NE 583 Radiation Transport

Final Exam (Take Home)

Due midnight, Monday, December 10, 2018

1. Using ONLY EXCEL (with no external macros—start with an empty worksheet) and the recurrence relation for Legendre polynomials:

$$P_n(x) = \frac{(2n-1)xP_{n-1}(x) - (n-1)P_{n-2}(x)}{n}$$

find the POSITIVE zeros of $P_{18}(x)$ in the range (0,1).

2. The 1D slab discrete ordinates equation relates the left, right, and average fluxes in a particle balance + auxiliary equation; therefore there is no explicit information about how the flux varies spatially across the cell. Your boss wants you to deduce the flux shape by finding the flux at the CENTER of each cell for the step, diamond difference, and weighted methods; the idea is that the flux shape will then be the combination of two lines (one on the left half of the cell and the other on the right) with:

$$\bar{\psi} = \frac{\psi_\ell + 2\psi_c + \psi_r}{4}$$

   a. Find the formula for ψ_c for step, diamond difference, and weighted diamond difference formulations.
   b. For each of the formulations (step, DD, and WDD), discuss the limits on cell width required to assure that each of the dependent fluxes ($\psi_r, \bar{\psi}, \text{and} \psi_c$) are positive.

3. Solve the Class Exercise defined in Slide 9-22 using slab discrete ordinates with the weighted auxiliary equation (alpha = 0.8) and S_{12} (from the text). Provide me the average flux in the range x=45 to x=50 cm. (Do NOT just run it so that the 45-to-50 cm range is a single spatial cell!)

4. Repeat the previous problem using integral transport theory with 5-cm-wide cells. (You may find the attached equations from Abramowitz and Stegun useful. NOTE: Now the 45-50 cm range CAN be a single cell.)

5. Verify Eqns. 5-38 and 5-39 in the text from Eq. 5.37. I am going to grade this fairly strictly. Do not skip steps. Specifically:
   a. Make no physically simplifying assumptions or physical arguments.
   b. Do not utilize the recurrence relation A-43 unless you have a term that fits the form EXACTLY.
   c. For change of variable from x to τ, you will need to use the Leibnitz formula:

$$\frac{d}{d\alpha}\int_{g(\alpha)}^{h(\alpha)} f(x,\alpha)dx = \int_{g(\alpha)}^{h(\alpha)} \frac{\partial f(x,\alpha)}{\partial\alpha}dx + f[h(\alpha),\alpha]\frac{dh(\alpha)}{d\alpha} - f[g(\alpha),\alpha]\frac{dg(\alpha)}{d\alpha}$$

Extra credit: Why did they choose mu_1=0.2182179 for the S_8 quadrature in Table 4-1?

IMPORTANT: Include with your submission a statement that this test is your OWN WORK, and you neither sought nor gave any help from/to anyone but Dr Pevey.

5.1.53 $0 \le x \le 1$

$$E_1(x)+\ln x=a_0+a_1x+a_2x^2+a_3x^3+a_4x^4+a_5x^5+\epsilon(x)$$

$$|\epsilon(x)|<2\times 10^{-7}$$

$a_0=-.57721\ 566$	$a_3=\ \ .05519\ 968$
$a_1=\ \ .99999\ 193$	$a_4=-.00976\ 004$
$a_2=-.24991\ 055$	$a_5=\ \ .00107\ 857$

5.1.54 $1 \le x < \infty$

$$xe^xE_1(x)=\frac{x^2+a_1x+a_2}{x^2+b_1x+b_2}+\epsilon(x)$$

$$|\epsilon(x)|<5\times 10^{-5}$$

$a_1=2.334733$	$b_1=3.330657$
$a_2=\ .250621$	$b_2=1.681534$

5.1.14

$$E_{n+1}(z)=\frac{1}{n}\left[e^{-z}-zE_n(z)\right]\quad (n=1,2,3,\ldots)$$