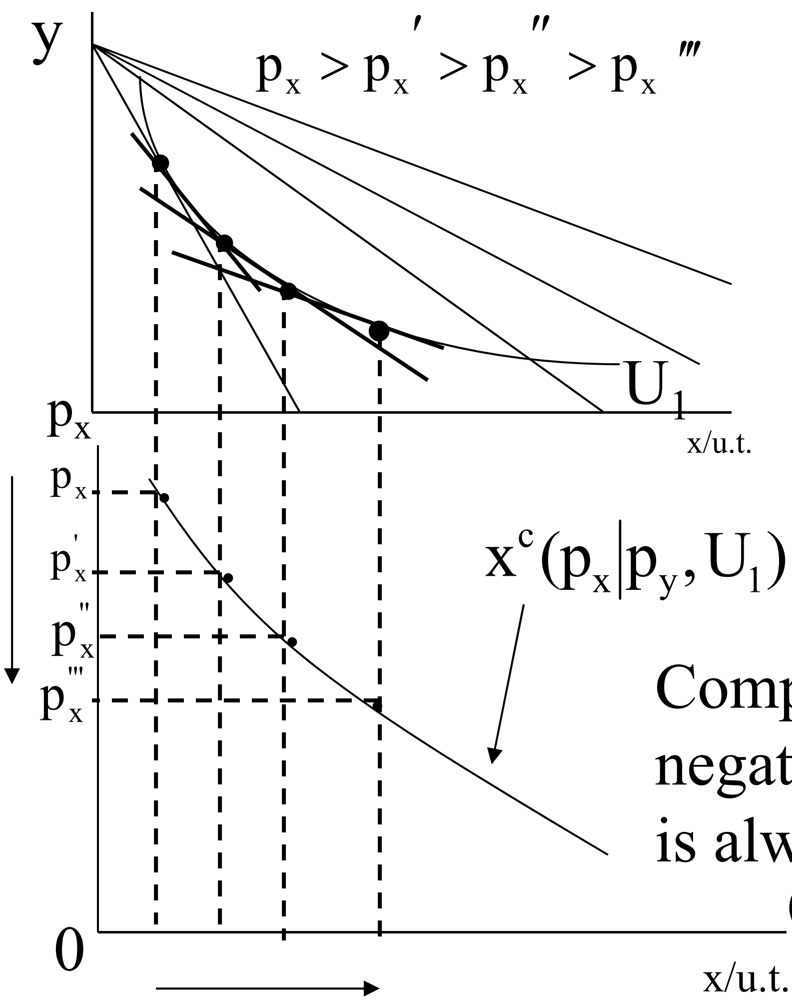
Increase I, the demand curve for x shifts up for normal good (down for inferior good).

Increase p_y, the demand curve for x shifts up for substitute good (down for a complement good).

Increase MU_x relative to MU_y, the demand curve for x shifts up. At any x, MRS_{xy} is larger. The absolute slope of the indifference curve is larger (steeper indifference curve at any level of x).

- Changes in p_x cause movements along the curve, i.e., "changes in the Quantity Demanded" rather than shifts in demand.

Compensated Demand Curves (Hicksian Demand Curve) – We could eliminate the income effects of changes in p_x and show the effects on x^*, holding utility or real income constant.



Income compensated demand curve (Hicksian) shows only the substitution effects of changes in p_x, while p_y, preferences and utility (real income) are held constant.

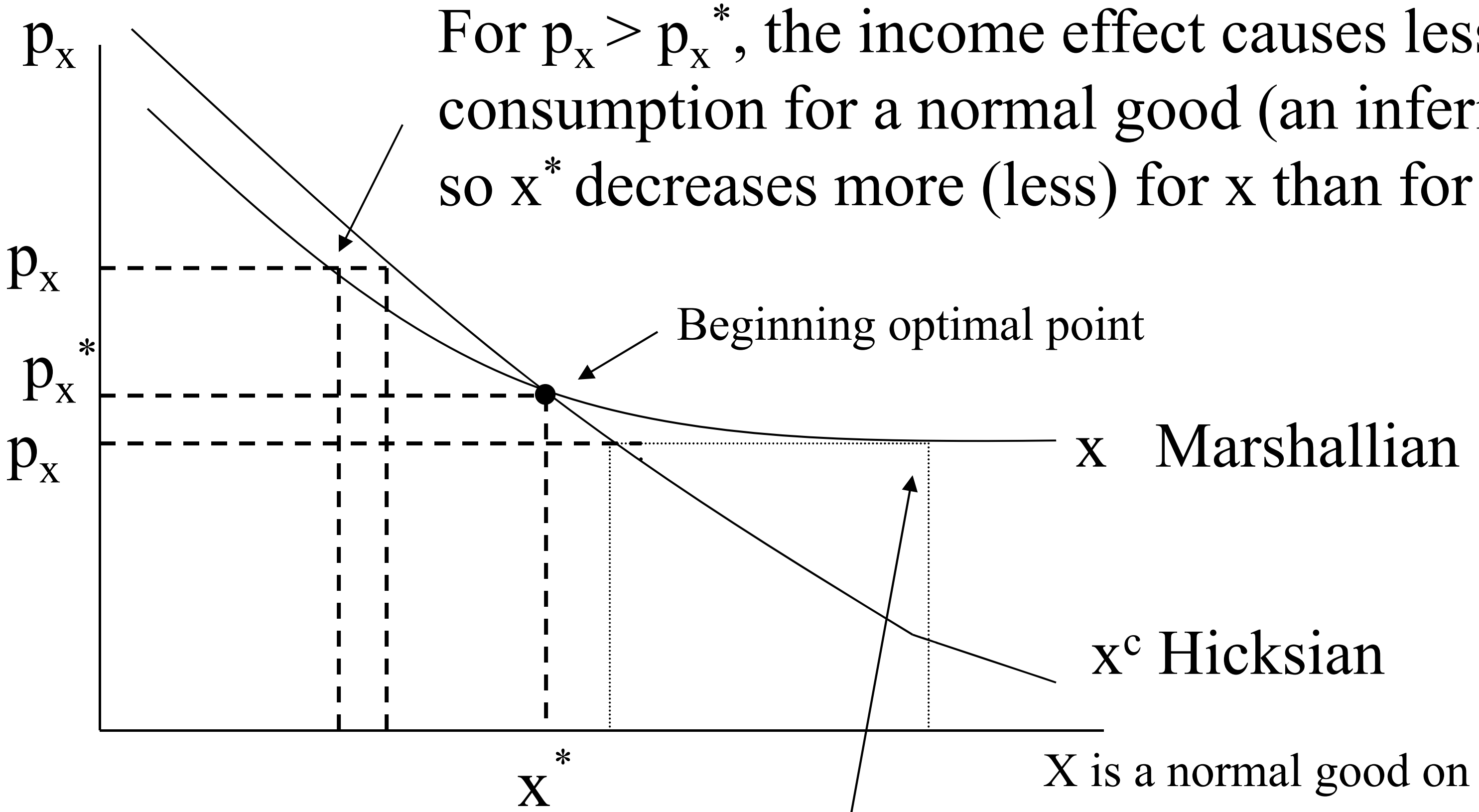
Means real income constant!

Compensated demand curve **always** has a negative slope because the substitution effect is always negative if MRS is diminishing.

($\partial x^c/\partial p_x < 0$ always if MRS_{xy} is diminishing)

<u>Compensated demand curve is **steeper (flatter)**</u> than the Marshallian demand curve for a normal (inferior) good.

For $p_x > p_x^*$, the income effect causes less (more) consumption for a normal good (an inferior good), so x^* decreases more (less) for x than for x^c.



X is a normal good on this graph.

For $p_x < p_x^*$, the income effect causes more (less) consumption for a normal good (an inferior good), so x^* increases more (less) for x than for x^c.

Mathematical Approach to Response to Price Changes

- Income and Substitution Effects (two-good world)

  - Use the compensated demand function

real income

$$x^* = x^c(p_x, p_y, U)$$

"Hicksian" or "Compensated"

and the ordinary demand function.

nominal income

$$x^* = x(p_x, p_y, I)$$

"Marshallian", "Ordinary", or "Uncompensated"

- These two demand functions are **equal** at the "beginning point" (U max point) where they cross.

$$x^c(p_x, p_y, U) = x(p_x, p_y, I)$$

Next, remember the Expenditure Function

$$E^* = E(p_x, p_y, U)$$

Substitute the Expenditure Function into x above and get the following (because I = E at U max).

$$x^c(p_x, p_y, U) = x(p_x, p_y, E(p_x, p_y, U))$$

Partially differentiate with respect to p_x to get

Chain rule

$$\frac{\partial x^c}{\partial p_x} = \frac{\partial x}{\partial p_x} + \underbrace{\frac{\partial x}{\partial E} \cdot \frac{\partial E}{\partial p_x}}_{\text{Chain rule}} \longrightarrow$$

Rearrange terms to get

$$\frac{\partial x}{\partial p_x} = \frac{\partial x^c}{\partial p_x} - \frac{\partial x}{\partial E} \cdot \frac{\partial E}{\partial p_x}$$

We will explore this relationship on next page.

(-) except for Giffen good

Always (-)

(-) for normal goods
(+) for inferior goods

Marshallian (total effect)

Hicksian (sub effect)

Income effect

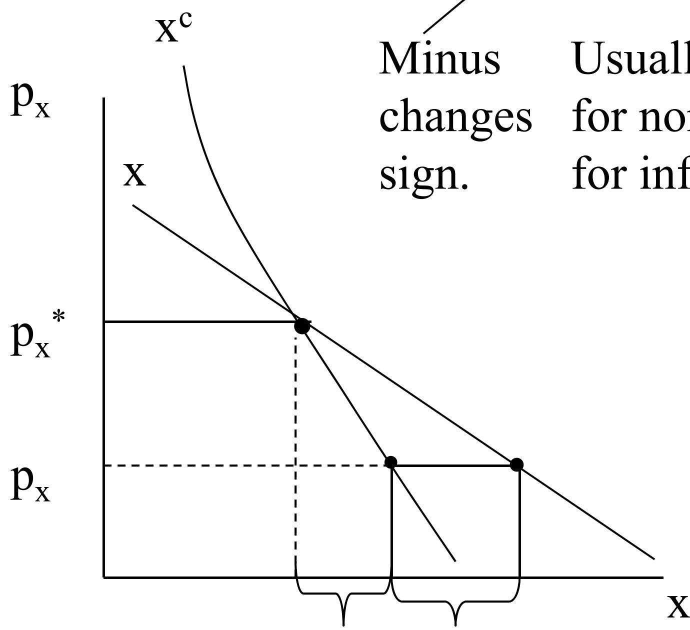
$$\underbrace{\frac{\partial x}{\partial p_x}}_{\text{Marshallian}} = \underbrace{\frac{\partial x^c}{\partial p_x}}_{\text{Hicksian}} - \frac{\partial x}{\partial E} \bullet \frac{\partial E}{\partial p_x}$$

This is not the slope of the Marshallian demand curve; it is the inverse of the slope. The more negative $\partial x/\partial p_x$, the flatter the demand curve. If x were on the vertical axis, this would be the slope of the demand curve.

Minus changes sign.

$\frac{\partial x}{\partial E}$: Usually (+); for normal (+), for inferior (-)

$\frac{\partial E}{\partial p_x}$: Always (+ or 0) because a decrease (increase) in p_x implies a decrease (increase) in E to maintain same utility level as before the price change. Also, the Expenditure Function is non-decreasing in prices.



$$\frac{\partial x}{\partial p_x} = \frac{\partial x^c}{\partial p_x} - \frac{\partial x}{\partial E} \cdot \frac{\partial E}{\partial p_x}$$

Summary of Equation on Previous Page

- $\partial x/\partial p_x$ is the gross change in x^* in response to a change in p_x (**total effect**). It is the inverse of the slope of the Marshallian demand curve.
- The first term on the right-hand side is the change in x^* in response to a change in p_x holding utility (real income) constant. It is the **substitution effect**! It is always negative because MRS is diminishing. It is the inverse of the slope of the compensated (Hicksian) demand curve.
- The second term $\left(-\frac{\partial x}{\partial E}\cdot\frac{\partial E}{\partial p_x}\right)$ shows the response of x^* to a change in p_x through the effect of p_x on income (I=E). It is the **income effect**!
- For a normal good, the substitution and income effects have negative signs; they reinforce each other.

The Slutsky Equation

Sub. effect = $\frac{\partial x^c}{\partial p_x} = \frac{\partial x}{\partial p_x}\Big|_U$ (real income) constant

Inc. effect = $-\frac{\partial x}{\partial E} \cdot \frac{\partial E}{\partial p_x} = -\frac{\partial x}{\partial I} \cdot \frac{\partial E}{\partial p_x} = -\frac{\partial x}{\partial I} \cdot x$

E=I

Differentiate the expenditure function with respect to p_x to get $x^c(p_x, p_y, U)$, which is the compensated demand function. See Shepard's Lemma and Envelope Theorem on page 137 of text.

<u>The Slutsky Equation</u> – Combine substitution and income effects.

(-) usually, except Giffen

(-) **always**

(-) normal good
(+) inferior good

$$\frac{\partial x}{\partial p_x} = \frac{\partial x}{\partial p_x}\Big|_U - x \frac{\partial x}{\partial I}$$

(-) **always**

(+) normal good
(-) inferior good

Elasticity - General definition

- The elasticity is the percentage change in Y for a 1% change in X.

- If Y = f(X,….) is some general function,

$$e_{Y,X} = \frac{\partial Y}{\partial X} \cdot \frac{X}{Y} = f_X \cdot \frac{X}{Y} \cong \frac{\Delta Y / Y}{\Delta X / X} = \frac{\Delta Y}{\Delta X} \cdot \frac{X}{Y}$$

Elasticity at a point on the function. Elasticity may be different at each point on the function.

Average elasticity over a segment of the function.

- The partial shows how Y changes as X changes. The partial is expressed in units of Y per unit of X. To remove the units, multiply by X/Y to get a pure percentage change. Units are gone.

Marshallian Demand Elasticities

$$\text{Price elasticity of demand} = e_{x,p_x} = \frac{\partial x}{\partial p_x} \cdot \frac{p_x}{x}$$

If $e_{x,p_x} < -1$, elastic; $e_{x,p_x} = -1$, unit elastic; $e_{x,p_x} > -1$, inelastic

$$\text{Income elasticity of demand} = e_{x,I} = \frac{\partial x}{\partial I} \cdot \frac{I}{x}$$

If $e_{x,I} \geq 0$, normal good; $e_{x,I} < 0$, inferior good

$$\text{Cross-price elasticity of demand} = e_{x,p_y} = \frac{\partial x}{\partial p_y} \cdot \frac{p_y}{x}$$

If $e_{x,p_y} > 0$, gross substitutes; $e_{x,p_y} < 0$, gross complements;

$e_{x,p_y} = 0$, independent

The use of partial derivatives indicates that all other demand determinants are held constant.

Effect of a Price Change on Total Expenditures for a Good

If ***total expenditures*** on good x = $p_x x$, then expenditures will react to a price change as follows:

$e_{x,px}$		$\uparrow p_x$	$\downarrow p_x$	
Elastic (< -1)	$\Rightarrow$	Exp. $\downarrow$	Exp. $\uparrow$	Because % increase in x is > than % decrease in p_x.
Unit elastic (= -1)	$\Rightarrow$	Con. Exp.	Con. Exp.	
Inelastic (> -1)	$\Rightarrow$	Exp. $\uparrow$	Exp. $\downarrow$	Because % increase in x is < than % decrease in p_x.

Mathematically:

$$\frac{\partial TE_x}{\partial p_x} = \frac{\partial p_x x}{\partial p_x} = \frac{\partial(p_x \cdot x(p_x))}{\partial p_x} = p_x \frac{\partial x}{\partial p_x} + x$$

Product Rule: First (p_x) times the derivative of the second (x) plus the second (x) times the derivative of the first (p_x).

Multiply this equation by x/x to get:

$$\frac{\partial TE_x}{\partial p_x} = x(e_{x,p_x} + 1).$$

So the change in total expenditures on the good is determined by the price elasticity of demand as follows:

If elastic ($e_{x,px}$ < -1), then $\left(\frac{\partial TE_x}{\partial p_x} < 0\right)$ (ΔTE is opposite the Δp_x).

If unit elastic ($e_{x,px}$ = -1), then $\left(\frac{\partial TE_x}{\partial p_x} = 0\right)$ (ΔTE = 0).

If inelastic $e_{x,px}$ > -1, then $\left(\frac{\partial TE_x}{\partial p_x} > 0\right)$ (ΔTE is same direction as Δp_x).

Compensated Demand Elasticities

If the compensated demand function is given by $x^c(p_x, p_y, U)$, then the compensated own price elasticity is: $e^c_{x,p_x} = \frac{\partial x^c}{\partial p_x} \cdot \frac{p_x}{x^c}$

and the compensated cross-price elasticity is:

$$e^c_{x,p_y} = \frac{\partial x^c}{\partial p_y} \cdot \frac{p_y}{x^c}.$$

The relationship between these compensated price elasticities and Marshallian price elasticities can be shown by putting the Slutsky Equation in elasticity form.

Relationships Among Elasticities

Slutsky Equation in Elasticity Form

$$\frac{\partial x}{\partial p_x} = \frac{\partial x}{\partial p_x}\bigg|_{U\ const} - x\frac{\partial x}{\partial I}$$

Original Slutsky

Multiply by p_x/x.

$$\frac{\partial x}{\partial p_x}\cdot\frac{p_x}{x} = \frac{p_x}{x}\frac{\partial x}{\partial p_x}\bigg|_{U\ const} - p_x x\frac{\partial x}{\partial I}\cdot\frac{1}{x}$$

Multiply the second term on the right-hand side by I/I.

$$\underbrace{\frac{\partial x}{\partial p_x}\cdot\frac{p_x}{x}}_{e_{x,p_x}} = \underbrace{\frac{p_x}{x}\frac{\partial x}{\partial p_x}\bigg|_{U\ const}}_{e^c_{x,p_y}} - \underbrace{\frac{p_x x}{I}}_{s_x}\underbrace{\frac{\partial x}{\partial I}\cdot\frac{I}{x}}_{e_{x,I}}$$

e^c_{x,p_y}: "Substitution Elasticity". The elasticity of the compensated demand curve.

$$e_{x,p_x} = e^{c}_{x,p_x} - s_x e_{x,I}$$

(-) except Giffen | (-) always | (-) for normal or (+) for inferior good.

The Slutsky Equation in elasticity form shows how the price elasticity of demand can be disaggregated into the substitution elasticity plus the expenditure proportion times the income elasticity of demand.

1) If substitution effect is 0 or close to 0 ($e^{c}_{x,p_x} = 0$), e_{x,p_x} is proportional to $e_{x,I}$.

2) If the expenditure share is small (s_x small), e_{x,p_x} is almost equal e^{c}_{x,p_x}.

3) If you can estimate, e_{x,p_x}, s_x, and $e_{x,I}$, you can derive e^{c}_{x,p_x}.

Euler's Theorem and the Homogeneity Condition

If $x_1 = f(p_{x_1}, p_{x_2}, \ldots, p_{x_n}, I)$ is homogeneous of degree m, then

$$\frac{\partial f}{\partial p_{x_n}}$$

$$f_1 p_{x_1} + f_2 p_{x_2} + \ldots + f_n p_{x_n} + f_I I = mf(p_{x_1}, p_{x_2}, \ldots, p_{x_n}, I) = mx_1.$$

When m = 0 (because f is homogeneous of degree zero in prices and income), then

$$f_1 p_{x_1} + f_2 p_{x_2} + \ldots + f_n p_{x_n} + f_I I = (0)x_1 = 0.$$

$$\frac{\partial f}{\partial p_{x_1}} \qquad \frac{\partial f}{\partial I}$$

Degree of homogeneity (m=0).

Homogeneity Condition

Because demand functions are homogeneous of degree zero, we can use Euler's theorem on the demand function for x (uncompensated) to get the following for a two-good world:

$$\frac{\partial x}{\partial p_x}\cdot p_x + \frac{\partial x}{\partial p_y}\cdot p_y + \frac{\partial x}{\partial I}\cdot I = 0$$

Dividing through by x gives:

$$\frac{\partial x}{\partial p_x}\cdot \frac{p_x}{x} + \frac{\partial x}{\partial p_y}\cdot \frac{p_y}{x} + \frac{\partial x}{\partial I}\cdot \frac{I}{x} = 0$$

$$\underbrace{e_{x,p_x}}_{\text{(-) except Giffen}} + \underbrace{e_{x,p_y}}_{\text{(+) for gross substitutes; (-) for gross complements}} + \underbrace{e_{x,I}}_{\text{(+) for normal good; (-) for inferior good}} = 0$$

(-) except Giffen

(+) for gross substitutes; (-) for gross complements

(+) for normal good; (-) for inferior good

Conclusion

The sum of own-price, cross-price, and income elasticities equals zero for any specific good. This reaffirms that demand is homogeneous of degree zero (equal percentage changes in prices and income leave the quantity demanded unaffected). If you know or can estimate two of the three terms, you can calculate the third. If you had more than two goods, you would have several cross-price elasticities, some of which could be < 0.

Engel Aggregation

Assuming a typical consumer (diminishing MRS) and two goods, the budget constraint is:

$p_x x^* + p_y y^* = I$ at optimallity.

For utility maximization, an increase in I must be accompanied by an increase in total expenditures because I = E.

The demand functions are:

$$x^* = x(p_x, p_y, I)$$
$$y^* = y(p_x, p_y, I)$$

Differentiate the budget constraint with respect to I assuming optimality:

$$\frac{\partial I}{\partial I} = p_x \frac{\partial x^*}{\partial I} + p_y \frac{\partial y^*}{\partial I} = 1$$

Multiply the first term on the left-hand side by $\frac{x}{I}$ and $\frac{I}{x}$ and multiply the second term by $\frac{y}{I}$ and $\frac{I}{y}$.

$$\frac{p_x x}{I} \cdot \frac{\partial x}{\partial I} \cdot \frac{I}{x} + \frac{p_y y}{I} \cdot \frac{\partial y}{\partial I} \cdot \frac{I}{y} = 1$$

$s_x e_{x,I} + s_y e_{y,I} = 1,$ where $s_x = \frac{p_x x}{I}$ is the proportion of I spent on x and s_y is the proportion of I spent on y.

For n goods $s_{x_1} e_{x_1,I} + s_{x_2} e_{x_2,I} + \ldots + s_{x_n} e_{x_n,I} = 1.$

Thus, the proportion of income spent on each good times its income elasticity of demand summed over all goods is 1.

If income increases by 10%, *certeris paribus,* total purchases must increase by 10% (because of the budget constraint), which implies that goods whose $e_{x,I} < 1$ must be offset by others whose $e_{x,I} > 1$ (assumes savings is a good). If you know $e_{x,I}$ and s_x and s_y in a two-good world, you can calculate $e_{y,I}$. Engle Aggregation applies to market demand as well as individual demand.

Cournot Aggregation

This concept deals with what happens to the demand for all goods when the price of a single good changes.

The budget constraint is $I = p_x x + p_y y$.

Differentiate the budget constraint with respect to p_x to get:

$$\frac{\partial I}{\partial p_x} = 0 = p_x \cdot \frac{\partial x}{\partial p_x} + x + p_y \cdot \frac{\partial y}{\partial p_x}$$ because nominal income and p_y do not change when p_x changes.

Multiply each term of this equation by p_x/I, the first term by x/x, and the last term by y/y to get:

$$0 = p_x \cdot \frac{\partial x}{\partial p_x} \cdot \frac{p_x}{I} \cdot \frac{x}{x} + x \cdot \frac{p_x}{I} + p_y \cdot \frac{\partial y}{\partial p_x} \cdot \frac{p_x}{I} \cdot \frac{y}{y}$$

$0 = s_x e_{x,p_x} + s_x + s_y e_{y,p_x}$ to give the Cournot result:

$$s_x e_{x,p_x} + s_y e_{y,p_x} = -s_x.$$

The cross-price effect of a change in p_x on y demanded is restricted by the budget constraint.

Linear Demand Functions

$$x = a + bp_x + cI + dp_y$$

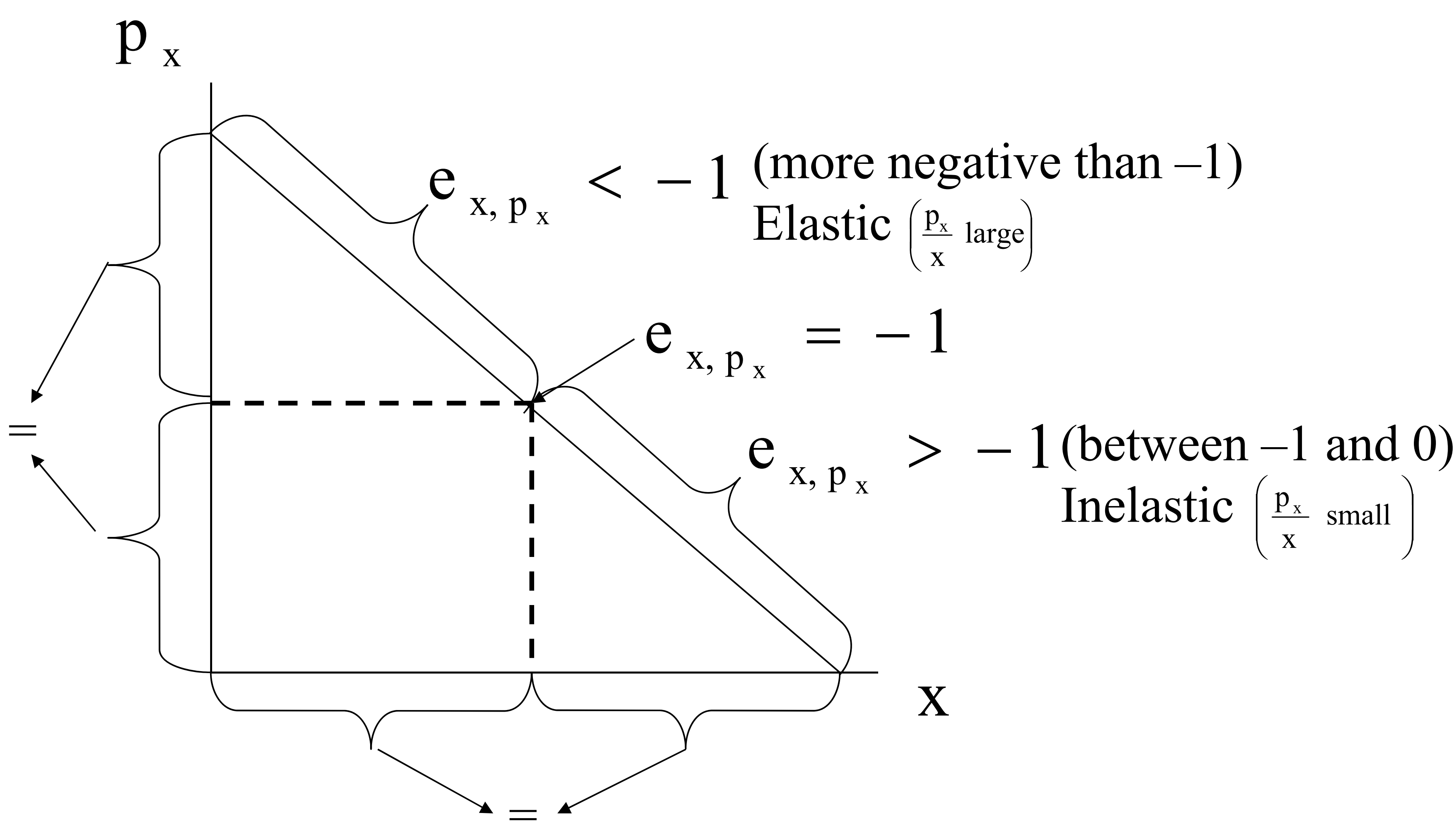
- b: (-) if not Giffen
- c: (+) normal; (-) inferior
- d: (+) Gross substitute; (-) Gross complement

Empirical (not theoretical) demand function, so it may not be homogeneous of degree zero because a ≠ 0 and b, c, and d are not indexed. Prices and income should be deflated by CPI or other index.

$$\frac{\partial x}{\partial p_x} = b \Rightarrow \quad e_{x,p_x} = b\frac{p_x}{x}, \qquad e_{x,I} = c\frac{I}{x}, \qquad e_{x,p_y} = d\frac{p_y}{x}.$$

Elasticities are typically evaluated at the means of x, p_x, p_y, and I.

e_{x,p_x} is not constant along the demand curve. $\frac{\partial x}{\partial p_x}$ is constant, but p_x/x is not constant!



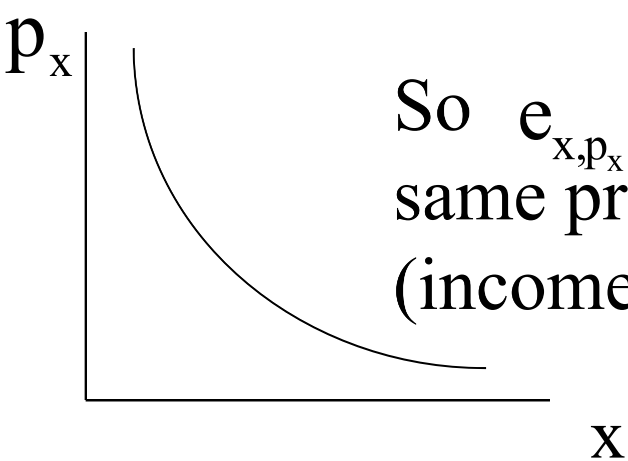
p_x
$e_{x, p_x} < -1$ (more negative than –1)
Elastic $\left(\frac{p_x}{x} \text{ large}\right)$
$e_{x, p_x} = -1$
$e_{x, p_x} > -1$ (between –1 and 0)
Inelastic $\left(\frac{p_x}{x} \text{ small}\right)$
x
=
=

Constant Elasticity Demand Functions

$$x = a p_x^b I^c p_y^d$$ Cobb-Douglas type

$$e_{x,p_x} = \frac{\partial x}{\partial p_x} \cdot \frac{p_x}{x} = ab{p_x}^{b-1} I^c p_y^d \cdot \frac{p_x}{a p_x^b I^c p_y^d} = b$$

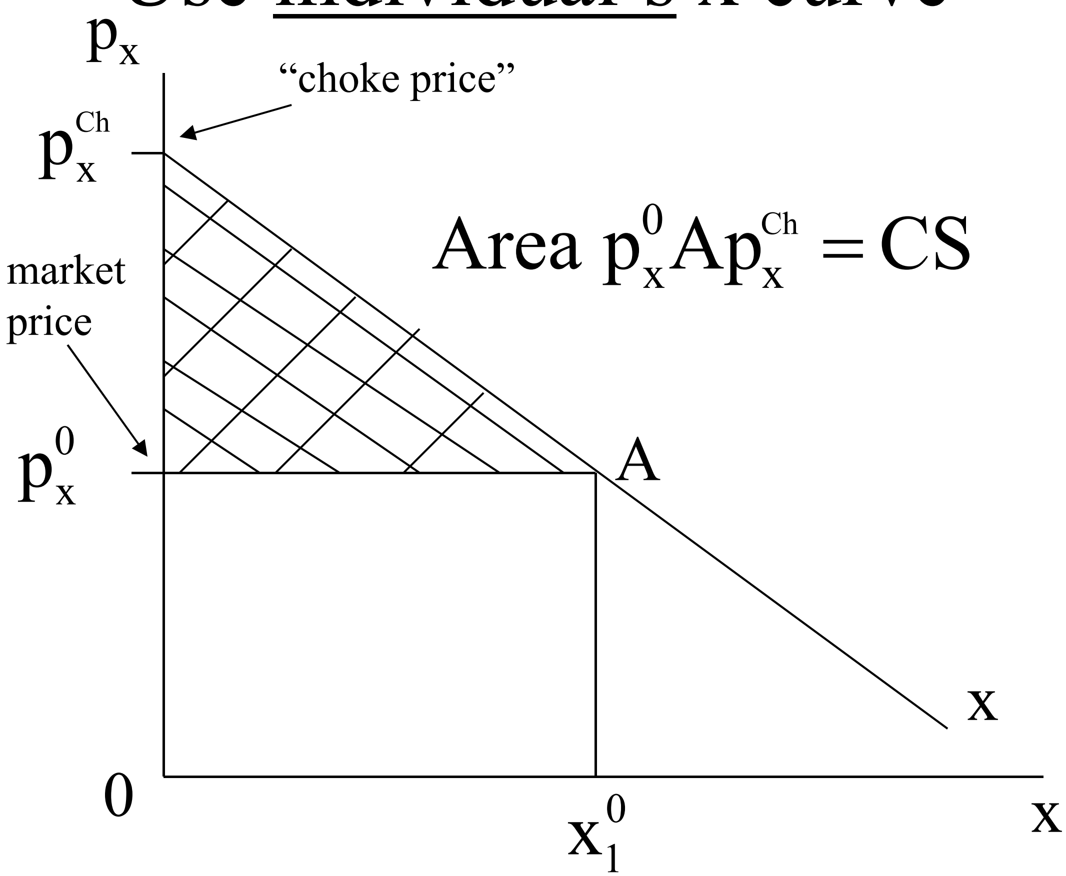
Or, $\ln x = \ln a + b \ln p_x + c \ln I + d \ln p_y$



So $e_{x,p_x} = b$ everywhere on the curve. The same property holds for other elasticities (income and cross-price elasticities).

Consumer Surplus

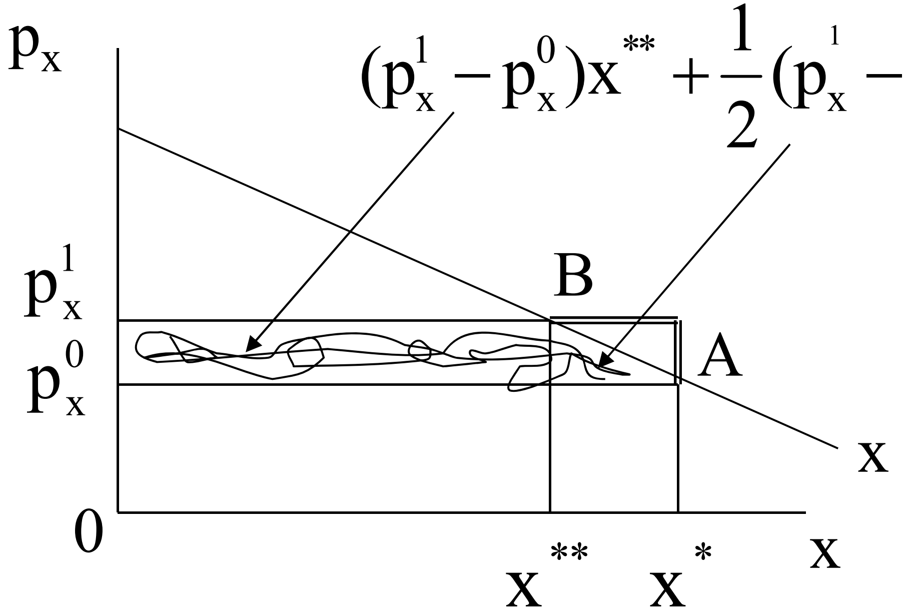
Use <u>individual's</u> x curve ---



Consumer surplus is the area under the x curve, above p_x^0. It is the amount of extra expenditures an individual would be willing to make above what he/she has to make to get each unit of the good.

- The concept of consumer surplus is used to evaluate the effects of price changes on consumers. If the price goes from p_x^0 to p_x^1, the consumer will suffer a net loss of consumer surplus equal to the area $P_x^0ABP_x^1$.

For linear demand function



$$(p_x^1 - p_x^0)x^{**} + \frac{1}{2}(p_x^1 - p_x^0)(x^* - x^{**}) = \text{net loss}$$

in consumer surplus caused by the price of x increasing from p_x^0 to p_x^1.

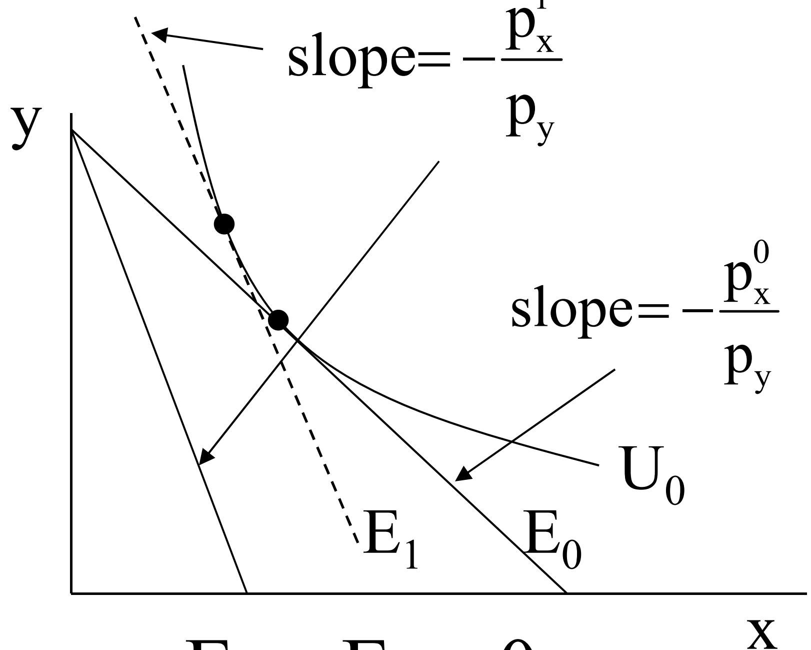
- We will use the compensated demand curve and the expenditure function to illustrate the measurement of consumer surplus.

- A change in consumer surplus can be measured by <u>expenditure differences</u> to maintain a <u>fixed level of utility, $U=U_0$</u>. Thus we can use the compensated demand function where utility is fixed.

$$E_0 = E(p_x^0, p_y, U_0) =$$ Expenditures at $p_x{}^0$, given U_0 and p_y.

$$E_1 = E(p_x^1, p_y, U_0) =$$ Expenditures at $p_x{}^1$, given U_0 and p_y.

$$p_x^1 > p_x^0$$



$$E_0 - E_1 < 0$$

The change in consumer surplus is $E_0–E_1$, which is the negative of the change in expenditures required to maintain the same level of utility when the price changes. For an increase in p_x, $E_0–E_1<0$, meaning real income has fallen and nominal income has to increase to maintain U_0. For a decrease in p_x, $E_0–E_1>0$, meaning real income has increased and nominal income has to decrease to maintain U_0.

- We can differentiate the expenditure function to get the compensated demand function.

$$\frac{\partial E}{\partial p_x} = x^c = x^c(p_x, p_y, U_0)$$

As shown earlier by envelope theorem, "<u>Shephard's Lemma.</u>"

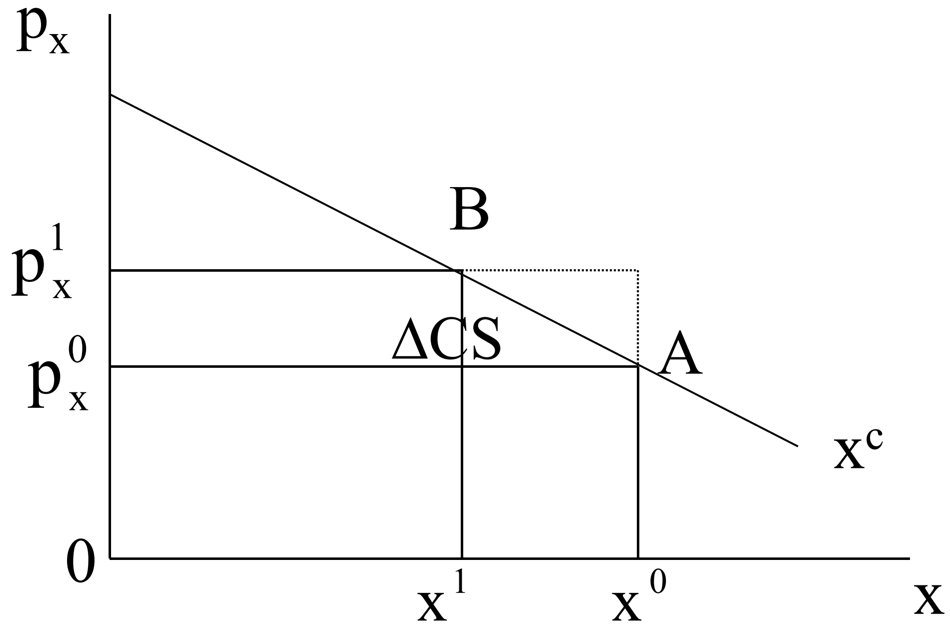
$\partial E/\partial p_x$ is the change in consumer surplus for a very small change in p_x.

The instantaneous change in E resulting from a very small change in p_x is equal to x^c (quantity demanded on the compensated demand curve).

Because the change from p_x^0 to p_x^1 covers some distance, <u>must integrate</u> $x^c(p_x, p_y, U_0)$ to get the change in consumer surplus.

$$\Delta CS = \int_{p_x^0}^{p_x^1} x^c(p_x, p_y, U_0)\,dp_x$$

Gives the area to left of the compensated demand curve between p_x^0 and p_x^1.



- The compensated demand function gives the "true" estimate of the change in consumer surplus resulting from a price change, but it is difficult to estimate, and once estimated, the estimate of consumer surplus is usually quite similar to the estimate obtained from the Marshallian demand curve, so using the Marshallian demand curve is more practical in the real world.