Vacuum Technology II

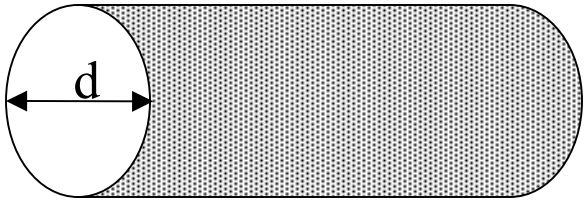
Dr. Philip D. Rack
Assistant Professor
Department of Materials Science and Engineering
University of Tennessee
603 Dougherty Engineering Building
Knoxville, TN 37931-2200
Phone: (865) 974-5344
Fax (865) 974-4115
Email: prack@utk.edu

Gas Flow

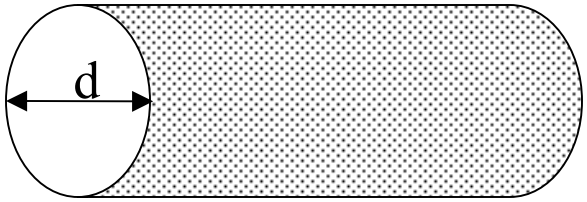
- Flow Regimes
  - Nature of the Gas (Knudsen's Number)
  - Relative Quantity (Reynolds' Number)
  - Turbulent
  - Laminar
  - Viscous
  - Molecular
  - Transition

Knudsen's Number (Kn)

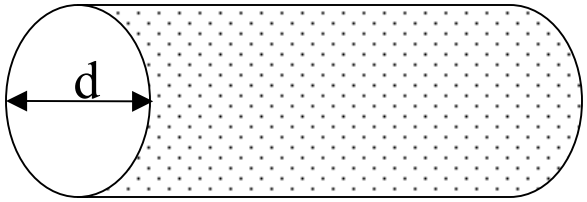
$$Kn = \frac{\lambda}{d}$$



Kn < 0.01 Viscous Flow - gas flow determined by gas-gas collisions



1 > Kn > 0.01 Transition Flow



Kn > 1.0 Molecular Flow - gas flow determined by gas-wall collisions

Reynolds' Number

$$R = \frac{U\rho d}{\eta}$$

where:

U = gas stream velocity

ρ = mass density of the gas

d = diameter of the tube

η = viscosity

R > 2200 Turbulent Flow

R < 1200 Laminar Flow

1200 < R < 2200 Turbulent or Viscous Depending on the Geometry

Flow Regimes

- Laminar Viscous Flow - Ordered flow of gas in streamlines. Dominated by gas-gas collisions
  - $Kn < 0.01$, $R < 1200$
- Molecular Flow - Dominated by gas-wall collisions. Assume diffuse reflection at surfaces (molecules are re-emitted in a direction independent of the incident direction). Gas molecules do not collide with eachother.
  - $Kn > 1$, $R < 1200$
- Turbulent Flow -
  - $Kn < 0.01$, $R > 2200$
- Transition Flow - Gas is neither viscous nor molecular. Flow is dominated by both gas-gas collisions and gas-wall collisions
  - $1 > Kn > 0.01$

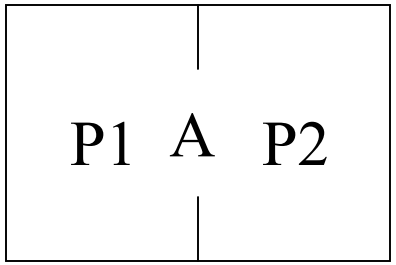
Definitions

- Throughput (Q) - Quantity of gas (the volume of gas at a known pressure) that passes a plane in a known time. Units = Pa-m^3/s or Watts (energy flow -- energy it takes to transport the molecules across a plane)
  - Q = P(dV/dt) where: P=pressure and dV/dt = volumetric flow rate
- Mass Flow - The quantity of a substance (kg) that passes a plane in a known time. Units = kg/s
- Molecular Flow - The quantity of a substance (number of molecules N) that passes a plane in a known time. Units = N/s
- Conductance - The ability of an object to transport gas between two pressures regimes. Units = m^3/s
  - Analogous to ohms law -- 1/R= I/V

$$C = \frac{Q}{(P_2 - P_1)}$$

Viscous flow -- Orifice

- Orifice – tube of zero length
  - As P2 drops (P2/P1) decreases which induces a gas flow through orifice A. As P2/P1 decreases the gas flow through the orifice will increase until it reaches a maximum. At this maximum, this ratio of inlet pressure (P1) to outlet pressure (P2) has reached a "critical pressure ratio." at this ratio the flow becomes "choked" at the speed of sound (343m/s)



Viscous Flow -- Orifice

$$Q = AP_1C'\left(\frac{2\gamma}{\gamma-1}\frac{kT}{m}\right)^{\frac{1}{2}}\left(\frac{P_2}{P_1}\right)^{\frac{1}{\gamma}}\left[1-\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}\right]^{\frac{1}{2}}$$

$for:$

$$1 > \frac{P_2}{P_1} \geq \left(\frac{2}{\gamma-1}\right)^{\frac{\gamma}{\gamma-1}}$$

$where:$

$$\gamma = \frac{C_P}{C_V}, \gamma = 1.4 \text{ (diatomic species)}, 1.667 \text{ for (monatomic)}, \text{ and } 1.333 \text{ (triatomic)}$$

C' is a factor which reduces the cross sectional area

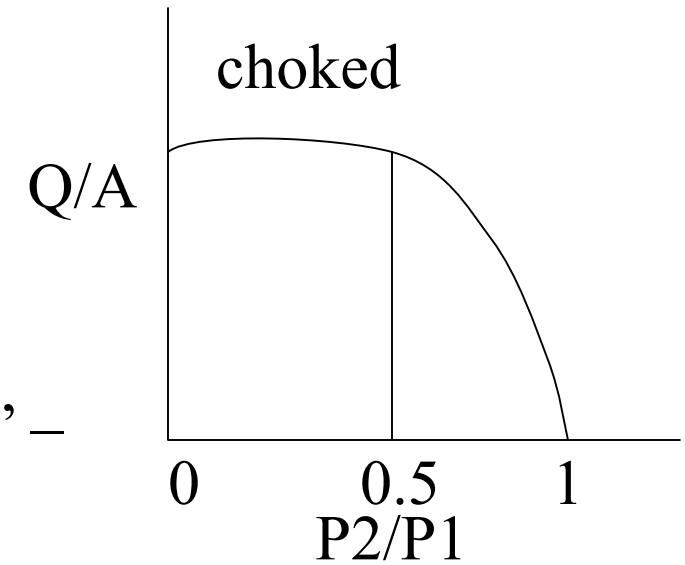
because of the high speed of gas flow "vena contracta" ≈ 0.85

Viscous Flow -- Orifices

$$for\ \frac{P_2}{P_1} \geq \left(\frac{2}{\gamma - 1}\right)^{\frac{\gamma}{\gamma-1}} = 0.525 \text{ for air } (\gamma = 1.4)$$

$$Q = AP_1C'\left(\frac{2\gamma}{\gamma+1}\frac{kT}{m}\right)^{\frac{1}{2}}\left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}}$$

Flow is independent of P2



Flow is said to be "critical" or "choked" – limited by gas stream traveling at the speed of sound

Viscous Flow -- Orifices

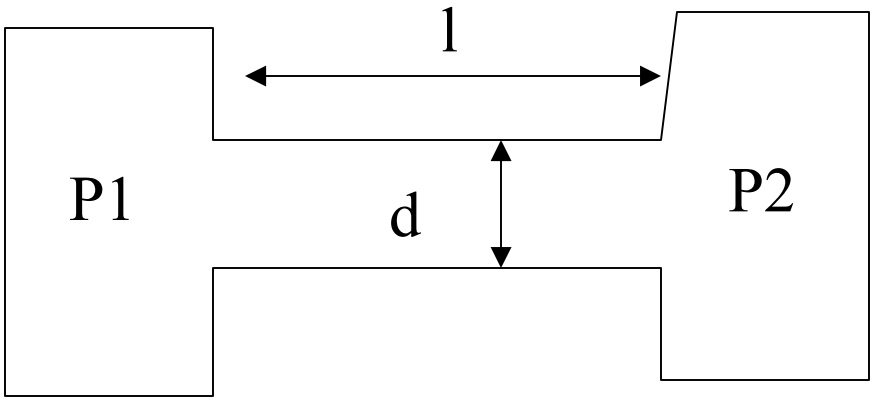
For Air at Room Temperature

$$C\left(\frac{l}{s}\right) = \frac{7.66x10^5 C' A(m^2)}{1-\frac{P_2}{P_1}}\left(\frac{P_2}{P_1}\right)^{0.714}\left[1-\left(\frac{P_2}{P_1}\right)^{0.286}\right]^{\frac{1}{2}}$$

$$for: 1 > \frac{P_2}{P_1} \geq 0.52$$

Viscous Flow – Long Round Tubes

Hagan-Poiseuille Equation



$$Q = \frac{\pi d^4}{128 \eta l} \frac{P_1 + P_2}{2} (P_1 - P_2)$$

For Air at Room Temperature

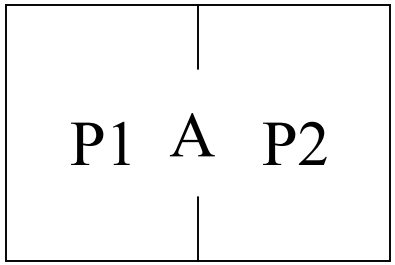
$$C\left(\frac{l}{s}\right) = 1.38x10^6 \frac{d(meters)^4}{l} \frac{P_1 + P_2(Pascals)}{2}$$

Viscous Flow – Long Round Tubes

- Assumptions for Hagan-Poiseuille Equation
  - Fully developed flow – ie velocity profile is not position dependent
  - Laminar flow
  - Zero wall velocity
  - Incompressible gas – U(mach number) = $U/U_{sound} < 1/3$ (ie non-choked flow)

Viscous flow, R<1200

Molecular Flow -- Orifice



$$Q_1 = P_1 \frac{dv}{dt}, PV = NKT$$

$$P_1 \frac{dv}{dt} = \frac{dN}{dt} kT,$$

$$Q_1 = kT \frac{dN}{dt}, \frac{dN}{dt} \frac{1}{A} = \Gamma$$

$$Q_1 = kT\Gamma A, \Gamma = \frac{1}{4} n v_{(avg)}, n = \frac{P}{kT}$$

$$Q_1 = \frac{1}{4} P_1 A v_{(avg)}, Q_2 = \frac{1}{4} P_2 A v_{(avg)}$$

$$Q = Q_1 - Q_2 : Q = \frac{1}{4} v_{(avg)} (P_1 - P_2)$$

For Air at Room Temperature

$$C\left(\frac{m^3}{s}\right) = 116 A(m^2)$$

$$C\left(\frac{m^3}{s}\right) = 11.6 A(cm^2)$$

Molecular Flow – Long Round Tube

$$C = \frac{\pi}{12} v \frac{d^3}{l}$$

For Air at Room Temperature

$$C\left(\frac{m^3}{s}\right) = 121 \frac{d^3(m)}{l(m)}$$