12.3 This problem asks us to show, using the rock salt crystal structure, that the minimum cation-to-anion radius ratio is 0.414 for a coordination number of six. Below is shown one of the faces of the rock salt crystal structure in which anions and cations just touch along the edges, and also the face diagonals.

From triangle $FGH$, $GF = 2r_A$ and $FH = GH = r_A + r_C$.

Since $FGH$ is a right triangle

$$(GH)^2 + (FH)^2 = (FG)^2$$

or

$$\left(r_A + r_C\right)^2 + \left(r_A + r_C\right)^2 = (2r_A)^2$$

which leads to

$$r_A + r_C = \frac{2r_A}{\sqrt{2}}$$

Or, solving for $r_C/r_A$

$$\frac{r_C}{r_A} = \left(\frac{2}{\sqrt{2}} - 1\right) = 0.414$$

12.15 This problem calls for us to determine the unit cell edge length for MgO. The density of MgO is 3.58 g/cm$^3$ and the crystal structure is rock salt.
(a) From Equation (12.1)

\[
\rho = \frac{n'(A_{Mg} + A_{O})}{V_{C}N_{A}} = \frac{n'(A_{Mg} + A_{O})}{a^{3}N_{A}}
\]

Or,

\[
a = \left( \frac{n'(A_{Mg} + A_{O})}{\rho N_{A}} \right)^{1/3}
\]

\[
= \left( \frac{(4 \text{ formula units/unit cell})(24.31 \text{ g/mol} + 16.00 \text{ g/mol})}{(3.58 \text{ g/cm}^{3})(6.023 \times 10^{23} \text{ formula units/mol})} \right)^{1/3}
\]

\[
= 4.21 \times 10^{-8} \text{ cm} = 0.421 \text{ nm}
\]

(b) The edge length is to be determined from the Mg^{2+} and O^{2-} radii for this portion of the problem. Now

\[
a = 2r_{Mg^{2+}} + 2r_{O^{2-}}
\]

From Table 12.3

\[
a = 2(0.072 \text{ nm}) + 2(0.140 \text{ nm}) = 0.424 \text{ nm}
\]

12.32 (a) For a Cu^{2+}O^{2-} compound in which a small fraction of the Cu^{2+} ions exist as Cu^{+}, for each Cu^{+} formed there is one less positive charge introduced (or one more negative charge). In order to maintain charge neutrality, we must either add an additional positive charge or remove a negative charge. This may be accomplished by either creating Cu^{2+} interstitials or O^{2-} vacancies.

(b) There will be two Cu^{+} ions required for each of these defects.

(c) The chemical formula for this nonstoichiometric material may be expressed as Cu_{1+x}O or CuO_{1-x}, where x is some small fraction.

12.38 We are asked for the critical crack tip radius for an Al_{2}O_{3} material. From Equation (8.1)
\[ \sigma_m = 2 \sigma_0 \left( \frac{a}{\rho_t} \right)^{1/2} \]

Fracture will occur when \( \sigma_m \) reaches the fracture strength of the material, which is given as \( \frac{E}{10} \); thus

\[ \frac{E}{10} = 2 \sigma_0 \left( \frac{a}{\rho_t} \right)^{1/2} \]

Or, solving for \( \rho_t \)

\[ \rho_t = \frac{400 a \sigma_0^2}{E^2} \]

From Table 12.5, \( E = 393 \) GPa, and thus,

\[ \rho_t = \frac{(400)(2 \times 10^{-3} \text{ mm})(275 \text{ MPa})^2}{(393 \times 10^3 \text{ MPa})^2} \]

\[ = 3.9 \times 10^{-7} \text{ mm} = 0.39 \text{ nm} \]