INTRODUCTION

Spreadsheets provide a powerful and flexible modeling platform, but they have a number of limitations. Perhaps the most serious limitation is that the row-and-column format does not allow for the development of model boundaries and structure independent of the numerical details. The boundaries of a model determine which factors are included and which are excluded. Model structure includes the key inputs and outputs, as well as the relationships that link outputs to inputs. In every spreadsheet, the numerical details are commingled with model boundaries and structure.

This commingling of boundaries, structure, and numbers is not a problem for experienced modelers, because they have learned to develop a model structure before approaching the spreadsheet. In contrast, we often observe novice modelers taking whatever information is at hand and entering it into a spreadsheet before they have sufficiently developed the model itself. In other words, novices generally approach a spreadsheet-modeling task from the bottom up, without deciding such essential modeling issues as what the key outputs are, how those outputs will be obtained from the inputs, and how essential relationships will be modeled. Experts, on the other hand, usually develop a top-down, or high-level, view of their model, often using a table, chart, or sketch. That is to say, experts concentrate on defining the boundaries and the essential structure of their models first. Experts may choose a bottom-up approach (starting with the details) when they are working on a familiar problem, but it is an ineffective approach for novices. It is also ineffective for anyone, novice or expert, who is confronting an unfamiliar type of modeling problem. Bottom-up modeling is also an ineffective approach for teams. An early task for a modeling team is to bring to light each other’s mental models of the situation at hand. Only then can they develop a shared understanding of model boundaries and structure.

Expert modelers use a variety of visual modeling tools in the early stages of model formulation and design. These tools are useful in providing a high-level structure to a model before implementing it in a spreadsheet. Individuals can use these tools to develop their understanding of the fundamental interconnections that drive the problem. Teams can use these tools to develop a shared understanding of the essential elements in the model. A number of these tools can also be used to present the essential features of a model to clients. In this chapter, we describe four visual modeling tools:

- Influence charts
- Outlines
- Decision trees
- Network diagrams

Influence charts are the most general and powerful of these tools. They identify the main elements and delineate the boundaries of a model. We recommend using them in the
early stages of any unstructured problem-formulation task. Outlines can be useful in organizing the components of a model and ultimately in laying out the rows of a spreadsheet. Many standard spreadsheets, such as financial statements, can be structured in outline form. Decision trees are particularly useful in situations where there are random variables and a sequential logic to events. Finally, network diagrams are handy for creating a specialized structure where material flows across space or time.

INFLUENCE CHARTS

We pointed out in Chapter 2 that model building and analysis are used within the broader context of problem solving. To be successful, this process must begin with the recognition of a problem and end with implementation of a solution. At a minimum, modeling should help in evaluating alternative solutions, but it can also provide the analyst with an enhanced intuitive understanding of the problem and the forces within it.

One of the key challenges modelers face in the problem-solving process is how to translate an initial, vague understanding of a problem into a concrete model. A mathematical model, of course, requires specific numerical inputs and outputs and also the precise relationships that connect them. As we have mentioned, many modelers make the mistake of plunging into the details of a model before they think through the role the model will play in the overall process. We recommend a different approach, using the power of visualization to develop a broad understanding of the critical inputs, outputs, and relationships in a model before building a prototype. An influence chart is a simple diagram that shows what outcome variables the model will generate and how these outputs are calculated from the necessary inputs. The process of building an influence chart is an instance of the decomposition heuristic that we described in Chapter 3. As with any form of decomposition, the benefit of using the heuristic is the clarity it brings to the task. Note, however, that influence charts are not designed to provide numerical results or even insights into which particular solutions are desirable.

Influence charts are particularly powerful in the early, conceptual stages of a modeling effort. They encourage the modeler or modeling team to focus on major choices, such as what is included and what is excluded, rather than on details that may ultimately turn out to be unimportant. Influence charts thus provide a high-level view of the entire model that can be comprehended at one glance. This high-level perspective, in turn, supports modeling in teams by facilitating communication among team members. As a result, areas of agreement and disagreement among team members surface early. Influence charts can also be highly effective in communicating the essence of the modeling approach to clients.

Influence charts are flexible, so they support the kind of frequent revision that effective modeling requires. We often encourage our student teams to devote the first hour in the life of a model to working out an influence chart. In addition, we ask them not to turn on the computer until all members of the team agree that their chart represents a suitable initial description of their model.

EXAMPLE

A Pricing Decision

The task at hand is to determine the price we should set for our product so as to generate the highest possible profit this coming year. Since our plan will ultimately be measured by its profitability, we define Profit as the outcome measure and enclose it in a hexagon (Figure 4.1A). Next we ask what we need to know to determine Profit. The necessary components, Total Revenue and Total Cost, are drawn as variables enclosed in circles to the left of Profit and connected to it by arrows (Figure 4.1B). These arrows
identify which variables are required to calculate the outcome. Next, Total Cost is determined by Fixed Cost and Variable Cost, which are drawn to the left of Total Cost (Figure 4.1C). Variable Cost in turn is the product of Quantity Sold and Unit Cost (Figure 4.1D). Now we turn to Total Revenue, which is the product of Quantity Sold and Price. We add Price and enclose it in a box to show it is our decision variable (Figure 4.1E). Finally, Price Elasticity, along with the price we set, determines Quantity Sold. So, in Figure 4.1F, we add the Price Elasticity variable and an arrow from Price to Quantity Sold.

Traditionally, influence charts are built from right to left, using diagrammatic conventions that distinguish the roles of different types of variables. For example, we use hexagons to represent outputs and boxes to represent decisions, as indicated in our example. We also use circles to represent other variables. As we complete the layout, we can identify certain of the variables as inputs. These are shown in the diagram as triangles. Later, we will also use double circles to represent variables that are random.

While this is a highly simplified example, its development does involve a number of modeling choices. For example, we can see in the influence chart that Fixed Cost is assumed to be a known quantity, since there are no variables that are needed to determine
Fixed Cost. In another situation, we might face a set of choices as to which production technology to choose for the coming year. In this case, Fixed Cost would not be known but would be influenced by our technology choices, and the chart would have to reflect those complexities. Another modeling choice is evident in how Quantity Sold is determined. In our chart, both Price and Price Elasticity influence Quantity Sold. This reflects our modeling judgment that we face a price-dependent market. In many situations, we might assume instead that Sales are independent of Price, at least within a reasonable range of prices. One final modeling decision is evident in our chart: since Quantity Sold determines Variable Costs, we are assuming that production and sales are simultaneous. If, on the other hand, it were our practice to produce to stock and to sell from inventory, we would need to modify the chart to reflect this process.

This example illustrates that influence charts help the modeler make explicit decisions about what is included in the model and how the variables interact to determine the output.
A Pro Forma Income Statement

An income statement is a standard accounting framework that is widely used for projecting the financial future of a company. The bottom line in an income statement is Retained Earnings, which is roughly the difference between revenue and costs, adjusted for taxes and dividends. A simple income statement is shown in the form of an influence chart in Figure 4.2.

If our purpose were simply to record the historical performance of a company, then the relationships depicted in Figure 4.2 would be sufficient. Moreover, the related spreadsheet would consist entirely of numbers; no formulas would be needed because all variables are already determined. However, Figure 4.2 would be inadequate if our purpose were to make projections into the future because it reveals nothing about how critical variables such as Sales Revenue and Cost of Goods Sold will be determined. In other words, Figure 4.2 represents only a static accounting framework and not a model of the future. To convert a static income statement into a model, we will need to determine how underlying variables such as Quantity Sold will evolve over time. In a simple model, we could assume that Unit Cost and Price will be constant and that Quantity Sold will be determined by Initial Sales and Sales Growth Rate. Figure 4.3 shows an influence chart for this model.

It is noteworthy that even in this case, where accounting rules determine much of the model structure, an influence chart is useful for depicting the underlying forces that drive the results.

PRINCIPLES FOR BUILDING INFLUENCE CHARTS

An influence chart is not a technical flowchart that must conform perfectly to a rigid set of rules. Rather, it is a somewhat free-form visual aid for thinking conceptually about a model. We offer the following guidelines for constructing such charts:
FIGURE 4.2
Influence Chart for a Static Income Statement

FIGURE 4.3
Influence Chart for an Income-Statement Model
Start with the outcome measure. To decide which variable this is, ask what single variable the decision maker will use to determine success or failure.

Decompose the outcome measure into a small set of variables that determine it directly. Each of these influencing variables should be independent of the others, and together they should be sufficient to determine the result.

Take each variable in turn and repeat this process of decomposition. For each variable, ask, “What do I need to know to determine . . . ?”

Identify input data and decisions as they arise.

A variable should appear only once in the diagram.

Highlight special types of elements with special symbols. For example, we use squares for decision variables and double circles for random variables, but any consistent code will work.

The most common error in drawing influence charts is to draw an arrow from the output back to the decisions. The motivation for this seems to be that the outcome will be used to determine the best decisions. Remember, however, that an influence chart is simply a description of how we will calculate outcomes for any set of decisions and other parameters. It is not intended to be used to find the best decisions. That is a separate process, requiring an actual model, not simply a diagram.

In what follows, we present two detailed exercises in building influence charts for unstructured problems. Read each case and draw an influence chart before proceeding. We will then describe the process of building an influence chart and discuss some of our modeling choices. Keep in mind, however, that there is no one correct diagram, just as there is no one correct model.

EXAMPLE

The SS Kuniang*

In the early 1980s, New England Electric System (NEES) was deciding how much to bid for the salvage rights to a grounded ship, the SS Kuniang. If the bid were successful, the ship could be repaired and fitted out to haul coal for the company’s power-generation stations. But the value of doing so depended on the outcome of a U.S. Coast Guard judgment about the salvage value of the ship. The Coast Guard’s judgment involved an obscure law regarding domestic shipping in coastal waters. If the judgment were to indicate a low salvage value, then NEES would be able to use the ship for its shipping needs. If the judgment were high, the ship would be considered ineligible for use in domestic shipping unless a considerable amount of money was spent in fitting her with fancy equipment. In effect, this would mean additional expenses for NEES. The Coast Guard’s judgment would not be known until after the winning bid was chosen, so there was considerable risk associated with submitting the winning bid. If the bid were to fail, the alternatives would include purchasing either a new ship or a tug/barge combination, both of which were relatively expensive alternatives. One of the major issues was that the higher the bid, the more likely that NEES would win. NEES judged that a bid of $2 million would definitely not win, whereas a bid of $12 million definitely would win. Any bid in between was possible.

The goal here is to select an amount to bid for the SS Kuniang that will allow NEES to supply coal to its plants in the most economical way. We assume that the amount of coal

to be shipped is fixed and that NEES will either use the Kuniang or buy a new ship or a tug/barge combination. That is, we explicitly rule out the possibility that NEES can avoid meeting the demand for shipped coal. We further assume that the outcome measure is the NPV of profits from this shipping operation over an appropriate time period (in the case of a ship, perhaps twenty years).

Our influence chart starts with an outcome measure for NPV and two influences: Costs and Revenues (Figure 4.4). Since the revenues are most likely independent of the ship chosen, that part of the diagram does not need to be developed further. The costs incurred in coal shipping depend on which option is chosen. Apparently, NEES can always buy a new ship or a tug/barge combination, and it may have the option to buy the Kuniang if its bid wins. So Costs will be calculated as the minimum of these three costs. The costs of the Kuniang are the essential part of the model. These costs are dependent on the salvage value set by the Coast Guard, which is unpredictable and is therefore shown as a random variable (a double circle). The cost is also influenced by our bid and whether we win the auction. In Figure 4.4, we have shown the outcome of the auction as the random variable “Win?” We have in mind a simple model in which the probability of winning increases as our bid increases. But this is an area of the diagram where further elaboration could be productive. We could, for example, add modules for the bids of our competitors. We could also add a module for the auction process itself. Whether to add further detail is always the modeler’s judgment. But this simple influence chart is sufficiently detailed to support the building of a prototype model.

One additional point to notice here is that the numerical information in the problem statement, which places some limits on reasonable bids, plays no role at all in constructing the influence chart. In fact, we routinely ignore all available numerical data when we build influence charts because the goal is to develop a problem structure, not to solve the problem. Problem structure is not influenced by the values of parameters. This principle conflicts with another that many of us learned in early math classes, which was to use all the given data to solve the problem. This may be an appropriate problem-solving heuristic for simple math problems in school, but it is not necessarily helpful in structuring real business decisions.

FIGURE 4.4
S.S. Kuniang Influence Chart
During the 1990s, leasing grew to 40 percent of new-car sales. Nowadays, the most popular leases are for expensive or mid-range vehicles and carry terms of twenty-four or thirty-six months. The most common form of leasing is the **closed-end** lease, where the monthly payment is computed based on three factors:

- **Capitalized Cost**: the purchase price for the car, net of trade-ins, fees, discounts, and dealer-installed options.
- **Residual Value**: the value of the vehicle at the end of the lease, specified by the leasing company (the “lessor”) in the contract. The customer has the right to purchase the vehicle at this price at the end of the lease.
- **Money Factor, or Rate**: the interest rate charged by the leasing company.

A lower residual value results in higher monthly payments. Therefore, a leasing company with the highest residual value usually has the lowest, and most competitive, monthly payment. However, if the actual end-of-lease market value is lower than the contract residual value, the customer is likely to return the car to the lessor. The lessor then typically sells the vehicle, usually at auction, and realizes a “residual loss.”

A low residual value results in a high monthly payment (which is relatively less attractive), but, at the end of the lease, if the actual market value is greater than the contract residual, the customer is more likely to purchase the vehicle. By then selling the vehicle for the prevailing market value, the customer in essence receives a rebate for the higher monthly payments. (Of course, the customer may also decide to keep the car.) When customers exercise their purchase option, the lessor loses the opportunity to realize “residual gains.”

The primary challenge for companies offering a closed-end lease is to select the residual value of the vehicle. Intelligent selection means offering competitive monthly payments on the front end without ignoring the risk of residual losses on the back end. In approaching this problem from a modeling perspective, the first task is to find ways to cut it down to size. After all, any leasing company offers leases on dozens of vehicles at any one time. Furthermore, unless it is just starting to do business, the company has an existing portfolio of hundreds of leases on its books, and the risk characteristics of this portfolio may influence the terms offered on new leases. We can become overwhelmed by complexity if we start by trying to model the entire problem. A modular, prototyping approach is vital here.

One reasonable approach is to develop a prototype for a specific lease on a single type of vehicle. Once a prototype model based on this diagram is tested and proved, we can expand on it by bringing in excluded aspects of the problem.

An example will make the problem more concrete. Consider new Honda Accord models, which sell for $25,000. Also consider only three-year leases, and assume the money rate is fixed at 5 percent. Given these assumptions, our goal is to determine the best contract residual value (CRV) for a single lease on a single class of vehicles.

The CRV is clearly our decision variable. How will we determine whether we have made a good choice? Once we have chosen the CRV (and the other terms of the lease), we will offer it to the leasing market. Some number of customers will purchase our lease and pay us the monthly lease payments for three years. (A few will default during this period, but we ignore that factor in our initial prototype.) Our monthly lease revenues will be the product of the monthly payment and the number of leases sold. The monthly payment, in turn, will depend on the term, the money factor, and the CRV.
At the end of three years, all our leases will expire. Some customers will buy their vehicles at the CRV; others will return their vehicles and take a new lease with us; still others will return their vehicles and not purchase another lease with us. (We ignore the value of follow-on leases in our initial prototype.) When all is said and done, we will have made some level of profit. Profit, then, is our outcome measure, and it is influenced by three factors: lease revenues, our cost of borrowing (to pay for new vehicles), and the residual value of vehicles at the end of the lease (Figure 4.5).

So far, this is a rather straightforward influence chart. But two parts of it deserve additional attention. First, what determines how many leases are sold? Presumably, customers are sensitive to the monthly payment, and that influence is shown in the diagram, but what else influences volume? One simple approach is to assume a value for demand elasticity: volume increases (or decreases) by $x$ percent when our monthly payments decrease (or increase) by 1 percent. This relationship is sufficient to generate some realistic aspects of the lease market—namely, a decline in volume with increasing payments—and it may be sufficient for a prototype model. But it does not explicitly include any information about our competitor’s monthly payments. In particular, the elasticity is probably different when our payments are above the competition than when they are below. This may be a fertile area for refinement in later prototypes.

We should also consider what factors determine the residual value of the vehicle to the leasing company. When a lease expires, the contract allows the customer to purchase the vehicle for the CRV or to return it to the leasing company. The customer’s decision at this point is crucial to determining the profitability of the lease. If used-car prices are high relative to the CRV, it is in the customer’s interest to buy the car at the CRV and then sell it for the higher market price. On the other hand, if used-car prices are low, customers will tend to return their leased vehicles and buy a cheaper equivalent used car. In this case, the leasing company will have to sell the vehicle at the low market price. And, of course, some customers will lease a new vehicle regardless of used-car prices, and some may not behave in an economically rational manner at all. Should we include all of these factors in our influence chart?

**FIGURE 4.5**
National Leasing Influence Chart
One approach would be to assume that all vehicles will be purchased if used-car prices exceed the CRV, and none will be purchased if the reverse holds. But how do we know how much used cars will be worth three years from now? In our chart, we model used-car prices as a random variable—for example, a normal distribution with a mean of $15,000 and a standard deviation of $2,000. Alternatively, we might assume that this class of vehicles loses a random amount of its value each year, where the annual loss is uniformly distributed between 8 and 12 percent. This slightly more detailed model will also generate a distribution of values three years from now. In further refinements of the chart, we might expand on these ideas and model the fundamental determinants of used-car values: new-vehicle quality, the macro economy, and so on. In any case, a random value for used-car prices captures one of the essential features of this problem—namely, the risk of residual losses and residual gains. This influence chart is probably sufficiently detailed to support construction of a prototype model. Working with this model will help us discover whether we have captured the essential trade-offs in the problem.

As we have stressed before, the influence chart documents the simplifying assumptions made during the modeling process. Here are some of the critical assumptions embodied in Figure 4.5:

- one vehicle/one lease term
- no lease defaults
- no follow-on leases
- rational behavior of customers at lease end
- random used-car prices

We recommend that the modeler or one member of the modeling team record each assumption as it is made during the process of developing an influence chart. This is useful for two reasons. First, it focuses attention and discussion on assumptions as they are being made. Second, each assumption should be viewed as a potential area for later refinement of the model.

OUTLINES

Influence charts and other visual modeling tools are powerful because they assist the modeler in separating the design of a model from the design of the spreadsheet that implements the model. As we have noted elsewhere, modelers too often begin the modeling process by entering information in a spreadsheet without first establishing the model’s essential relationships. This approach is generally ineffective for novice modelers and for unstructured problems. Learning to use visual modeling tools such as influence charts is an important step in the development of modeling expertise.

The influence chart is the tool of choice for complex, unstructured problems. These are problems in which the highly structured row-and-column format of the spreadsheet would limit the creativity of the modeler during the initial phases of problem structuring. However, there are circumstances in which the row-and-column layout of the spreadsheet corresponds so closely to the desired model logic that it can provide a useful visual modeling paradigm. Usually, the model involves projecting a set of variables over time, so that the column headings naturally represent the time dimension (months, quarters, years, etc.). The rows represent variables in the model. Since we know that the columns will correspond to time periods, it is effective to concentrate initially on determining the appropriate row headings. Two things must be determined at this stage: what are the appropriate variables, and what is the most logical sequence in which to calculate them?
an outline of a model by concentrating on the headings of rows in a spreadsheet can be an effective way to conceptualize a model without becoming buried in details.

### Outline for the Pricing Decision

To illustrate the use of outlines as a modeling tool, we return to the pricing example discussed in the previous section. Since we are interested in setting a price that maximizes our profits over the coming year, the time dimension across the columns of the spreadsheet will most likely consist of either twelve months or four quarters. The different variables will appear down the rows. Initially, we might simply describe Profit as determined by Total Revenue and Total Cost, as follows:

**Initial Pricing Model in Outline Form**

<table>
<thead>
<tr>
<th>Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Revenue</td>
</tr>
<tr>
<td>Total Cost</td>
</tr>
</tbody>
</table>

At a more detailed level, we can decompose Total Revenue into Quantity Sold and Unit Price, and Total Cost into Fixed Cost and Variable Cost:

**Second Pricing Model in Outline Form**

<table>
<thead>
<tr>
<th>Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Revenue</td>
</tr>
<tr>
<td>Quantity Sold</td>
</tr>
<tr>
<td>Unit Price</td>
</tr>
<tr>
<td>Total Cost</td>
</tr>
<tr>
<td>Fixed Cost</td>
</tr>
<tr>
<td>Variable Cost</td>
</tr>
</tbody>
</table>

At an even more detailed level, we can add the determinants of these components:

**Third Pricing Model in Outline Form**

<table>
<thead>
<tr>
<th>Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Revenue</td>
</tr>
<tr>
<td>Quantity Sold</td>
</tr>
<tr>
<td>Unit Price</td>
</tr>
<tr>
<td>Price Elasticity</td>
</tr>
<tr>
<td>Unit Price</td>
</tr>
<tr>
<td>Total Cost</td>
</tr>
<tr>
<td>Fixed Cost</td>
</tr>
<tr>
<td>Variable Cost</td>
</tr>
<tr>
<td>Quantity Sold</td>
</tr>
<tr>
<td>Unit Cost</td>
</tr>
</tbody>
</table>

As we develop these outlines in more and more detail, we often find that some variables begin to repeat. For example, Unit Price repeats under Total Revenue, because it directly influences both the Quantity Sold and Total Revenue. Similarly, Quantity Sold
repeats because it influences both Total Revenue and Total Cost. This is not necessarily bad practice when outlining a model, but, as we might expect, repeating calculations in a spreadsheet model is to be avoided. Here, we can avoid the repetition by adding formulas for calculating variables that use information from previous rows, as shown below. We also add headings to group related variables. Finally, we have reordered the rows to ensure that inputs appear before the variables that depend on them. As discussed in Chapter 5, when we cover the principles of good spreadsheet design, it is advantageous to have all rows calculated from inputs that appear above them.

**Final Pricing Model in Outline Form**

- **Revenue**
  - Unit Price
  - Price Elasticity
    - Quantity Sold = function of Unit Price and Elasticity
    - Total Revenue = \( \text{Quantity Sold} \times \text{Unit Price} \)

- **Cost**
  - Fixed Cost
  - Unit Cost
    - Variable Cost = \( \text{Quantity Sold} \times \text{Unit Cost} \)
    - Total Cost = \( \text{Fixed Cost} + \text{Variable Cost} \)

- **Profit**
  = Total Revenue – Total Cost

**Outline for the Pro Forma Income Statement**

A pro forma income statement is another common example of a model structure that takes the form of an outline. In the previous section, we described the process of creating an influence chart for an income statement. In its simplest form, an income statement is a list of financial variables:

**Income Statement in List Form**

- Sales
- Cost of Goods Sold
- Depreciation
- Interest
- Profit before Taxes
- Taxes
- Profit after Taxes
- Dividends
- Retained Earnings

While all the necessary variables are included here, the list form disguises some of the structure behind the income statement. A more useful format is an outline, in which we indent items that are components of other calculations. We also add the heading Expenses to show that three items (Cost of Goods Sold, Depreciation, and Interest) all belong to the same category.
Income Statement in Outline Form

Sales
Expenses
  Cost of Goods Sold
  Depreciation
  Interest
Profit before Taxes
  Taxes
Profit after Taxes
  Dividends
Retained Earnings

One advantage of standard formats such as the income statement is that many of the variables are determined by standard definitions and therefore require no explicit modeling. For example, Profit before Taxes always equals the difference between Sales and Expenses. On the other hand, a variable such as Cost of Goods Sold could be represented as a percentage of Sales (where the percentage is an input parameter that could be modeled as constant or as varying with time), or as the sum of Fixed Costs and Variable Costs. Likewise, Variable Costs could be modeled in turn as a percentage of Sales. If an element in the outline is not determined by a definition, then it must be modeled in some way. Such choices should be made taking into account the specifics of the situation at hand. The definitional relationships, by contrast, require no explicit modeling; in effect, they come for free. We have added these definitional relationships to the outline below:

Income Statement with Definitional Relationships

Sales
Expenses = Cost of Goods Sold + Depreciation + Interest
  Cost of Goods Sold
  Depreciation
  Interest
Profit before Taxes = Sales – Expenses
  Taxes
Profit after Taxes = Profit before Taxes – Taxes
  Dividends
Retained Earnings = Profit after Taxes – Dividends

In our final version, we have formulated equations for each of the variables not given by a definition. For example, we have projected Sales each year by multiplying the previous year’s sales by a constant growth rate.

Fully Specified Income Statement

Sales = Previous sales × (1 + Sales growth rate)
Expenses = Cost of Goods Sold + Depreciation + Interest
  Cost of Goods Sold = Sales × Cost of Goods Sold percentage
  Depreciation = Current assets × Depreciation rate
  Interest = Current year’s debt × Interest rate
Profit before Taxes = Sales – Expenses
  Taxes = Profit before Taxes × Tax rate
Profit after Taxes = \( \text{Profit before Taxes} - \text{Taxes} \)
Dividends = \( \text{Profit after Taxes} \times \text{Dividend rate} \)
Retained Earnings = \( \text{Profit after Taxes} - \text{Dividends} \)

It might seem that outlines are useful only for accounting models, in which much of the model structure is predefined. However, we have found that outlines can be used more broadly to structure the thinking of an individual or a group in open-ended situations.

**EXAMPLE**

**Buying a Cottage**

Bruce and Amy, both fifty-eight, are considering buying a cottage at a nearby lake. Amy expects to inherit a substantial sum of money within the next ten to fifteen years, which she can use to help pay the costs of the cottage. However, she is concerned that their income will be insufficient to meet their other needs after Bruce retires. Bruce’s main concern is that his retirement may have to be postponed if their income does not grow as expected.

The couple has tried to discuss this situation several times but found it difficult. One problem is that each of them focuses on different aspects of the problem, such as whether the inheritance will become available, how quickly other assets will appreciate, when Bruce will decide to retire, and so on. Another complication is that each person evaluates the risks differently. For example, Amy fears becoming strapped for cash ten years from now and having difficulty paying for the cottage. Bruce, on the other hand, feels they can always sell the cottage if necessary, so the financial risks seem much smaller to him. The two need some way to agree on a set of assumptions and to trace through their implications. A simple but flexible financial model would be of considerable help.

Any model for this situation will probably involve projections of the couple’s cash flows and assets from the present into the future. Since the time dimension is obvious, attention should focus on the variables to project. An initial outline could simply track income, expenses, and assets:

**Initial outline**

- Income
- Expenses
- Excess cash = Income – Expenses
- Assets

At the next level of detail, the couple might want to expand these general categories with an eye toward representing the major issues, such as the inheritance and cottage expenses.

**First refinement**

\[ \text{Income} = \text{Bruce’s salary} + \text{Social Security} + \text{Retirement-plan income} + \text{Inheritance} \]
\[ \text{Expenses} = \text{Normal living expenses} + \text{Cottage expenses} \]
\[ \text{Mortgage} + \text{Taxes} + \text{Upkeep} \]
Taxes
Upkeep
Excess cash = Income – Expenses
Assets = Retirement savings + Inheritance assets
Retirement savings
Inheritance assets

Obviously, this outline can be further elaborated if that is needed. In the end, the model may look like a standard spreadsheet, but the end product will not necessarily reveal the value the process brings to the couple making the decision. Elaborating this outline should help the two understand each other's assumptions and values. For example, when is Bruce planning to retire, and how does he feel about retiring a year or two later, if that proves necessary? When does Amy think her inheritance is likely to be available, and how does she feel about using it to cover their expenses? Which categories of living expenses do they feel will change during retirement? Many of these questions can be answered fully only by building a more complete model and using it to analyze the situation. But the process of constructing the outline itself begins to raise questions about what is known, what can be assumed, what is random, and what is a decision. Answering such questions is the essence of modeling. Outlines can be a useful tool in the process.

### DECISION TREES

Decision-tree models offer another visual tool that can complement influence charts and provide a more detailed picture of the uncertain elements in decision making. Decision trees are especially useful in situations where there are multiple sources of uncertainty and a sequence of decisions to make. They help organize the elements in a problem by distinguishing between decisions (controllable variables) and random events (uncontrollable variables). As a first step in describing decision trees, we introduce a simpler structure that is useful in its own right.

A **probability tree** depicts one or more random factors. For example, if we believe that demand for our product is uncertain, we might model that uncertainty using the probability tree in Figure 4.6. In this simple tree, we assume that demand may take on one of the three alternative values: High, Medium, or Low. The node from which the branches emanate is called a **chance node**, and each branch represents one of the possible **states** that could occur. Each state, therefore, is a possible resolution of the uncertainty represented by the chance node. Later, when we make calculations to analyze the model, we specify probabilities for each of the states, thus creating a probability distribution to describe the uncertainty at the chance node.

![Simple Probability Tree](image-url)
In other circumstances, we might want to show greater detail than just three qualitative states for demand, so we might use a tree with more branches, perhaps quantifying the states as $10 million, $20 million, $30 million, $40 million, and $50 million, as shown in Figure 4.7. Again, the tree is meant to show that demand is uncertain and that one of the alternative states will actually occur. When we specify the probabilities for each of these five states, we create a probability distribution for the dollar value of demand.

Probability trees can accommodate more than one source of uncertainty. In addition to the demand uncertainty in Figure 4.6, suppose we face uncertainty in both the number of competing products and the competitive effectiveness of our advertising. Now we can draw a tree with three chance nodes. The first chance node represents demand states, characterized as High, Medium, or Low. For each demand state, one of several possible numbers of competitors will occur. Likewise, for each combination of demand and number of competitors, one of several levels of advertising effectiveness will occur. We can depict this situation either in a telegraphic form (Figure 4.8), in which only one chance node of each type is displayed, or in exhaustive form (Figure 4.9), where all possible combinations are displayed. In either case, the tree conveys the idea that there are forty-five possible
FIGURE 4.9
Three Chance Nodes in Exhaustive Form
states, corresponding to (three demand levels) × (five competitors) × (three levels of advertising effectiveness). Once we realize this fact, we could equivalently represent the outcomes with one chance node and forty-five states. To perform the steps in analyzing the tree, we would need to examine all of the details. For that purpose, the telegraphic form is not sufficiently detailed, and we would instead need to represent the entire set of forty-five alternatives, as in Figure 4.9.

While probability trees are helpful for displaying chance events and their outcomes, they can quickly become complex when there are several sources of uncertainty, as our forty-five-state example suggests. At an early stage in building a tree, it is not necessary to be precise in specifying the number of alternatives at a chance node. It is more important to recognize which outcomes are uncertain and how to structure the tree to represent them.

In a decision tree, we represent decisions as well as chance nodes. For example, suppose that we are introducing a new product and that the first decision determines which channel to use during test-marketing. When this decision is implemented, and we make an initial commitment to a marketing channel, we can begin to develop estimates of demand based on our test. At the end of the test period, we might reconsider our channel choice, especially if the demand has been low, and we may decide to switch to another channel. Then, in the full-scale introduction, we attain a level of profit that depends, at least in part, on the channel we chose initially. In Figure 4.10, we have depicted (in telegraphic form) a situation in which we choose our channel initially, observe the test market, reconsider our choice of a channel, and finally observe the demand during full-scale introduction.

Decision trees are used to describe the choices and uncertainties facing a single decision-making agent. This usually means a single decision maker, but it could also mean a decision-making group or a company. Decisions are represented as boxes, and we can think

![Decision Tree Diagram](image-url)
of them as controllable variables. For each decision, the alternative choices are represented as branches emanating from the decision node. These are potential actions that are available to the decision maker. Uncertainties are represented as circles, and we can think of them as uncontrollable variables. For each source of uncertainty, the possible alternative states are represented as branches emanating from a chance node. If there are other players or agents in the scenario, their actions can be depicted as chance nodes, although this may be an oversimplification. For example, if we were to sue a competitor for patent infringement, it is unlikely that they would toss a coin to decide whether to settle or go to court against us. More likely, they would act as we would in that situation—that is, they would study the situation and make a decision aimed at reaching the best outcome for them. The analysis of the interrelated decisions of two (or more) actors is quite difficult, and we will not discuss it at length except to mention that representing a competitor’s actions as random probably overstates the value we can extract from the situation.

**Decision Tree for the S.S. Kuniang**

To illustrate the use of a decision tree, we return to the problem of the SS *Kuniang*, which was introduced earlier. Two decisions must be made: how much to bid for the salvage rights and which ship to use for hauling coal (a new ship, a tug/barge combination, or the salvaged *Kuniang*). Two of the uncertainties are whether NEES will win the auction and whether the Coast Guard will award a low or high salvage value.

In order to draw a decision tree, it is essential to understand the sequence in which these decisions and uncertainties occur. NEES must first decide on an amount to bid, before it can know the results of the auction or the Coast Guard judgment. Furthermore, the decision as to which ship to use can be delayed until after these uncertainties are resolved. It is immaterial whether the results of the auction are resolved before or after the outcome of the Coast Guard judgment, so we assume that the auction result is known first.

We begin the *Kuniang* tree (Figure 4.11) with a decision node representing the amount NEES will bid for the salvage rights. For simplicity, we distinguish three choices of bid level: $2 million, $8 million, and $12 million. These three choices encompass the high and low extremes, along with an intermediate option. Later, we can return to this part of the model and add more detail.

After making its bid, NEES finds out whether it won the auction. We represent this result with a chance node at the end of each of the branches coming from the decision node. Each of these chance nodes generates two states: win the auction or lose it. Sometime after the auction, the result of the Coast Guard judgment also becomes known. This uncertain event can be represented by another chance node, appearing on each path through the auction nodes, except where there is no chance of winning the auction. Chance nodes for the Coast Guard judgment also have two states: low salvage value and high salvage value. Finally, NEES must decide which of three alternatives to adopt for transporting coal: buying a new ship, buying a tug/barge combination, or salvaging the *Kuniang*. As shown in the tree, this decision is represented by a decision node on each path, although the *Kuniang* is not an option on some of the paths. Note that the decision about how to transport coal occurs after NEES has determined whether the company has won the bid and after NEES has learned whether the salvage rights are low or high.

This completes the building phase for this tree. In principle, we have thirty-six (= 3 × 2 × 2 × 3) paths through the tree, although some combinations cannot occur. For example,
FIGURE 4.11 Decision Tree for the S.S. Kuniang Example
we cannot lose the bid and salvage the Kuniang. The tree diagram shows only nineteen distinct paths through the tree. Each of these paths represents one distinct combination of choices and states; each path thus represents one way the story can turn out. The question now is which of these stories does NEES wish will occur? To put it another way, which bid level should NEES choose, and when the time comes, which ship should it use?

To answer these questions requires some quantitative analysis and two kinds of detailed information. First, we must specify the probabilities corresponding to the states at each chance node. Second, we must determine an overall value to NEES of arriving at the end of the tree along any of the possible paths. Once we have this information, we can analyze the decisions represented in the tree.

The probability of winning the bid was thought to increase from 0 at a bid of $2 million to 1.0 with a bid of $12 million. In our tree diagram, we have simplified the situation by including only three possible bids that NEES might make for the salvage rights to the Kuniang: $2 million, $8 million, or $12 million. As the company’s bid increases, so, too, does the probability that NEES will win the auction. For example, if they bid $2 million, they will not win; if they bid $8 million, the estimated chance of winning is 60 percent; and if they bid $12 million, they are sure to win. The other uncertainty is the legal decision regarding the salvage value. As this is independent of who wins the auction, the relevant probabilities do not change within the tree. Experts within the company have estimated that the Coast Guard would assign a low salvage value with probability 0.3, and a high value with probability 0.7, regardless of which firm wins the auction. At the end of each path through the tree, we place the dollar value of the profits to NEES (in millions). These values, developed from economic details available to the company’s analysts, include the cost of the bid as well as the estimated future net revenues. The numerical values are shown in Figure 4.12.

To evaluate this situation and determine which bid level is best, we analyze the tree using a procedure called rollback. Rolling back the tree involves two operations: choosing the best alternative at decision nodes and evaluating the expected value at chance nodes. We proceed from right to left, or backward in time, since we will need the results of later stages to evaluate the implications at earlier stages.

Starting with the decision node labeled A in Figure 4.12, we compare the profits from the three alternatives, and we choose the Kuniang over the other alternatives because it offers the highest profit. In effect, we are saying that if we bid $8 million, and if we win the auction, and if the salvage value is low, then we will use the Kuniang rather than build a new ship or employ a tug/barge combination. We can now erase node A and its branches and simply replace it with the value $7.5 million. Moving to node B, also a decision node, we see that the Kuniang is again the best choice. So if we bid $8 million, and if we win, and if the salvage value is high, then we also choose the Kuniang, making a profit of $4.5 million.

Node C is a chance node. From this node, we have a 30 percent chance of going to node A, where the value is $7.5 million, and a 70 percent chance of going to node B, where the value is $4.5 million. We evaluate the chance node at the expected value of its outcomes, which is $5.4 million ($= 0.3 \times 7.5 + 0.7 \times 4.5$).

Moving back to node E, we see we have a 60 percent chance of going to node C, worth $5.4 million, and a 40 percent chance of going to node D, where the best choice (New Ship) is worth $3.2 million. The expected value of these outcomes is $4.52 (= 0.60 \times 5.4 + 0.40 \times 3.2)$. Thus, if we bid $8 million, our expected outcome, given all the uncertainties and decisions yet to come, is $4.52 million.
FIGURE 4.12  S.S. Kuniang Tree with Quantitative Information
Similar analyses lead to an expected value of $3.2 million for a bid of $2 million and an expected value of $3.29 million for a bid of $12 million. Thus, the best decision (among these three bid levels) is to bid $8 million. The completed tree is shown in Figure 4.13, where the expected values are shown next to each node. We have also used arrows to show which decisions should be taken.

We should point out that the expected profit of $4.52 million is not the payoff that NEES should expect to receive. Rather, like all expected values, it is the probability-weighted average of all the outcomes that can occur. Expected values represent a way of summarizing the results of an uncertain process in just one figure. They are the most frequent means of summarizing the value of a chance node, given the values and probabilities corresponding to each of its states. But this is not done simply for convenience. Faced with a series of decisions to make where uncertainty plays a role, the decision maker’s best choices are the ones that maximize the expected value of uncertain outcomes, provided that none of the states represents a threat to the viability of the company.

Should we wish to look beyond the expected value as a measure of value, the tree also lets us determine the full range of outcomes and their associated probabilities. Examining the tree in Figure 4.13, we can see that if we lose the auction, we make $3.2 million, and the probability that this will occur is 0.4. If we win and the salvage value is low, we make $7.5 million. The probability of this occurring is the probability of winning (0.6) times the probability of a low salvage value (0.3), or 0.18. On the other hand, if we win and the salvage value is high, we make $4.5 million with probability 0.42 (= 0.6 × 0.7). These three possible outcomes are shown as a probability distribution in Figure 4.14, along with the expected value, which lies at the center of gravity of the three possible outcomes. Note that the probabilities of the three outcomes sum to 1.0, which is a check to confirm that we have constructed a valid probability distribution.

The detailed analysis of the SS Kuniang decision in Figure 4.13 illustrates all the important features of a decision-tree analysis. First, the expected value of the optimal decision is the probability-weighted average of the outcomes, taking into account future optimal decisions. Second, even under the set of optimal decisions, there is a range of potential economic outcomes with corresponding probabilities. This probability distribution can be used to understand the risks associated with the optimal decision. For example, there is a 40 percent chance in this case that we will make only $3.2 million; on the other hand, there is a 18 percent chance that we will make $7.5 million. We refer to the probabilities of extreme outcomes as tail probabilities.

Decision trees can be evaluated by hand, as we have done in this example, or within a spreadsheet. Specialized add-ins, which preserve the tree structure of the problem, are available for this purpose but are beyond the scope of this book. Alternatively, we can build a simple spreadsheet model for the Kuniang example. Only two specialized mathematical operations are needed for the tree: calculating the maximum value at decision nodes and calculating the expected value at chance nodes. One advantage of the spreadsheet approach in this case is that we can introduce the bid level as a parameter and optimize its value directly, without considering each discrete value separately. Our spreadsheet is shown in Figure 4.15.

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1 See, for example, Treeplan at the Web site www.treeplan.com.
FIGURE 4.13  Analysis of the S.S. Kuniang Tree
Given a bid level and the other parameters, the model first calculates the probability of winning the auction (cell F5). We use a simple formula for the probability of winning for any particular bid:

\[ P(\text{Win}) = \frac{\text{Bid} - 2}{10}, \quad \text{for } 2 \leq \text{Bid} \leq 12 \]

where the variable \( \text{Bid} \) represents the size of the bid in millions of dollars. This formula is consistent with the assumption that we lose for sure with a bid of $2 million and that we win for sure with a bid of $12 million. In cells I4 and I5, we calculate the gross profit for the \textit{Kuniang}, given that we win, for the two salvage levels. In cell I8, we calculate the expected value of these two outcomes, while in I10 we calculate the expected profit if we lose the auction. Finally, in cell F9, we calculate the expected profit based on the winning and losing states and their probabilities. By comparing different values of the bid, as shown in the accompanying table and graph, we can estimate that the best bid level is about $6 million.
Principles for Building and Analyzing Decision Trees

Decision trees can be built using a fairly standard procedure. Judgment is required, however, in determining which decisions and uncertainties are critical to the situation and therefore must be captured in the tree, as well as in selecting the specific choices and states to recognize. Beginners tend to draw overly complex trees. The more experienced analyst starts with a small number of nodes and a small number of outcomes and expands only when the results suggest that more detail is required. Here is a general procedure for constructing trees:

- Determine the essential decisions and uncertainties.
- Place the decisions and uncertainties in temporal sequence.
- Start the tree with a decision node representing the first decision.
- Select a representative but not exhaustive number of possible choices for the decision node.
- For each choice, draw a chance node representing the first uncertain event that follows the initial decision.
- Select a representative but not exhaustive number of possible states for the chance node.
- Continue to expand the tree with decision and chance nodes until the overall outcome can be evaluated.

While decision trees can be used to develop a purely qualitative understanding of a situation, they usually lead to a quantitative analysis. This analysis can identify which decisions are best and can construct the probability distribution of results. To carry out this analysis, we need two types of information: the probability for each possible state and the overall value of arriving at the end of the tree along a particular path. Here is the rollback procedure for analyzing trees:

- Start from the last set of nodes—those leading to the ends of the paths.
- For each chance node, calculate the expected value as a probability-weighted average of the values corresponding to the branches.
- Replace each chance node by its expected value.
- For each decision node, find the best value (maximum benefit or minimum cost) among the choices corresponding to the branches.
- Replace each decision node by the best value and note which choice is best.
- Continue evaluating chance nodes and decision nodes, backward in sequence, until the first node is resolved.

In the rollback procedure, any chance node, or any decision node, can be evaluated once the nodes connected to its emanating branches have been evaluated. In that way, the calculations move from the end of the tree toward the beginning, ultimately identifying the optimal choice at the initial decision node.

EXAMPLE

A Patent-Infringement Suit

One of our corporate competitors is threatening us with a lawsuit for patent infringement. The competitor is already in court in a similar lawsuit against another firm. One option open to us is to settle out of court now; the alternative is to wait until the other
suit is settled before taking action. If the competitor loses the other suit, it will not pursue its action against us. If, on the other hand, the competitor wins, it is likely (but not certain) to sue us. If the competitor sues at that point, we can settle, go to trial and contest the patent-infringement claim, or go to trial and concede the patent infringement but fight the settlement amount. In either case, of course, we will either win or lose in court.

Figure 4.16 shows a decision tree for this case. The first decision is whether to settle now or wait for the outcome of the competitor’s current suit. If that suit fails, we will not be sued, and the tree ends at that point. However, if the competitor wins that suit, it may sue us. This outcome is shown as a chance node. Again, if there is no suit, the tree ends. If the opponent does sue, we have the three choices shown on the tree. Finally, the outcome of each of the trial branches is shown as random.

One essential feature of legal proceedings is that both parties have a series of options. This example illustrates how decisions typically alternate with random events. In the specific scenario of the example, the random events are either the decisions of opponents or the results of trials. The objective of a quantitative analysis in this case is not simply to determine the optimal decision, but also to choose an appropriate value for settling the suit now. The first step is to evaluate the expected outcome if we wait. In Figure 4.16, we have
included the necessary probabilities and outcome values. Here, the outcomes are all costs, so the objective is to minimize expected cost.

We roll back the tree from right to left, calculating expected values at chance nodes and minimizing costs at decision nodes. If the opponent wins the current suit and sues us, we have three choices (node A): Settle, Go to Trial / Fight the Patent, or Go to Trial / Concede the Patent. The expected values for the last two options are $7 million and $11 million, respectively. Since settling at this stage costs us $8 million, the lowest-cost choice is to Go to Trial / Fight the Patent.

At node B, we will either be sued and lose $7 million in expected value, or not be sued and lose nothing. Given that the probability of a suit is 0.8, the expected value at node B is $5.6 million (= 0.8 × 7 + 0.2 × 0). At node C, we have a 50 percent chance of losing $5.6 million and a 50 percent chance of losing nothing. The expected value here is $2.8 million (= 0.5 × 5.6 + 0.5 × 0).

Now we can see that if we could settle the suit today for $2.8 million, we would be no better or worse off than if we were to wait, at least in terms of expected costs. But the risks of these two choices are quite different. No risks attach to settling now. On the other hand, if we wait, then we have a 72 percent chance of losing nothing and a 28 percent chance of losing $10 million. [We lose nothing in three cases: if the opponent loses the current suit (probability 0.5), if the opponent wins and does not sue (probability 0.1 = 0.5 × 0.2), and if the opponent wins and sues and we win (probability 0.12 = 0.5 × 0.8 × 0.3). The probabilities of these outcomes sum to 0.72.] Given the 28 percent chance of an outcome we’d like to avoid, it might even be preferable to offer somewhat more than $2.8 million to settle immediately.

Decision trees are particularly appropriate for situations like those we have illustrated, where there are just a few uncertainties and a few decisions. By contrast, trees with many decisions and events can become unwieldy. When there are many uncertainties, the calculations become tedious if done by hand, and they become error-prone if done on a spreadsheet. In principle, we can imagine the limiting case in which a large number of discrete branches at a chance node are instead represented by a continuous distribution. Unfortunately, the process of calculating expected values, while straightforward for simple discrete distributions, can become impractical with continuous distributions. Simulation is the tool of choice when we have a large number of uncertainties, especially when these are represented by continuous distributions. Simulation is also a practical method for analyzing decisions with uncertainty when the underlying model is highly complex, as we illustrate in Chapter 9.

**NETWORK DIAGRAMS**

Network diagrams (or flow diagrams) provide a visual modeling tool for situations in which material flows through a system. One example would be consumer products flowing through a physical distribution system; another would be cash in an investment account accumulating over time periods. Thus, the material that “flows” could be physical or nonphysical, and the system could be dispersed geographically or in time. As with influence charts, network diagrams show the relationships among elements of a problem, and they delineate the boundaries of the system under study. For the analyst, network diagrams can also be useful in organizing the analysis by making sure that no important elements are overlooked. A diagram may not provide all of the analysis, but it can often help identify the elements that have to be considered.
In general terms, a network is a collection of connected elements. The elements could be factories, bank accounts, people, and so on. The connections among factories could be delivery routes, the connections among bank accounts could be electronic funds-transfer routes, and the connections among people could be organizational reporting relationships. In a network diagram, the elements are shown as circles and boxes (or nodes) and the connections as arrows (or arcs). Many different kinds of systems lend themselves to network modeling, where the analysis of the model provides insight into the nature of the activities or flows that occur in the network. In what follows, we present three examples where a network diagram can organize the modeling effort as well as provide a convenient place to perform some of the analysis. For each example, we give a systematic procedure for developing the network diagram.

**EXAMPLE**

**Distribution at Western Paper**

The Western Paper Company manufactures paper at two factories (F1 and F2) on the West Coast. Their products are shipped by rail to a pair of depots (D1 and D2), one in the Midwest and one in the South. At the depots, the products are repackaged and sent by truck to three regional warehouses (W1, W2, and W3) around the country, in response to replenishment orders.

Each of the factories has a known monthly production capacity, and the three regional warehouses have placed their demands for next month. Knowing the costs of transporting goods from factories to depots and from depots to warehouses, Western Paper is interested in planning its logistics operations and costs for the month.

The distribution network for Western Paper contains three kinds of elements—factories, depots, and warehouses. The routes of flow for paper include rail routes from factories to depots and truck routes from depots to warehouses. In what follows, we specify the steps in building a flow diagram for the distribution network (Figure 4.17).

**Inputs and outputs.** At Western Paper, input flows originate at the factories, and we can view the capacities as inputs. The outputs are deliveries to meet demand at the warehouses. Thus, we will have two input nodes and three output nodes in our diagram, each

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**FIGURE 4.17**

Network Diagram for Western Paper

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**CAPACITY**

- F1: 4,000
- F2: 6,000
- D1: 4,500
- D2: 500
- D1W1: 2,400
- D1W2: 2,100
- D2W1: 2,400
- D2W2: 1,500
- D2W3: 3,000
- W1: 2,400
- W2: 3,600
- W3: 3,000

**DEMAND**

- F1D1: 4,000
- F2D1: 500
- F2D2: 4,500
- W1: 2,400
- W2: 3,600
- W3: 3,000

---
indicated by a triangle. In addition, the depots represent intermediate nodes. Material flows into the depots from the factories and out of the depots to the three warehouses.

*Capacities and demands.* The capacities at the factories limit the flow. We might list the capacities in a simple table such as the following:

<table>
<thead>
<tr>
<th>Factory</th>
<th>F1</th>
<th>F2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capacity</td>
<td>4,000</td>
<td>6,000</td>
</tr>
</tbody>
</table>

With a similar table, we can record the demands at each of the warehouses:

<table>
<thead>
<tr>
<th>Warehouse</th>
<th>W1</th>
<th>W2</th>
<th>W3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand</td>
<td>2,400</td>
<td>3,600</td>
<td>3,000</td>
</tr>
</tbody>
</table>

Certain elements may not have capacities. That would be the case for the nodes representing depots in this problem. At these intermediate nodes, there is apparently unlimited space available, the assumption being that Western Paper is able to obtain whatever space it needs in a public warehouse.

*Decisions.* Once the nodes of the diagram are determined, the next step is to identify the arcs, or the routes on which flows occur. While some flows in a network may be dictated by inputs and outputs, or by technology, other flows are redirected at the discretion of the decision maker. Thus, at some points in the system, flows are determined by conscious decisions. We refer to these as decision nodes, or simply *decisions,* and we show them as boxes in the diagram. Decisions determine the flow along some or all of the arcs.

In the case of Western Paper, all possible factory-depot pairs represent possible routes of flow by rail. Similarly, all possible depot-warehouse pairs represent possible routes of flow by truck. Each of these arcs should be labeled with a distinct name. For example, the flow from factory F1 to depot D2 can be labeled F1D2.

At this stage, we have produced a network diagram that reflects the nodes and arcs we have identified, along with the relevant capacities and demands. Showing nodes as boxes underscores the fact that there are decisions affecting the flows at those locations. On the arcs is a set of shipment quantities that make up a proposal for next month’s distribution plan at Western Paper. At this point, the diagram makes it possible to perform some basic analyses related to this proposed plan.

*Feasibility check.* The first step is to confirm the feasibility of this plan. This means making sure that the flow pattern can be implemented. At factory F2, for instance, we compare the total flow out of the factory node (5,000) with the capacity (6,000), to confirm that the plan is workable. A similar comparison can be done, on the network diagram itself, for F1.

A similar check must be made for the outputs. At warehouse W2, for instance, we compare the total flow into the warehouse (3,600) with the demand (3,600), to confirm that the planned deliveries meet the demand. A similar comparison can be done for the other two warehouses.

Finally, we must check that the flows at the depots are feasible. At depot D1, we compare the total flow into the depot (4,500) with the total flow out of the depot (4,500). A requirement that the total flow into a node must equal the total flow out of the node is sometimes called a *material-balance* constraint. Another material-balance constraint applies at depot D2.

*_costs and revenues.* Another part of the analysis traces the financial implications of the proposed flow pattern. In general, this type of analysis requires an accounting of the various costs and revenues in order to determine profit. In a distribution problem, such as Western Paper’s, we find ourselves dealing with a database that includes the unit cost for each route in the system, so the focus tends to be on costs, not on revenues. The unit costs can be displayed in a pair of tables, as shown below. The first table shows the unit cost of
rail shipment from factory to depot, while the second table shows the unit cost of truck shipment from depot to warehouse.

### Depot

<table>
<thead>
<tr>
<th>Factory</th>
<th>D1</th>
<th>D2</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1</td>
<td>$0.56</td>
<td>$0.58</td>
</tr>
<tr>
<td>F2</td>
<td>$0.60</td>
<td>$0.66</td>
</tr>
</tbody>
</table>

### Warehouse

<table>
<thead>
<tr>
<th>Depot</th>
<th>W1</th>
<th>W2</th>
<th>W3</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1</td>
<td>$1.25</td>
<td>$1.50</td>
<td>$1.60</td>
</tr>
<tr>
<td>D2</td>
<td>1.45</td>
<td>1.30</td>
<td>1.28</td>
</tr>
</tbody>
</table>

In Figure 4.18, we have relabeled the same diagram with the costs and quantities in order to make the cost calculations. The figure ignores the zero shipments on some of the arcs. By multiplying the volume on each route by its unit cost, we can compute the total rail cost at $5,510 and the total truck cost at $11,940—for a total distribution cost of $17,450.

Western Paper’s distribution system lends itself to a particular type of network diagram that is often called a transshipment network. To demonstrate some other features of network diagrams, we next look at a somewhat different example.

### EXAMPLE

**Planning for Tuition Expenses**

Two parents want to provide for their daughter’s college expenses with some of the $80,000 they have recently inherited. They hope to set aside part of the money and establish an account that would cover the needs of their daughter’s college education, which begins four years from now, with a one-time investment. Their estimate is that first-year college expenses will come to $24,000 and will increase $2,000 per year during each of the remaining three years of college. The following investment instruments are available:

<table>
<thead>
<tr>
<th>Investment</th>
<th>Available</th>
<th>Matures</th>
<th>Return at Maturity</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Every year</td>
<td>in 1 year</td>
<td>5%</td>
</tr>
<tr>
<td>B</td>
<td>In years 1, 3, 5, 7</td>
<td>in 2 years</td>
<td>11%</td>
</tr>
</tbody>
</table>

**FIGURE 4.18**

Cost Calculation for Western Paper
In this case, a key concern is the timing of various financial flows. The nodes in the diagram of Figure 4.19 correspond to points in time, and we can think of them as representing the start of this year, the start of next year, and so on. (For convenience, we treat the first year as year 1 and its starting point as time 0.)

*Inputs and outputs.* The one input flow is the initial fund (shown as IF), which will be established at time 0, corresponding to the start of this year. The four outputs are the tuition expenses anticipated at the start of years 5–8.

*Capacities and demands.* The initial fund is actually a decision as well as an input, and it does constrain the initial investments. Therefore, we show it as a box in the diagram to emphasize that it is a decision. However, once the initial fund is determined, its size will limit the choice of the four initial investments. The four outputs are the tuition expenses anticipated at the start of years 5–8. These are known quantities, analogous to the demands at Western Paper’s warehouses, and they are indicated by output triangles.

*Decisions.* The flows in this network correspond to the amounts invested. Investment A can be selected in each year, so we use A1 to represent the amount invested in year 1, A2 for the amount invested in year 2, and so on, up to A7. Investment B can be selected only every other year, so we use B1 to represent the amount invested in year 1, B3 for the amount invested in year 3, and B5 for the amount invested in year 5. Similarly, we use C1, C4, and D1 to complete the investment alternatives. In the network diagram, a one-year investment instrument connects two successive nodes, while a two-year instrument skips a node, and so on.

All nodes in this model correspond to decisions about the allocation of financial assets. Not only is there an allocation decision needed initially, to distribute funds among the instruments available at the start, but there are also subsequent decisions needed as...
investments mature. On occasion, as at the start of year 2, there is only one investment alternative given, so the allocation decision is trivial. However, in most years, an allocation is needed.

Each arc represents the investment in a particular financial asset over a specified period of time. As a result, each such asset grows in value by accumulating interest at a rate given in the problem scenario. Thus, on the diagram, we can think of the head of the arc as being worth more than the tail, reflecting the appreciation of the corresponding asset. While we write A1 to represent the investment in A at time 0, we know that when the arc arrives at the start of year 2, it is worth $1.05 \times A1$, due to the 5 percent annual return. In order to make the tracking of funds on the diagram more convenient, we can split each arc into two parts, as shown in Figure 4.20. Here, a circle depicts the transformation of its input flow to a corresponding output flow. When such a node represents investment A, the output flow is 5 percent larger than the input flow; when the node represents investment B, the output flow is 11 percent larger, and so on. In order to label the outflows from such nodes, we may need to make some brief side calculations.

As an example, suppose the initial investment is $80,000, split equally among the four instruments available at the outset. Furthermore, suppose that we follow a rule that calls for an investment of $20,000 in A whenever there is an allocation to make. We can now determine whether this plan is feasible.

**Feasibility check.** Given the equal split dictated by our rule, the output flows from the initial node are $20,000 each in A1, B1, C1, and D1. A1 is transformed into $21,000 by its 5 percent return, and the only alternative at the start of year 2 is to reinvest this amount as A2. Then, A2 is transformed into $22,050, and B1 into $22,200, so that the total input into year 3 comes to $44,250. Following our rule, $20,000 is allocated to A3, and the rest ($24,250) to B3. At the start of year 4, the input flows are $21,000 from A3 and $23,200...
from C1—for a total of $44,200. Following our rule, $20,000 is allocated to A4, and the rest ($24,200) to C4.

At the start of year 5, there are three input flows, totaling about $76,720. The output flows are $24,000 to cover the first tuition payment, $20,000 for A5 under our rule, and the remainder ($32,720) for B5.

At the start of year 6, there is only one input flow, $21,000 from A5. However, there is a need to cover a tuition payment of $26,000. Thus, the calculations on the diagram allow us to see that our investment rule will not be feasible. On the other hand, if we simply raise A5 to $25,000, all tuition payments will be covered, and we will wind up with a surplus of more than $3,500 (Figure 4.21).

The flow diagrams in Figures 4.20 and 4.21 contain inputs, outputs, capacities, and demands—just like the Western Paper diagram—as well as allocation decisions. The new element is a **transformation node**, for which the output flow is larger than the input flow due to financial returns. In principle, transformation nodes can work to reduce flows as well as to increase them. The next example provides a case in point.

**EXAMPLE**

**Production at Delta Oil**

The refining process at Delta Oil Company separates crude oil into components that eventually yield gasoline, heating oil, jet fuel, lubricating oil, and other petroleum products. In particular, gasoline is produced from crude oil by either a distillation process alone or by a distillation process followed by a catalytic-cracking process. The outputs of these processes are subsequently blended to obtain different grades of gasoline.

![Network Diagram](image-url)
The distillation tower at Delta’s refinery uses five barrels of crude oil to produce three barrels of distillate and two barrels of other “low-end” by-products. Some distillate is blended into gasoline products; the rest becomes feedstock for the catalytic cracker.

The catalytic-cracking process produces high-quality catalytic gasoline (or catalytic, for short) from the feedstock. Delta’s catalytic cracker requires 2.5 barrels of distillate to produce 1.6 barrels of catalytic and 1 barrel of “high-end” by-products. (The cracking process creates output volume that exceeds input volume.)

Finally, distillate from the distillation tower is blended with catalytic to make regular gasoline and premium gasoline. The blend of distillate and catalytic must be at least 50 percent catalytic to meet the quality requirements of regular and at least 65 percent catalytic to meet the quality requirements of premium.

Planning and scheduling operations at Delta’s refinery must take into account the capacity requirements of the equipment while matching product flows with demands in the various product markets. A diagram for Delta Oil is shown in Figure 4.22. In this diagram, the nodes represent stages of the manufacturing process (purchase, production, and distribution).

**Inputs and outputs.** At Delta Oil, there is only one input, described as crude oil, but there are several outputs. These include by-products (broken down into low-end and high-end categories) and gasoline products (consisting of regular and premium). Thus, we can distinguish four outputs. The one input and four outputs are represented by triangles in the diagram.

**Transformation processes.** In our previous example, flows were transformed by the process of financial appreciation. In a production environment, transformation nodes are associated with changing one kind of flow into another. At Delta Oil, there are two processes—the distillation tower and the catalytic cracker—which are represented as nodes in the network. Distillation takes crude oil as input, and it produces distillate and low-end by-products. Cracking takes feedstock (which in this case happens to be distillate) as input, and it produces catalytic and high-end by-products.

---

**FIGURE 4.22**  
Network Diagram for Delta Oil
**Capacity levels.** In some cases, capacities may be unlimited, as we saw in the depots of the Western Paper network, but for production processes, capacities are usually limited. When we come to the analysis, we will want to know the restrictions these processes impose on the rates of flow through the network. In the case of Delta Oil, the distillation tower has a capacity of 50,000 barrels per day, while the catalytic cracker has a capacity of 20,000 barrels per day. These capacities are noted in brackets on the diagram.

**Decisions.** Delta Oil’s production system allows for a decision regarding how much of the distillate generated by the distillation tower should be sent as feedstock to the catalytic cracker and how much should be sent directly to gasoline blending. There are also two decisions regarding the blending of distillate and catalytic. Since the outputs are “regular” and “premium,” we must decide how much distillate and how much catalytic should be allocated to each. Decisions are again represented in the diagram by boxes.

Note the distinction between transformation nodes and decision nodes. For a transformation node, outputs may take a different form than the inputs, and the total output quantity may not equal the total input quantity. For example, crude oil is an input to the distillation tower, but the outputs are distillate and by-products. On the other hand, for a decision node, the outputs are the same material as the inputs. For example, the distillate that comes out of the tower is still distillate when routed to the cracker as feedstock or when blended with gasoline. In addition, a material-balance requirement holds at each decision node: the sum of the quantities flowing in must equal the sum flowing out.

Once the nodes have been determined, we can place arcs in the diagram to represent every possible flow route. Each of these arcs should be labeled with a distinct name. For example, the distillate that we allocate to the cracker is labeled Feed, while the distillate that we allocate for blending is labeled Blend. Premium gasoline is made from a combination of BP (from Blend) and CP (from Catalytic), while regular gasoline involves a combination of BR and CR. Labeling each arc makes it possible to trace the entire pattern of flows in the network.

The network diagram allows us to analyze various production plans for Delta Oil. For example, we might simply check a proposed plan for consistency. We would need to know, of course, how much crude and how much of the four outputs were to be produced. But, having created the diagram and having identified the embedded decisions, we also know that we must obtain information on the feedstock quantity and how the gasoline is split between regular and premium.

**Feasibility check.** Suppose we were asked to evaluate a plan to buy 36,000 barrels of crude oil and produce approximately 16,000 barrels of gasoline and 21,000 barrels of by-products. These figures would not be sufficient to label all the flows in the diagram. However, if further inquiry revealed that 16,000 barrels would be fed to the catalytic cracker and that regular gasoline would be made up of 2,000 barrels of distillate blend along with 3,000 barrels of catalytic, then we could label the entire diagram. First, we calculate the outflow from the tower, using the input volume of 36,000 and the fact that distillate represents 60 percent of the output. This places the distillate volume at 21,600. Since 16,000 is fed into the cracker, the remaining 5,600 must be allocated as Blend. Furthermore, we can calculate the outflow from the cracker, using the input volume of 16,000 and the fact that Catalytic represents 64 percent of the output. This places the catalytic volume at 10,240. With Blend and Catalytic known, and with BR and CR specified at 2,000 and 3,000, respectively, we can use material-balance arithmetic to calculate BP as 3,600 and CP as 7,240. Figure 4.22 contains the numerical summary.

With some side calculations, we can also confirm that this plan calls for regular gasoline consisting of 60 percent catalytic and premium gasoline consisting of 66.8 percent catalytic, both above the minimum levels that are required. In addition, the crude-oil vol-
ume and the feedstock quantity are well within the capacity limits of the distillation tower and the catalytic cracker, so the proposed plan is a feasible one.

Costs and revenues. In the case of Delta Oil, there are purchase costs for crude oil and operating costs for the two processes. There are also revenues for each of the four products sent to market. These unit costs and revenues are shown in the following table:

<table>
<thead>
<tr>
<th>Activity</th>
<th>Item</th>
<th>Cost per bbl.</th>
<th>Price per bbl.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Purchase</td>
<td>Crude</td>
<td>$28</td>
<td></td>
</tr>
<tr>
<td>Distillation</td>
<td>Crude</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Cracking</td>
<td>Feed</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>Sales</td>
<td>Low-end</td>
<td>$25</td>
<td></td>
</tr>
<tr>
<td>Sales</td>
<td>High-end</td>
<td>38</td>
<td></td>
</tr>
<tr>
<td>Sales</td>
<td>Regular</td>
<td>40</td>
<td></td>
</tr>
<tr>
<td>Sales</td>
<td>Premium</td>
<td>42</td>
<td></td>
</tr>
</tbody>
</table>

In Figure 4.23, we label the diagram with these values, in order to evaluate the profitability of the proposed plan. We multiply purchase quantity by unit purchase cost, and we multiply operating levels by unit operating costs; these calculations give us a figure for total cost. When we multiply sales quantities by unit prices, we obtain total revenue. Finally, we can calculate profit as the difference between total revenue and total cost. As shown in the figure, the proposed plan leads to a loss of nearly $64,000 for the month. Evidently, Delta Oil will want to use the diagram to search for a more profitable plan.

**SUMMARY**

Although we emphasize the use of spreadsheets in modeling, there are often many useful steps that can be taken before opening up a workbook. In this chapter, we have discussed a number of tools that reinforce the visual aspects of modeling activity. The
ability to visualize model structure (and to communicate that visualization) can be a powerful and effective factor in an expert’s set of modeling skills.

Influence charts provide a way to begin organizing a model. They start with the model outputs and break them down into constituent parts. By following the logical trail from what we want to learn back to what we already know, or what we can readily find out, an influence chart allows us to map out the main structure of our model. At the same time, the chart helps us distinguish among inputs, outputs, and decisions. Moreover, an influence chart reinforces a top-down approach, which helps us avoid the traps associated with bottom-up analysis.

Outlines serve to organize our models into categories, subcategories, and individual elements. They are particularly helpful when our models resemble accounting statements, because we can draw on accounting definitions and relationships in structuring our outline. Thus, the elements of income statements, balance sheets, and cash-flow statements can often serve as a framework for our modeling logic. In addition, these model forms communicate effectively with clients who are more familiar with accounting information than they are with model output.

A decision tree is a somewhat specialized tool for recognizing the role of random factors in a model. Trees help us distinguish between decisions and random events, and, more importantly, they help us sort out the sequence in which they occur. Probability trees provide us with an opportunity to consider the possible states in a random environment when there are several sources of uncertainty, and they can become components of decision trees. Using the rollback procedure, we can identify those decisions that optimize the expected value of our criterion. Furthermore, we can produce information in the form of a probability distribution to help assess the risk associated with any decision in the tree.

Network diagrams represent another tool that helps us specify the boundary of a model and sort out its major elements. In networks, we trace the progress of a certain material as it moves through a complicated system. We begin by identifying inputs and outputs and by noting whatever capacities and demands occur at those points. We then look within the network for decisions that reallocate flows at the various stages of progress. Network diagrams allow us to represent transformations that commonly occur in production or financial systems, as well as material-balance relationships that apply to particular allocation decisions. The diagrams themselves serve as tools to help test the feasibility of any proposed plan, and they provide a laboratory for basic cost and profitability analysis.

These four kinds of visual tools can be useful at the outset of a modeling project, to help build an initial structure and to identify key elements of that structure. In addition, in the case of network diagrams, optimization techniques can build on basic diagramming tools. We highlight this approach in Chapter 8.

REFERENCES

Some related material can be found in the following books:


EXERCISES

Influence Charts

1. The Boeing Company faces a critical strategic choice in its competition with Airbus S.A.S. for the long-haul flight segment: Should it design and build a super-747 model that can carry 550
passengers at speeds around 350 mph, or a plane that can fly at 95 percent of the speed of sound but carry only about 350 passengers? Draw an influence diagram for this decision.

2. Many forms of cancer can be cured or controlled if identified early. Blood tests exist for many cancers, but they are costly and do not provide 100 percent reliable results. Each test would involve drawing one blood sample and testing it for between five and ten forms of cancer. Each test has its own accuracy, both for false positives and for false negatives. Some cancers can be cured with high probability when detected by blood test, while others either cannot be cured or can only be slowed. Would it be cost-effective for HMOs to offer cancer screening every year to patients more than, say, fifty years old? Draw an influence diagram.

3. The Red Cross provides about 40 percent of the replacement blood supply for the United States. The available donor base has been shrinking for years, and although increased advertising has kept Red Cross supplies adequate, the time is approaching when demand will outstrip supply. For many years, the Red Cross has refused to pay donors for blood, on the grounds that to do so would “put the blood supply of the country at risk.” Evaluate a policy under which the Red Cross would pay each donor a set fee. Draw an influence diagram.

4. Refer to the Retirement Planning case. Review the problem statement that was generated for this case in conjunction with the corresponding exercises in Chapters 2 and 3. (If this has not yet been done, develop the problem statement as a first step.) Draw an influence diagram for the case based on the problem statement already developed, modifying it if necessary.

5. Refer to the Draft TV Commercials case. Review the problem statement that was generated for this case in conjunction with the corresponding exercises in Chapters 2 and 3. (If this has not yet been done, develop the problem statement as a first step.) Draw an influence diagram for the case based on the problem statement already developed, modifying it if necessary.

6. Refer to the Icebergs for Kuwait case. Review the problem statement that was generated for this case in conjunction with the corresponding exercises in Chapters 2 and 3. (If this has not yet been done, develop the problem statement as a first step.) Draw an influence diagram for the case based on the problem statement already developed, modifying it if necessary.

7. Refer to the Racquetball Racket case. Review the problem statement that was generated for this case in conjunction with the corresponding exercises in Chapters 2 and 3. (If this has not yet been done, develop the problem statement as a first step.) Draw an influence diagram for the case based on the problem statement already developed, modifying it if necessary.

Outlines

8. You have been asked to serve as chair of a professional conference that will take place six months from now. You must arrange for the physical requirements of the event, which include hotel and room space, meals, vendor space, and breakout space. You must also arrange for advertising in professional and public media. You must hire and train staff to accept reservations and communicate with attendees, both before and during the event. Finally, you must anticipate and budget for unforeseen events. Develop the outline of a budget for this project.

9. Refer to the XYZ Company case. Compose a list of categories that will allow an analyst to predict monthly cash needs and profitability for the first six months of the year, for the case where there are payment lags and the firm is using accrual accounting.

10. Refer to the Damon Appliances case. Compose a list of categories for the pro forma income statement and the pro forma balance sheet, as desired by the CFO.

Decision Trees

11. Copy Makers Inc. (CMI) has just received a credit request from a new customer who wants to purchase a copying machine. As input to its decision of whether to grant credit, CMI has made the following estimates and assumptions:
If CMI denies the customer credit, there is a 20 percent chance that the customer will buy the copying machine with cash anyway.

If CMI grants credit, there is a 70 percent chance the customer will be a good credit risk.

If CMI grants credit and the customer is a good credit risk, CMI will collect 100 percent of the purchase price.

If CMI grants credit and the customer is a bad credit risk, CMI has two options. Under the first option, CMI would continue to send the customer a bill and hope it is eventually paid. Under this option, CMI will collect 100 percent, 50 percent, or 0 percent of the amount owed, with probabilities 0.3, 0.5, and 0.2, respectively. Under the second option, CMI would vigorously pursue the collection of the amount owed. To do so would cost CMI 25 percent of the amount owed, regardless of the amount eventually collected. Under this second option, CMI will again collect 100 percent, 50 percent, or 0 percent of the amount owed, with probabilities 0.1, 0.2, and 0.7, respectively.

The copy machine sells for $8,000 and costs CMI $5,000. Nonvigorous enforcement has no cost, while vigorous enforcement costs $2,000.

12. TCS Corporation has recently decided to manufacture a product in its own facilities rather than outsource to Asian manufacturers. Its new plant will last about ten years. TCS is considering two options: build a large plant now that will have sufficient capacity to handle demand into the foreseeable future, or build a small plant that can be expanded two years later, after demand is better known. TCS will not expand the small plant unless demand in the first two years exceeds a threshold level.

TCS assumes that the level of demand in subsequent years will be the same as in the first two years (e.g., if demand is high in the first two years, it will continue to be high in the next eight years).

Draw a decision tree for TCS’s problem. What additional data are needed to determine the optimal decision?

13. A small manufacturer uses an industrial boiler in its production process. A new boiler can be purchased for $10,000. As the boiler gets older, its maintenance expenses increase while its resale value declines. Since the boiler will be exposed to heavy use, the probability of a breakdown increases every year.

Assume that when a boiler breaks down, it can be used through the end of the year, after which it must be replaced with a new one. Also, assume that a broken-down boiler has no resale value.

Some basic data are given in the table below:

<table>
<thead>
<tr>
<th>Year of Operation</th>
<th>Expenses</th>
<th>Resale Value</th>
<th>Breakdown Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1,500</td>
<td>7,000</td>
<td>0.1</td>
</tr>
<tr>
<td>2</td>
<td>2,000</td>
<td>5,000</td>
<td>0.2</td>
</tr>
<tr>
<td>3</td>
<td>3,000</td>
<td>4,000</td>
<td>0.4</td>
</tr>
<tr>
<td>4</td>
<td>4,500</td>
<td>2,000</td>
<td>0.5</td>
</tr>
<tr>
<td>5</td>
<td>6,000</td>
<td>500</td>
<td>0.8</td>
</tr>
</tbody>
</table>

Draw a decision tree for this problem. Using the rollback procedure, determine the optimal decision and its expected value.

14. Delta Electric Service is an electrical-utility company serving parts of several states. It is considering replacing some of its equipment at generating substations and is trying to decide whether it should replace an older, existing PCB transformer. (PCB is a toxic chemical known formally as polychlorinated biphenyl.) Although the PCB generator meets all current regulations, if an incident such as a fire were to occur, and PCB contamination caused harm either to neighboring businesses or farms, or to the environment, the company would be liable for dam-
ages. Recent court cases have shown that simply meeting regulations does not relieve a utility of liability if an incident causes harm to others. In addition, courts have been awarding very large damages to individuals and businesses harmed by incidents involving hazardous material. If Delta replaces the PCB transformer, no PCB incidents will occur, and the only cost will be the cost of the new transformer, estimated to be $85,000. Alternatively, if the company elects to keep the existing PCB transformer in operation, then, according to their consultants, there is a 50/50 chance that there will be a high likelihood of an incident or a low likelihood of an incident. For the case of a high likelihood of an incident, there is also a 0.004 probability that a fire will occur sometime during the remaining life of the transformer, and a 0.996 probability that no fire will occur. If a fire occurs, there is a 20 percent chance that it will be severe and the utility will incur a very high cost, whereas there is an 80 percent chance that it will be minor and the utility will incur a low cost. The high- and low-cost amounts, including both cleanup and damages, are estimated to be $100 million and $10 million, respectively, based on results from other incidents in the industry. For the case of a low likelihood of an incident, there is a 0.001 probability of a fire during the remaining life of the transformer, and a 0.999 probability of no fire. If a fire does occur, then the same probabilities exist for the severe and minor outcomes as in the previous case. In both cases, there will be no cost if no fire occurs.

Draw a decision tree for this problem. Using the rollback procedure, determine the optimal decision and its expected value.

**Network Flow Diagrams**

15. Refer to the Gulfport Oil case. Draw a network diagram showing the inputs, outputs, and flows in this problem.

   **Questions**
   a. Find a feasible set of flows consistent with the given information on input capacities, quality constraints, production capacities, and output requirements.
   b. For the set of flows in (a), find the total profit.

16. Refer to the Coastal Refining Company case. Draw a network diagram showing the inputs, outputs, and flows in this problem.

   **Questions**
   a. Find a feasible set of flows consistent with the given information on input capacities, quality constraints, production capacities, and output requirements.
   b. For the set of flows in (a), find the total profit.

17. Refer to the Quincy Chocolate case. Draw a network diagram showing the inputs, outputs, and flows in this problem.

   **Questions**
   a. Find a feasible set of flows consistent with an output of 2 million pounds of milk chocolate.
   b. For the set of flows in (a), find the total profit.

18. Refer to the Workforce Management case. Draw a network diagram showing the inputs, outputs, and flows in this problem.

   **Questions**
   a. Find a feasible set of flows consistent with the given information on initial workforce size, monthly requirements, and other features of the problem.
   b. For the set of flows in (a), find the total profit.