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Do shocks last forever? Local persistency in economic time series

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Abstract

While it is recognized that many macroeconomic time series are highly persistent over certain range, less persistent results are also found around very long horizons, indicating the existence of local or temporary persistency. In this paper, we study locally persistent behavior in economic time series. A test for stationarity against locally persistent alternative is proposed. Asymptotic analysis of the test statistic are provided under both the null and the alternative hypothesis of local persistency. Monte Carlo experiment is conducted to study the power and size of the test. An empirical application reveals that many US economic variables may exhibit local persistency.

JEL classification: C22; E30; E40

Keywords: Time series; Local persistence; Mean reversion; Unit root

1. Introduction

Since the influential article by Nelson and Plosser (1982), hundreds of economic time series have been examined by unit root tests and empirical evidence has accumulated that many economic and financial time series contain a unit root. However, as argued elsewhere (see for example Kwiatkowski et al., 1992), many standard testing procedures

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consider the null hypothesis of a unit root which ensures that the null hypothesis is accepted unless there is strong evidence against it. Indeed, different results have been obtained from other approaches.

While it is recognized that many economic time series are persistent, less persistent results are also found around very long horizons (see, e.g., Beaudry and Koop, 1993; Hess and Iwata, 1997; Koenker and Xiao, 2003), indicating the existence of "local persistency" in economic time series. For example, output fluctuations may be persistent over a long range of time, but not forever and will eventually disappear (Cochrane, 1988).

In recent 10 years, a large amount of literature has emphasized that many economic time series are better characterized by a process with root near unity rather than an exact unit root. In effect, Cheung and Lai (1998) claim that most of real exchange rates of the G-7 countries has root near unity. Lanne (2000) claims that the dynamic of interest rates is better characterized by a process with a root near unity rather than a process with an exact unit root. Dutkowsky and McCoskey (2001) show that near-unit root is also present in the spread between Federal Funds rate and the discount rate during the post-1987 period and they use this fact to show that structural restrictions are compatible with stationary borrowing and a spread with root near unity.

The simplest local to unity model is a triangular array for a time series y_i of the form

$$y_i = \alpha y_{i-1} + u_i, \quad \alpha = 1 + \frac{c}{n}, \quad i = 1, \ldots, n \tag{1}$$

with i.i.d. $(0, \sigma^2)$ innovations u_i. While the autoregressive coefficient $\alpha \to 1$ as $n \to \infty$, it is apparent that for any given sample size n in (1), the model accommodates a wider range of autoregressive coefficients as the localizing parameter c varies. This flexibility has helped to make the model popular in studying economic time series for which roots near unity are considered highly plausible but roots at unity are considered too restrictive. However, in the traditional local to unit root model, shocks are still permanent and cannot capture the feature of local persistency.

The current paper aims to provide a first step of study on locally persistent processes. In this paper, we use a new time series model proposed by Phillips et al. (2001) to capture local persistence. This new formulation of local to unity model offers more flexibility than the traditional model (1). The new model leads to a class of different limit processes beyond simple diffusions and also provides a more complete interface between $I(0)$ and $I(1)$ models and between $O(\sqrt{n})$ and $O(n)$ asymptotics. We may call this model a block local to unity model or a weak unit root model. The serial correlation in this model is stronger than that in the conventional stationary ergodic process, but weaker than the unit root, or traditional local-to-unit root time series, providing a model that displays local persistency.

This paper is organized as follows: Section 2 presents the econometric model with local persistency, based on Phillips et al. (2001). The locally persistent process is compared with the fractionally integrated process, which is a related but different process. In Section 3, we introduce a test through which we test the null hypothesis of stationarity against local persistency and derive its asymptotic distribution under the null and alternative hypothesis. Section 4 presents some results of Monte Carlo experiments. An empirical illustration on the presence of local persistency in some US time series is conducted in Section 5. Section 6 concludes.

A word on notation. We will use the symbols " $\Rightarrow$ ", " $\to$ " and ":=" to signify weak convergence, convergence in probability and equality in distribution, respectively. Following the standard stochastic order of magnitude notation, we write $X_n = O_p(1)$ and $X_n = o_p(1)$ to signify that the sequence of random variable X_n is bounded and converges to zero, respectively, as the sample size, n, goes to infinite.

2. A model with local persistency

2.1. Locally persistent process without drift

Consider the time series (y_i) generated as:

$$y_i = \alpha y_{i-1} + u_i, \quad i = 1, \ldots, n \tag{2}$$

where $\{u_i\}$ is a general covariance stationary ARMA process which satisfies an invariance principle. In the above model, the autoregression coefficient "α" measures the persistency in time series y_i. When $|\alpha| < 1$, y_i is covariance stationary and the innovations are not persistent; when $|\alpha| > 1$, y_i is an explosive process; and when $\alpha = 1$ (or close to 1), y_i is called as a (near) unit root process and the innovations are persistent.

Many macroeconomic time series display persistent behavior. For this reason, we are particularly interested in the case where the coefficient "α" is close to unity (in the sense that α is in a shrinking neighborhood of unity). For convenience to study the near-unit-root property, we may reparameterize α so that

$$\alpha = 1 + \delta$$

where $\delta \to 0$ as the sample size $n \to \infty$. The new parameter δ represents the deviation of α from unity.

Since α is in a shrinking neighborhood of unity ($\delta \to 0$ as the sample size $n \to \infty$), there are some advantages to directly write δ as a function of the sample size n:

$$\delta = \delta(n) \to 0, \quad \text{as } n \to \infty.$$

A general class of local deviation can be modelled in the following way:

$$\delta = \frac{c}{n^d} \tag{3}$$

where $c < 0$, and $\delta = \delta(n) \to 0$ at rate n^d and $d \in D \subset (0,1]$, where D is an interval in the range of $(0, 1)$, see more discussion in the asymptotic analysis in Section 3. The above reparameterization (3) provides a very general model with different types of persistency. However, the generality of such a representation also brings an identification issue in practice when we estimate the autoregression coefficient based on (2): in the original autoregression model (2), there is only one unknown parameter characterizing persistency, α; the reparameterization (3) introduces two unknown parameters c and d. One way to solve this problem is to fix the value of one of the two parameters c and d, so that an one-to-one relationship can be maintained between the original model (2) and the reparameterized model (3) over certain range of parameter values.

If we fix the value of d by setting $d = 1$, we obtain exactly the traditional local to unit root model:

$$\delta = \frac{c}{n},$$

where $\delta = \delta(n) \to 0$ at rate n. In this model, the effect of innovation is permanent and never disappears. However, as mentioned above, this traditional setting does not provide a model that display *local persistency*. The reason is that δ ($=c/n$) converge to 0 too fast and thus α is too close to 1. Consequently, the effect of persistency in such a time series is similar to an exact unit root model. Intuitively, the convergence rate of $\delta(n)$ reflects the level of persistency in y_i.

In order to capture the property of local persistency, we need to consider a time series model with the local deviation parameter $\delta = \delta(n) \to 0$ at rate $o(n)$. For this reason, instead of fixing $d = 1$, we consider in this paper the alternative standardization by fixing c and set the localizing parameter $c = -1$. Thus

$$\delta = -1/n^d, \tag{4}$$

where d takes values between 0 and 1. The process provides a new form of persistent behavior. It has autoregressive coefficient near unity, but it is not the conventional stationary or unit root process. In particular, under appropriate regularity conditions, $y_{[nr]} = O(n^{d/2})$, implying that the process will ultimately diverge at rate $n^{d/2}$ as $n \to \infty$.

The above device provides a statistical model for what may be described as "locally persistent behavior" for macroeconomic time series. Many macroeconomic time series are now well known to display a form of persistence whereby economic shocks have long-run effects. However, it is possible that shocks may affect an economy for a long period of time but not forever. In other words, the effects of a shock may be highly persistent over a certain range (the range of persistent behavior), but then may begin to disappear outside this range. In the above model, the largest autoregressive root of time series y_i is close to unity, and thus persistency can be found in y_i. On the other hand, the series evolves over time in such a way that there is persistency over a range of time (of order $O(n^d)$, compared to the full sample range n), but the effect of shocks will eventually disappear over time horizon longer than order $O(n^d)$. The region of persistent behavior may constitute a 'little infinity' relative to the full sample, since there is persistent memory within a time horizon of order $O(n^d)$, but there is only short memory over longer periods. For this reason, we call the above process a locally persistent process with persistent parameter d.

Locally persistent processes are not covariance-stationary and, as we will see soon, they may be used to model the dynamics of economic time series that display persistence as well as transitory shocks.

2.2. Locally persistent process with a deterministic component

The locally persistent process can be extended to include an intercept term or a deterministic component. Such an extension is important because many economic time series display tendency of growth. We may consider a locally persistent process y_i^τ with deterministic component

$$y_i^\tau = \tau_i + y_i; \quad i = 1, \ldots, n \tag{5}$$

where stochastic part y_i is a locally persistent process as described in the previous section, and τ_i is a deterministic component. The leading cases being (i) a constant term where $\tau_i = \varphi_o$, and (ii) a linear time trend where $\tau_i = \varphi_o + \varphi t = \varphi_y' \Upsilon_i$.

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Note that the trend coefficients are unknown and thus, in practice, appropriate detrending is needed. We may estimate y_i from the residuals of the following detrending (or demeaning, if there is only an intercept term) regression

$$y_i^{\tau} = \hat{\varphi}_y' \Upsilon_i + \hat{y}_i \tag{6}$$

where $\hat{\varphi}_y$ is the least squares estimator of φ_y. The detrended time series $\hat{y}_i$ has properties similar to the process with no drift y_i (see Phillips et al. (2001) for more discussion).

2.3. Local persistency versus long memory

A related but different model is the long-memory, or fractional integrated process with order of integration equal to d_l, that is, FI(d_l) with $d_l \in (0,1)$. Both the locally persistent process that we consider in this paper and the conventional fractional integrated process are between the conventional covariance stationary process and unit root process. However, these two processes have important differences. We next illustrate the differences between the two models in terms of impulse response function.

An impulse response function traces the effect of a shock in the innovation u_i on current and future values of the endogenous variable y_i. If the process is stationary, then its impulse response will converge to zero as the response horizon k increases and we say that the shocks are transitory. On the other hand, when the process y_i has a unit root the impulse response never converges to zero. Thus, when the process is a random walk we say that the shocks are permanent implying that an initial shock never dies out. As an illustration, consider the following models (In all these models we assume that u_i is an i.i.d sequence of innovations):

Model 1: Stationary process: $y_i = \alpha y_{i-1} + u_i$, $|\alpha| < 1$.
Model 2: Fractionally integrated process: $(1-L)^d(y_i - \mu) = u_i$
Model 3: Locally persistent process: $y_i = \alpha y_{i-1} + u_i$, $\alpha = (1 - \frac{1}{n^d})$.

In Model 1, y_i is stationary when $|\alpha| < 1$. One can show that the k period impulse response $IR_k = \frac{\partial y_k}{\partial u_1} = \alpha^{k-1} \to 0$ as $k \to \infty$. Therefore, if y_i is stationary, then the shock will be totally absorbed as k increases. Also notice that, according to Model 1, the total impact of a shock is finite, that is, $\sum_{k=0}^{\infty} IR_k < \infty$. Now, for comparison purpose, we re-consider Model 1 with $\alpha = 1$. In this random walk specification, it is well known that $IR_k = 1$ for any k. (If u_i is not an i.i.d. sequence, then the impulse response function may move up and down but with no convergence towards zero.) The shocks never vanish when there is a unit root and, more importantly, the total impact goes to infinity, that is, $\sum_{k=0}^{\infty} IR_k = \infty$.

Model 2 represents a fractional white noise process. This process can be expressed as an infinite order moving average representation,

$$y_i = \sum_{j=0}^{\infty} \boldsymbol{\Psi}_j u_{i-j}.$$

Therefore, $IR_k = \frac{\partial y_k}{\partial u_1} = \Psi_k \approx 1/(k^{1-d_l})$ for $0 < d_l < 1$. Because of this hyperbolic decaying, the impact of a innovation vanishes in the long run, but vanishes slowly. Also notice that since $0 < 1 - d_l < 1$, we have $\sum_{k=0}^{\infty} IR_k = \infty$ so that the total impact of a shock is infinite, yielding a result similar to the one obtained in the unit root case.

Now, we analyze the behavior of the impulse response function of a locally persistent process. Consider the standardized locally persistent process given in Model 3 with $0 < d < 1$. The k period impulse response is given by $IR_k = \alpha^{k-1} = (1 - \frac{1}{n^d})^{k-1}$ and we can see that $IR_k \to 0$ if $k = O_p(n^{d+\epsilon})$ for any $\epsilon > 0$ ($IR_k \nrightarrow 0$ if $k = O_p(n^d)$ or $o_p(n^d)$). For example, if we set $n = n_o + k$ where n_o is the fixed sample size when the shock occurs, then $k = O_p(n)$ satisfies the condition above. At the local persistence model, the autoregressive coefficient gets closer to unity as new k observations become available. Therefore, if the process is locally persistent, its impulse response eventually converges to zero and the shocks are globally transitory. Moreover, one can see that $\sum_{k=0}^{\infty} IR_k < \infty$ as $k \to \infty$ meaning that, unlike what happens to fractional and unit root processes, the total impact of a shock is finite. Notice that the degree of persistency is determined by the parameter d: the larger d, the more persistent the shocks are. Therefore, we can affirm that shocks take much more time to die out when the process is locally persistent than when it is stationary. In the degenerate case of Model 3 where $d = 1$, the k-period impulse response function $\frac{\partial y_k}{\partial u_1} = \alpha^{k-1}$. In this case, $k = O_p(n^d)$ with $d = 1$: the autoregressive coefficient converges to one at rate n which is the same rate by which the exponent $k - 1$ goes to infinity. Thus, the coefficient is close enough to unity to avoid the impulse response function converging to zero, and therefore $\sum_{k=0}^{\infty} IR_k \to \infty$ as $k \to \infty$. This happens because a traditional nearly integrated process display the same type of persistency as a unit root process, that is, the shocks are not transitory.

Overall, we may say that a locally persistent process and a fractionally integrated process are similar in the sense that they are sitting in between the stationary and unit root extremes and their impulse responses converge to zero. However, it is important to stress that the total impact of a innovation will never be less than infinite if the process is fractionally integrated and this represents an important difference between local persistence and long memory. In practice, it may be more appropriate to think at a mean-reverting economic variable as a process in which the total impact of a unit innovation is finite. For example, technological innovations (shocks) might trigger a persistent economic growth, but it would be hard to believe that the impact of such innovations on the GDP is going to persist forever.

We have seen so far that the persistency parameter d is important to determine the extension of region of persistency of a locally persistent process. Hence, it turns out to be important to discuss estimation of d as well as testing related hypothesis. In the next two sections, we discuss estimation of d and propose a test for stationarity ($d = 0$) against the alternative hypothesis of local persistency.

3. Estimation of the local-persistence parameter

The most convenient approach to obtain some basic asymptotic properties of this model that are useful for our empirical analysis is to use the sequential asymptotic analysis of Phillips et al. (2001). It can be verified that by a re-parameterization (# of observations in each block = $m = n^d$, # of blocks = M) we may re-write the locally persistent time series in the format of a block local to unit root model which was first introduced by Phillips et al. (2001). The sequential asymptotic result can be obtained by taking $m \to \infty$ first, followed by $M \to \infty$. Following the proofs of Phillips et al. (2001), we can show that the process y_i yields the standard law of large numbers and central limit theorem type results. In particular, under the assumptions of Phillips et al. (2001), given the model (2) and standardization (4), we obtain the following normal asymptotics:

$$n^{\frac{1}{2}+\frac{d}{2}}(\hat{\alpha}-\alpha) \Rightarrow \xi = N(0,2), \tag{7}$$

where

$$\hat{\alpha} = \hat{\alpha}_{\mathrm{OLS}} - \frac{n\hat{\lambda}}{\sum y_{i-1}^2}.$$

$\hat{\alpha}_{\mathrm{OLS}}$ is the ordinary least squares estimator of α in the model (2), and $\hat{\lambda}$ is the consistent estimator of the one sided long run covariance parameter.[1] Therefore, $\hat{\alpha}$ is made up of two components: the first one corresponds to the usual least-square estimator of α, and the second component is a nonparametric correction that uses the consistent estimator of the one sided long run covariance parameter. The nonparametric correction is needed whenever we have a non i.i.d. innovation sequence. When the innovation sequence $\{u_i\}$ is independent and identically distributed, we have $\lambda = 0$. The result of Phillips et al. (2001) uses an sequential asymptotic analysis which does not impose restriction on $0 < d < 1$, and thus no additional assumptions on the relative magnitude of number of blocks and sample size within each block. As discussed in their paper (p. 36), under somewhat stronger conditions, the same result (normal asymptotic theory) holds for joint limits. Recently, Phillips and Magdalinos (2004) developed an general asymptotic theory for this model with i.i.d. innovations and $0 < d < 1$.

According to a local persistence process, the extension of region of persistency is given by the magnitude of the parameter d. The greater the value of d, the longer the persistent range and the longer the persistent effect will last. Therefore, it turns out to be important to estimate the parameter d in order to identify the degree of local persistence of the stochastic process. Notice that $1-\alpha = n^{-d}$, and after taking the logarithm, one obtains

$$d = -\frac{\ln(1-\alpha)}{\ln(n)}. \tag{8}$$

From (7), we have that:

$$n^{\frac{1}{2}-\frac{d}{2}}[n^d(\hat{\alpha}-1)+1] \Rightarrow \xi. \tag{9}$$

The above result implies that

$$n^d(1-\hat{\alpha}) \xrightarrow{P} 1 \quad \text{or} \quad \ln[n^d(1-\hat{\alpha})] \xrightarrow{P} 0. \tag{10}$$

Hence, one can propose the following consistent estimator for d:

$$\hat{d} = -\frac{\ln(1-\hat{\alpha})}{\ln(n)} = -\frac{\ln[n^d(1-\hat{\alpha})] - d\ln(n)}{\ln(n)} = d - \frac{\ln[n^d(1-\hat{\alpha})]}{\ln(n)} \to d. \tag{11}$$

[1] $\hat{\lambda} = \frac{1}{2}(\hat{\omega}_y^2 - \hat{\sigma}_y^2)$, where $\hat{\sigma}_y^2$ is a consistent estimator of the variance of y_i and $\hat{\omega}_y^2$ is a consistent estimator of the long-run variance of y_i. In this paper, we consistently estimate ω_y^2 by using nonparametric kernel smoothing.

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4. Testing the null hypothesis of stationarity against local persistency

In this section, we consider hypothesis testing for locally persistent process. We focus our attention on two classes of models: (H_0) under the null hypothesis, the time series is covariance stationary (or trend stationary); (H_1) under the alternative hypothesis, the time series is locally persistent as described by models (4) and (2). We construct a test for the null hypothesis (H_0) of covariance stationarity, against the alternative of local persistency. Notice that under the null, the order of magnitude of the partial sum process $\sum_{i=1}^{\upsilon} y_i$ should be proportional to ($\upsilon^{1/2}$) (although y_i may have high variance, it is not large in order of magnitude and can be normalized). Under mild conditions $n^{-1/2}\sum_{i=1}^{[nr]} y_i$ $(0 < r < 1)$ satisfies an invariance principle, i.e., $n^{-1/2}\sum_{i=1}^{[nr]} y_i \Rightarrow B(r) = w_y W(r)$, where $B(r)$ is a Brownian motion with variance $\omega_y^2 =$ long-run variance of $y_i = \lim_{n\to\infty} E(\frac{1}{\sqrt{n}}\sum_{i=1}^{n} y_i)^2$ and $W(r)$ is the standardized Brownian motion.[2] On the other hand, if the time series is locally persistent as described by (4), then the cumulated sum process $\sum_{i=1}^{\upsilon} y_i$ diverges to ∞ more rapidly than rate $\upsilon^{1/2}$. This observation suggests that it is possible to design a test by looking at the order of magnitude of the partial sum process.

We consider the following quantity as a measurement of the magnitude of the cumulated sum

$$\underset{1\leqslant \upsilon\leqslant n}{\mathrm{Max}}\ \frac{1}{\sqrt{n}}\left|\sum_{i=1}^{\upsilon} y_i - \frac{\upsilon}{n}\sum_{i=1}^{n} y_i\right|.$$

Under H_0 and regularity conditions, the above quantity converges weakly to $\sup_{0\leqslant r\leqslant 1}|\widetilde{B}(r)|$, where $\widetilde{B}(r) = B(r) - rB(1)$ is a Brownian bridge which is tied down to the origin at the end of the $[0,1]$ interval, with variance ω_y^2. Under the alternative hypothesis, y_i is a locally persistent, it is easy to verify that the corresponding statistic has much larger order of magnitude, diverging to ∞ as $n \to \infty$.

Notice that in practical analysis the limiting distribution depends on the long-run variance parameter ω_y^2 which is unknown and thus the above quantity can not be used directly. However, ω_y^2 can be consistently estimated using nonparametric kernel smoothing. In this paper, we consider the following nonparametric kernel estimator for ω_y^2 given by $\hat{\omega}_y^2 = 2\pi\hat{f}_{yy}(0)$, where

$$\hat{f}_{yy}(0) = \frac{1}{2\pi}\sum_{h=-q}^{q} k\left(\frac{h}{q}\right)\hat{\gamma}(h) \tag{12}$$

is the conventional spectral density estimator. In (12), $\hat{\gamma}(h)$ is the sample variance defined as $n^{-1}\sum' \hat{y}_i\hat{y}_{i+h}$, where $\sum'$ signifies summation over $1 \leqslant i,\ i+h \leqslant n$, $k(.)$ is the lag window defined on $[-1,1]$ with $k(0)=1$, and q is the bandwidth parameter satisfying the property that $q \to \infty$ and $q/n^d \to 0$ (say $q = o_p(n^d)$) as the sample size $n \to \infty$. Then, $\hat{\omega}_y^2$ is a consistent estimator of ω_y^2 under H_0. Candidate kernel functions can be found in standard texts (e.g. Hannan, 1970; Priestley, 1981). For example, when we use $k(x) = 1 - |x|$, we get the Bartlet estimator, that is

[2] Definition 1. A standard Brownian motion $W(\cdot)$ is a continuous-time stochastic process satisfies the following properties: (1) For any given t, $W(t)$ is normal with mean zero and variance t. (2) For any partition $\{r_1,\ldots,r_k\}$ of $[0,1]$, the increments $W(r_1)-W(0), W(r_2)-W(r_1),\ldots,W(r_k)-W(r_{k-1})$ are independent Gaussian. (3) For any given realization, $W(r)$ is continuous with probability one.

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$$\hat{\omega}_y^2 = \sum_{h=-q}^{q} \left(1 - \frac{|h|}{q}\right) \hat{\gamma}(h).$$

We denote the estimator for ω_y^2 as $\hat{\omega}_y^2$. We propose the following two statistic for testing the null hypothesis of stationarity or trend stationarity against local persistent nonstationarity

$$Q_n = \underset{1 \leqslant \upsilon \leqslant n}{\text{Max}} \frac{1}{\sqrt{n}} \frac{1}{\hat{\omega}_y} \left| \sum_{i=1}^{\upsilon} y_i - \frac{\upsilon}{n} \sum_{i=1}^{n} y_i \right| \tag{13}$$

$$\widehat{Q}_n = \underset{1 \leqslant \upsilon \leqslant n}{\text{Max}} \frac{1}{\sqrt{n}} \frac{1}{\hat{\omega}_y} \left| \sum_{i=1}^{\upsilon} \hat{y}_i - \frac{\upsilon}{n} \sum_{i=1}^{n} \hat{y}_i \right| \tag{14}$$

where Q_n is the test statistic evaluated at the observable time series, y_i, and $\widehat{Q}_n$ is the test statistic evaluated at the detrended time series $\hat{y}_i$.

The next three theorems present the behavior of the test statistics Q_n and $\widehat{Q}_n$ under the null hypothesis of stationarity and the alternative of local persistence. Proofs are found in Appendix.

Theorem 1 (Asymptotic behavior of the test statistic Q_n under the null). *Let y_i be a process without a time trend (t) as defined in (2). Under H_0 of covariance stationarity and the assumption of Phillips* et al. (2001),

$$Q_n = \underset{1 \leqslant \upsilon \leqslant n}{\text{Max}} \frac{1}{\sqrt{n}} \frac{1}{\hat{\omega}_y} \left| \sum_{i=1}^{\upsilon} y_i - \frac{\upsilon}{n} \sum_{i=1}^{n} y_i \right| \Rightarrow \sup_{0 \leqslant r \leqslant 1} |W(r) - rW(1)| \tag{15}$$

where $W(.)$ is a standard Brownian motion.

Remark 1. In the case y_i corresponds to the demeaned value of the observed time series, Theorem 1 still holds.

Table 1 reproduces the critical values for the test statistic Q_n.

Theorem 2 (Asymptotic behavior of the test statistic $\widehat{Q}_n$ under the null). *Let y_i be a process with a time trend (t) as defined in (5). Under H_0 and assumptions of Phillips* et al. (2001),

$$\widehat{Q}_n = \underset{1 \leqslant \upsilon \leqslant n}{\text{Max}} \frac{1}{\sqrt{n}} \frac{1}{\hat{\omega}_y} \left| \sum_{i=1}^{\upsilon} \hat{y}_i - \frac{\upsilon}{n} \sum_{i=1}^{n} \hat{y}_i \right|$$
$$\Rightarrow \sup_{0 \leqslant r \leqslant 1} \left| W(r) - rW(1) + 6(1-r)\left\{\frac{1}{2}W(1) - \int_0^1 W(s)\, \mathrm{d}s\right\} \right| \tag{16}$$

where $\hat{y}_i$ is the detrended value of y_i.

Table 1
Upper tail critical values for Q_n

Level of significance	0.1	0.05	0.01
Critical value	1.22	1.36	1.63

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Table 2
Upper tail critical values for $\widehat{Q}_n$

Level of significance	0.1	0.05	0.01
Critical value	0.827	0.901	1.041

In Theorem 1, the test statistic converges to a functional of Brownian bridge. Theorem 2, on the other hand, states that the test statistic $\widehat{Q}_n$ converges to the sup of the Brownian bridge plus a second term brought of a time trend t. Xiao (2001) calculated, via simulation, the critical values for the test statistic $\widehat{Q}_n$ which is reproduced in Table 2.

It is critical that a statistical test be able to discriminate between the null and the alternative in large sample. The following theorem gives properties of the tests under the alternative.

Theorem 3 (Consistency). *Under the alternative hypothesis of local persistency,*

(i) $\frac{1}{\hat{w}_y}\frac{1}{\sqrt{n}}\sum_{i=1}^{[nr]} y_i = \mathrm{O}_p\left(\sqrt{\frac{n^d}{q}}\right)$,

(ii) $\frac{1}{\hat{w}_y}\frac{1}{\sqrt{n}}\sum_{i=1}^{[nr]} \hat{y}_i = \mathrm{O}_p\left(\sqrt{\frac{n^d}{q}}\right)$,

(iii) *Assuming that the bandwidth parameter $q = \mathrm{o}_p(n^d)$, then Q_n (and $\widehat{Q}_n$) $\to \infty$, indicating that under the alternative hypothesis, the test statistic will reject the null with probability one.*

Theorem 3 shows that if we choose $q = \mathrm{o}_p(n^d)$, say, $q = k * [\ln(n)]$, where k is a constant and [.] an integer number, the statistical test proposed in this section is consistent since Q_n and $\widehat{Q}_n$ diverge under the alternative hypothesis as $n \to \infty$.

5. Monte Carlo results

A Monte Carlo experiment was conducted to examine the finite performance of the test statistic Q_n under H_0 and H_1.[3] From the construction of Q_n we know that such statistics depends on the sample size n, the parameter of persistency d, and the bandwidth parameter q that is used to calculate $\hat{w}_y^2$. Consequently, we paid special attention to the effects of n, d, and, q on the performance of this test. We considered the following sample sizes: $n = 200$, 500, 1000, and 2000. These sample sizes represent the most relevant range of sample sizes in many empirical analyses involving financial and economic data. Three bandwidth choices were considered, $q_1 = [\ln(n)]$, $q_2 = 3 * [\ln(n)]$, and $q_3 = 6 * [\ln(n)]$, where $[\cdot]$ indicates the integer part. All experiments contain 5000 replications. For the Kernel function, following Kwiatkowski et al. (1992), we used the Bartlett window $k(x) = 1 - |x|$, so that the nonnegativity of $\hat{w}_y^2$ was guaranteed.

5.1. Power of test

Under H_1, the data are generated from $y_i^\tau = \varphi_o + \varphi t + y_i$, with $\varphi_o = \varphi = 0$, $y_i = (1 - \frac{1}{n^d})y_{i-1} + u_i$, with $u_i \equiv$ i.i.d.$N(0,1)$. We considered three values of the persistency

[3] Results for $\widehat{Q}_n$ are available under request. They are qualitatively similar to the results obtained using Q_n.

Table 3
Power of test, 5% level

Local persistence parameter	Sample size (n)			
	$n = 200$	$n = 500$	$n = 1000$	$n = 2000$
Bandwidth parameter $= q_1 = [\ln(n)]$				
$d = 0.5$	0.531	0.753	0.862	0.936
$d = 0.8$	0.817	0.966	0.994	0.999
$d = 1$	0.864	0.980	0.996	1.000

parameter: $d = 0.5$, 0.8, and 1.0. The first and second value represent processes with local persistence. The degenerate case, $d = 1$, represents near integration and this process is supposed to diverge to infinity at the same rate as a process with unit root, i.e., $O(n^{1/2})$. In each replication, we computed the statistic Q_n and compared it with the 5% critical value from Table 1. Theorem 2 tells us that the null hypothesis will be rejected with probability one when the sample size goes to infinity and H_1 is true. For the three values of the local-persistence parameter d, the tables below confirm what is predicted by the theory. In particular, the test exhibits low power when $d = 0.5$, $n = 200$ and $q = q_3$. However, the power increases substantially for larger samples. For example, even if $d = 0.5$ and $q = q_3$, the power rises from 0.071 to 0.369 as sample size increases from 200 to 2000. One can also see that the power is reduced as the bandwidth parameter q increases because, as showed by Theorem 3, the power of our test depends upon n^d/q. In other words, a large q will reduce power, whereas a large n and a large d will increase power. All this is confirmed by the Monte Carlo results, see Table 3.

5.2. Size of test

We next examined the properties of Q_n under the null hypothesis. Note that under H_0 $y_i = u_i$, a general covariance stationary time series that satisfies an invariance principle. The data were generated from $u_i = \beta u_{i-1} + \epsilon_i$, with $\epsilon_i \equiv \text{i.i.d.}N(0,1)$. Note that similarly to other testing procedures in the unit root literature, size distortion in finite sample exists. One way to improve the performance of the tests is to use appropriate bandwidth selection. Thus, if we choose an appropriate bandwidth parameter q, we could expect that the size of test would converge to the nominal size as the sample size increases. The bandwidth parameter q corresponds to the number of lags used to calculate $\hat{\omega}_y^2$. Intuitively, for $\beta > 0$, the larger β is, the longer lags we need. In the case that $\beta = 0$, y_i is an independent sequence and the long-run variance of y_i equals the variance of y_i. Thus, we expect that for small β a small bandwidth parameter would be more appropriate than a large one. On the other hand, if β is large then we need to increase q in order to account for the existence of serial correlation in y_i. Table 4 presents the empirical size of 5% tests for the same bandwidth parameters considered in Table 5 and $\beta = 0.0, 0.4, 0.9$ and 0.95.

Results in Table 6 suggest that when $\beta = 0$ and $\beta = 0.4$, the size distortion is very small no matter the bandwidth parameter employed.[4] On the other hand, if $\beta = 0.90$ or $\beta = 0.95$ and one uses q_1 or q_2, then the test will be oversized. However, Table 6 shows that, unlike what happens to the power of test, the size distortion does not converge to one as sample

[4] Note, however, that the test becomes very conservative when q_3 is used and $\beta = 0.0$ or $\beta = 0.4$.

Table 4
Power of test, 5% level

Local persistence parameter	Sample size (n)			
	$n = 200$	$n = 500$	$n = 1000$	$n = 2000$
Bandwidth parameter $= q_2 = 3 * [\ln(n)]$				
$d = 0.5$	0.152	0.272	0.420	0.568
$d = 0.8$	0.346	0.689	0.891	0.974
$d = 1$	0.459	0.774	0.938	0.987

Table 5
Power of test, 5% level

Local persistence parameter	Sample size (n)			
	$n = 200$	$n = 500$	$n = 1000$	$n = 2000$
Bandwidth parameter $= q_3 = 6 * [\ln(n)]$				
$d = 0.5$	0.071	0.125	0.234	0.369
$d = 0.8$	0.093	0.415	0.650	0.843
$d = 1$	0.118	0.503	0.798	0.914

Table 6
Size of test, 5% level

β	n	q_1	q_2	q_3
0.0	200	0.032	0.017	0.00
	500	0.042	0.034	0.025
	1000	0.042	0.039	0.033
	2000	0.045	0.042	0.039
0.4	200	0.051	0.018	0.00
	500	0.063	0.036	0.024
	1000	0.064	0.042	0.033
	2000	0.063	0.046	0.040
0.90	200	0.416	0.066	0.00
	500	0.454	0.115	0.037
	1000	0.44	0.135	0.050
	2000	0.423	0.135	0.065
0.95	200	0.639	0.154	0.01
	500	0.72	0.242	0.071
	1000	0.719	0.283	0.110
	2000	0.718	0.276	0.113

size increases. This happens because under the null, even in the extreme case in which β is large and close to 1 (say $\beta = 0.9$, 0.95), the standardized cumulated sums of the time series still satisfy an invariance principle. We can use this fact to reduce dramatically the size distortion by choosing a wider range of bandwidth selection. For example, if $\beta = 0.95$ and q_3 is used, then the test may still be oversized but the size distortion will be much smaller than when one uses q_1 or q_2.

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In sum, if we combine the results in Table 5 with the results in Table 6, we can conclude that the proposed test is an asymptotic test which discriminate between the null and the alternative in large samples. In the next section, we illustrate the applicability of this test using samples with 934, 2,501 10,619 observations. For this range of sample sizes, the trade-off between size and power may be very small even when q_3 is used.

6. Local persistency in economic time series

In this section, we illustrate the applicability of the proposed test using macroeconomic time series. In order to avoid the trade-off between size and power, we only consider time series with large samples. In particular, we considered four US time series: (i) 1-year treasury constant maturity rate (NIR-12m) with daily observations ranging from 1962-02-01 to 2004-08-20, totalizing 10,619 observations; (ii) nominal exchange rate for Canadian dollar–US dollar (NER (Can–US)) with daily observations from 1992-07-24 to 2002-06-28, corresponding to 2501 observations; (iii) nominal exchange rate for United Kingdom pound–US dollar (NER (UK–US)) with daily observations from 1992-07-24 to 2002-06-28, corresponding to 2501 observations and; (iv) inflation rate (INF) measured by the consumer index price with monthly observations from 1926-01-30 to 2003-10-31, totalizing 934 observations. All aforementioned data were collected from the Board of Governors of the Federal Reserve System.

Figs. 1–3 show graphs of NIR-12m, NER (UK–US) and INF centered with respect to their sample mean. Fig. 4 shows graphs of NER (Can–US) with centered and detrended values. One can see that all the variables display wide fluctuations about mean (trend), but there seems to be a mean (trend) reversion in all cases. Therefore, we could expect unit root test to reject the null hypothesis of a unit root for these cases.

However, as suggested by past studies, this visual impression of mean reversion (or trend reversion) has been hard to establish statistically using traditional unit root tests. Table 7 shows the results for the ADF test.[5] Unlike the visual evidence, we reject the unit root hypothesis at 5% level of significance only for the time series INF.

Another interesting aspect displayed in Table 7 is that most of the series has roots near unity[6] and, as documented by Campbell and Perron (1991) and Dejong et al. (1992), near unity roots may explain the failure to reject the unit root null in the ADF test. Since processes with local persistence have roots too close to unity, we should employ a more powerful test to reject the null hypothesis of unit root. Elliott et al. (1996) introduced a modified unit root test (DF-GLS) that has better power when the AR coefficient is close to unity. Table 7 shows the results from the DF-GLS test with the notation '*', '**', and '***' indicating that the null of unit root is rejected at 10%, 5% and 1% level of significance. By using the DF-GLS test, we reject the null hypothesis of unit root for all variables of our sample.

Nonetheless, it is important to mention that the rejection of the null hypothesis of unit root does not necessarily imply that the process is ergodic stationary, for instance, it can also be locally persistent.

[5] We used the same lag choice for the ADF and DF-GLS tests, that is, the choice based on the Modified Information Criteria (MIC) suggested by Perron and Serena (2001). The main advantage of this criteria is that it imposes the null hypothesis of unit root into the objective function used to calculate the optimal lag.

[6] Estimates of the autoregressive coefficients were computed according to Eq. (7), that is, $\hat{\alpha} = \hat{\alpha}_{\mathrm{OLS}} - \frac{n\hat{\lambda}}{\sum y_{t-1}^2}$.

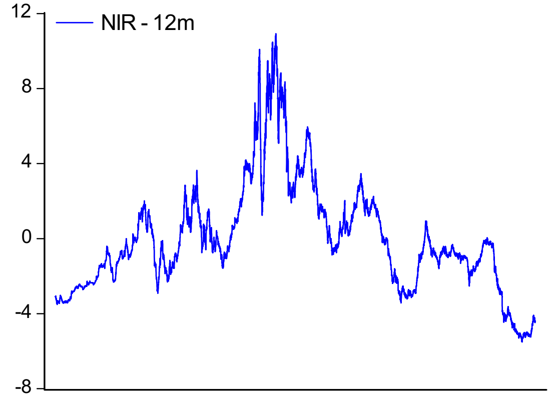

Fig. 1.

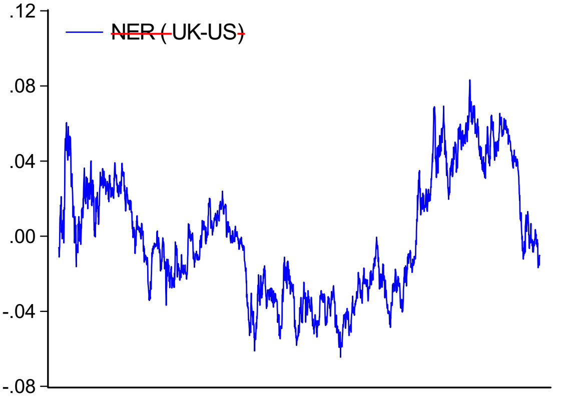

Fig. 2.

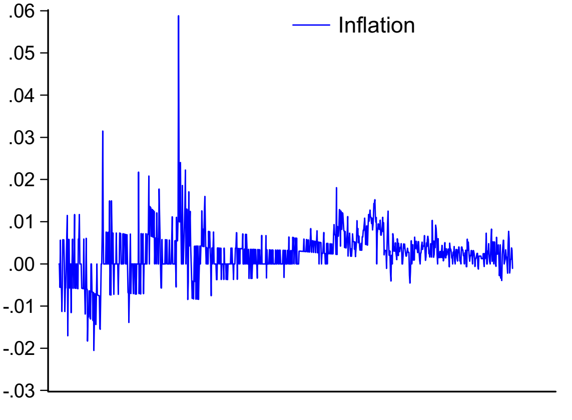

Fig. 3.

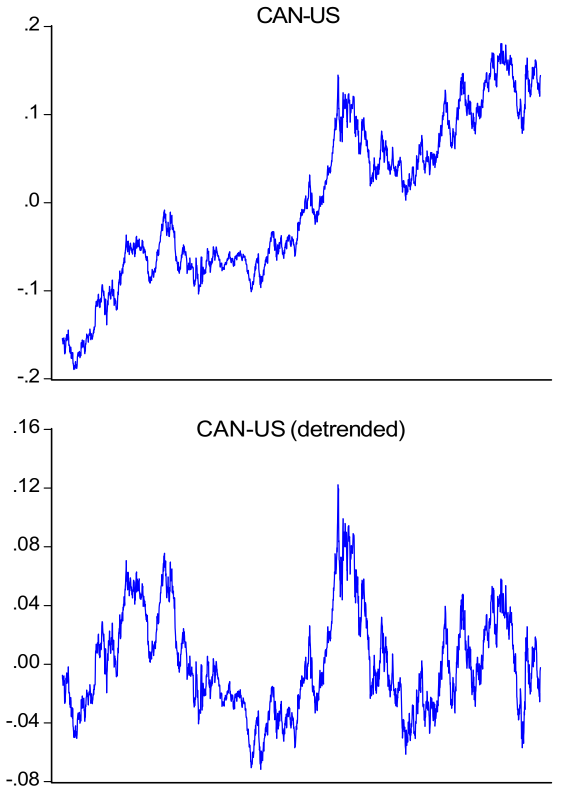

Fig. 4.

Table 7
Unit root tests

Series	Specification	Sample size	Lags	ADF	$\hat{\alpha}$	DF-GLS
NER (Can–US)	trend	2501	0	−3.11	0.990	-3.41^{**}
NER (UK–US)	Intercept	2501	0	−2.52	0.996	-2.51^{**}
INF	Intercept	934	3	-8.11^{***}	0.935	-6.96^{***}
NIR-12m	Intercept	10,619	12	−2.26	0.999	-1.62^{*}

Table 8 reports the results for test of the null against the alternative of local persistency, as well as point estimates of the local persistence parameter, d. We used three sample dependent bandwidth parameters $q_1 = [\ln(n)]$, $q_2 = 3*[\ln(n)]$ and $q_3 = 6*[\ln(n)]$ where [.] signifies an integer number. Again, we are considering time series with large samples, and this is particularly important because as pointed out by the Monte Carlo experiments in the previous section, the proposed test is an asymptotic test and, therefore, it needs large samples to be able to discriminate between local persistence and stationarity. The notation '*', '**', and '**' suggests that the null hypothesis is rejected at 10%, 5%, and 1% level of significance, respectively. First, we notice that the results reported in Table 8 indicate that the data uniformly reject the stationarity null hypothesis against the alternative. In addi-

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Table 8
Results from local persistency analysis

Series	$\hat{d}$	$\hat{d}_L$	Model	Q_n		
				$q_1 = [\ln(n)]$	$q_2 = 3*[\ln(n)]$	$q_3 = 6*[\ln(n)]$
NER (Can–US)	0.68	0.90	Trend	1.90***	1.19***	0.87*
NER (UK–US)	0.70	0.92	Intercept	4.92***	3.05***	2.19***
INF	0.54	0.37	Intercept	2.16***	1.50**	1.26*
NIR-12m	0.78	0.95	Intercept	7.23***	4.26***	3.05***

tion, all series have an estimated local persistence parameter, $\hat{d}$, different from zero. Combining this result with the previous evidence in Table 7, our empirical analysis indicates that these time series are sitting between the conventional stationary and the unit root processes, supporting the existence of local persistency in US data of inflation, interest rate and exchange rate.

The existence of local persistence implies that shocks affecting NER (Can–US), NER (UK–US), INF, and NIR-12m are long lasting, but the time series exhibits mean (trend) reverting behavior in the sense that the impulse response converges to zero.

In order to illustrate the differences between a local persistence model and other models largely used in the literature to capture persistence in macroeconomic time series, such as fractional integration and linear autoregressive models with large autoregressive coefficient, we computed values for the k-period impulse response function (IR_k). As mentioned in Section 2.3, if the time series y_i is integrated of order d_l, $I(d_l)$, with $0 < d_l < 1$, its $IR_k \approx 1/(k^{1-\hat{d}_l})$ where $\hat{d}_l$ is an estimate of d_l. Table 8 contains $\hat{d}_l$ obtained by applying the non-linear least squares method available at Arfima package (Doornick and Ooms, 2001) for Ox programming language.[7] The estimates of d_l reported in Table 8 are all significant at 5%. On the other hand, if we assume that the time series is a simple linear autoregressive model of order one, $AR(1)$, then its $IR_k = \hat{\alpha}^{k-1}$ where $\hat{\alpha}$ is reported in Table 7. Finally, if we assume that the time series is locally persistent, then its $IR_k = (1 - 1/n^{\hat{d}})^{k-1}$, where $n = n_o + k$ and n_o is the sample size used to estimate $\hat{\alpha}$ displayed in Table 7.

Table 9 displays the k-period impulse response function computed using the estimates of α, d, and d_l reported in the above tables. One can see, for example, that if we model exchange rates as a fractionally integrated process, then the impact of a unit innovation vanishes in the long run, but vanishes very slowly: the shocks are not totally absorbed even when $k = 10{,}000$, which corresponds to about 30 years!. [8] On the other hand, if exchange rates are modeled as locally persistent processes, then the shocks are still long lasting, but the persistence is much weaker than in a model with fractional integration: shocks are almost totally absorbed in about 3 years, ten times less than predicted by a fractional model. Finally, if we ignore the existence of local persistence and proceed modelling exchange rate as a linear autoregressive process with root near unity, then we would be led to believe that the shocks are rapidly absorbed. Note that the above discussion also applies to the series of inflation and interest rate, with the latter being much more persistent than the former. We argue that, using the numbers in Table 9, although fractional

[7] In order to estimate d_l, we considered a ARFIMA $(0, d_l, 0)$ specification for all time series appearing in Table 8.
[8] We are using daily observations of exchange rates. So $k = 10{,}000$ corresponds to about 30 years.

Table 9
The *k*-period impulse response function

Model	NER (Can–US)	NER (UK–US)	INF	NIR-12m
k = 5				
Local persistence	0.98	0.98	0.77	0.99
Fractional integration	0.85	0.88	0.36	0.92
Linear	0.96	0.97	0.76	0.97
k = 30				
Local persistence	0.86	0.88	0.15	0.98
Fractional integration	0.71	0.76	0.12	0.84
Linear	0.74	0.75	0.14	0.97
k = 300				
Local persistence	0.26	0.31	0.00	0.81
Fractional integration	0.56	0.63	0.03	0.76
Linear	0.05	0.05	0.00	0.74
k = 1000				
Local persistence	0.02	0.04	0.00	0.51
Fractional integration	0.50	0.57	0.02	0.70
Linear	0.00	0.00	0.00	0.367
k = 10000				
Local persistence	0.00	0.00	0.00	0.00
Fractional integration	0.40	0.48	0.01	0.63
Linear	0.00	0.00	0.00	0.00

models also imply mean (trend) reversion, the full reversion may take a very long time to occur. Thus, for reasonable finite horizons, we might say that full reversion does not occur if exchange rate, inflation and interest rate are fractionally integrated. On the other hand, local persistency may rather lead to full reversion within reasonably long finite horizons. In fact, while it is recognized that many economic time series are highly persistent over certain range, less persistent results are also found around very long horizons. In this sense, we believe that the local persistence model provides a useful alternative to the traditional unit root and stationary models, and it is a useful complement to the fractional integrated model.

7. Conclusion

We study local persistence of macroeconomic time series. We have proposed statistical tests for the null hypothesis of stationarity (or trend stationarity) against local persistence. The test statistics converge to nonstandard limiting distributions that are functions of Brownian motions, involving higher order Brownian bridges. Tables of critical values are provided based on the asymptotic null distributions and a Monte Carlo experiment was conducted to examine the finite performance of these test, with special emphasis to the study of the finite sample size and power. The test is applied to several important variables of the US economy: interest rate, inflation, and exchange rate. Our results suggest that these macroeconomic time series may be locally persistent and, therefore, display a pattern of temporal dependence that is different from the one generated by a traditional unit root and fractionally integrated process.

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Appendix

Proof of Theorem 1 and Theorem 2. By definition

$$\widehat{Q}_n = \max_{1\leqslant \upsilon\leqslant n} \frac{\upsilon}{\hat{\omega}_y n^{1/2}}\left|\frac{1}{\upsilon}\sum_{i=1}^{\upsilon}\hat{y}_i - \frac{1}{n}\sum_{i=1}^{n}\hat{y}_i\right| = \max_{0\leqslant r\leqslant 1}\frac{1}{\hat{\omega}_y}\left|\frac{1}{n^{1/2}}\sum_{i=1}^{[nr]}\hat{y}_i - \frac{[nr]}{n}\left(\frac{1}{n^{1/2}}\sum_{i=1}^{n}\hat{y}_i\right)\right|$$

where $r\in[0,1]$ and $\hat{y}_i$ is the (detrended) time series. Notice that

$$\frac{1}{n^{1/2}}\sum_{i=1}^{[nr]}\hat{y}_i = \frac{1}{n^{1/2}}\sum_{i=1}^{[nr]}y_i - \frac{1}{n^{1/2}}(\hat{\varphi}_y-\varphi_y)'\sum_{i=1}^{[nr]}\Upsilon_i$$

where $n^{-1/2}\sum_{i=1}^{[nr]}y_i$ is a stochastic process in $D[0,1]$, the space of functions on $r\in[0,1]$ that are right continuous with left-hand limits. We endow the space $D[0,1]$ with the Skorohod topology (Billingsley, 1968). Under the null hypothesis of stationarity, $H_o{:}d=0$, and the assumption of Phillips et al. (2001), the partial sum process $n^{-1/2}\sum_{i=1}^{[nr]}y_i$ satisfies the invariance principle, that is:

$$\frac{1}{n^{1/2}}\sum_{i=1}^{[nr]}y_i \Rightarrow B(r).$$

Assume that there is a standardizing matrix D such that $D^{-1}\Upsilon_{[nr]}\to\Upsilon(r)$ as $n\to\infty$, uniformly in $r\in[0,1]$. For the case of a linear trend, $D=\mathrm{diag}[1,n]$ and $\Upsilon(r)=(1,r)'$. Thus, by the continuous mapping theorem,

$$\begin{aligned} n^{1/2}D(\hat{\varphi}_y-\varphi_y) &= \left(n^{-1}\sum D^{-1}\Upsilon_i\Upsilon_i'D^{-1}\right)^{-1}\left(n^{-1/2}\sum D^{-1}\Upsilon_i y_i\right)\\ &\Rightarrow \left\{\int_0^1 \Upsilon(s)\Upsilon(s)'\,\mathrm{d}s\right\}^{-1}\left\{\int_0^1 \Upsilon(s)\,\mathrm{d}B(s)\,\mathrm{d}s\right\}\end{aligned}$$

and

$$\begin{aligned}\frac{1}{n^{1/2}}\sum_{i=1}^{[nr]}\hat{y}_i &= \frac{1}{n^{1/2}}\sum_{i=1}^{[nr]}y_i - \frac{1}{n^{1/2}}(\hat{\varphi}_y-\varphi_y)'\sum_{i=1}^{[nr]}\Upsilon_i\\ &= \frac{1}{n^{1/2}}\sum_{i=1}^{[nr]}y_i - \{n^{1/2}(\hat{\varphi}_y-\varphi_y)'D\}\left(\frac{1}{n}\sum_{i=1}^{[nr]}D^{-1}\Upsilon_i\right)\\ &\Rightarrow B(r) - \left\{\int_0^1 \mathrm{d}B(s)\Upsilon(s)'\right\}\left\{\int_0^1\Upsilon(s)\Upsilon(s)'\,\mathrm{d}s\right\}^{-1}\left\{\int_0^r\Upsilon(s)\,\mathrm{d}s\right\}\\ &= \omega_y\left[W(r) - \left\{\int_0^1 \mathrm{d}W(s)\Upsilon(s)'\right\}\left\{\int_0^1\Upsilon(s)\Upsilon(s)'\,\mathrm{d}s\right\}^{-1}\left\{\int_0^r\Upsilon(s)\,\mathrm{d}s\right\}\right]\\ &= \omega_y\widetilde{W}(r)\end{aligned}$$

where $\widetilde{W}(r) = W(r) - \{\int_0^1 \mathrm{d}W(s)\Upsilon(s)'\}\{\int_0^1 \Upsilon(s)\Upsilon(s)'\,\mathrm{d}s\}^{-1}\{\int_0^r \Upsilon(s)\,\mathrm{d}s\}$. Thus, by the fact that $[nr]/n \to r$ and the continuous mapping theorem,

$$\max_{0\leqslant r\leqslant 1}\left|\frac{1}{n^{1/2}}\sum_{i=1}^{[nr]}\hat{y}_i - \frac{[nr]}{n}\left(\frac{1}{n^{1/2}}\sum_{i=1}^{n}\hat{y}_i\right)\right| \Rightarrow \sup_{0\leqslant r\leqslant 1}|\omega_y\{\widetilde{W}(r) - r\widetilde{W}(1)\}|$$

and

$$\widehat{Q}_n = \max_{1\leqslant \upsilon\leqslant n}\frac{1}{\sqrt{n}}\frac{\upsilon}{\hat{\omega}_y}\left|\frac{1}{\upsilon}\sum_{i=1}^{\upsilon}\hat{y}_i - \frac{1}{n}\sum_{i=1}^{n}\hat{y}_i\right| \Rightarrow \sup_{0\leqslant r\leqslant 1}|\{\widetilde{W}(r) - r\widetilde{W}(1)\}|.$$

The limiting process $\omega_y\widetilde{W}(r)$ is a generalized Brownian bridge process. If the time series y_i is observed, then $\widetilde{W}(r) = W(r)$, and therefore $Q_n \Rightarrow \sup_{0\leqslant r\leqslant 1}|\{W(r) - rW(1)\}|$. This proves Theorem 1. □

When Υ_i has a constant element, the process $\widetilde{W}(r)$ is tied down to the origin at the ends of the $[0,1]$ interval, just like a Brownian bridge. Thus, $\widetilde{W}(1) = 0$, and $\widehat{Q}_n \Rightarrow \sup_{0\leqslant r\leqslant 1}|\widetilde{W}(r)|$. In the case that Υ_i is a constant, $\widetilde{W}(r) = W(r) - rW(1)$ is a standard Brownian bridge and $\widehat{Q}_n \Rightarrow \sup_{0\leqslant r\leqslant 1}|\{W(r) - rW(1)\}|$. This proves Remark 1.

If Υ_i is a linear trend, i.e. $\Upsilon_i = (1,t)'$, then

$$\widetilde{W}(r) = \{W(r) - rW(1)\} + 6r(1-r)\left\{\frac{1}{2}W(1) - \int_0^1 W(s)\,\mathrm{d}s\right\}$$

which is a sum of a standard Brownian bridge plus another factor

$$6r(1-r)\left\{\frac{1}{2}W(1) - \int_0^1 W(s)\,\mathrm{d}s\right\}$$

brought by the time trend t. This process is usually called a second-level Brownian bridge (MacNeill, 1978). Thus,

$$\widehat{Q}_n \Rightarrow \sup_{0\leqslant r\leqslant 1}|\widetilde{W}(r)| = \sup_{0\leqslant r\leqslant 1}\{W(r) - rW(1)\} + 6r(1-r)\left\{\frac{1}{2}W(1) - \int_0^1 W(s)\,\mathrm{d}s\right\}.$$

This proves Theorem 2. □

Proof of Theorem 3. For the estimation of w_y^2, we consider the estimator

$$\hat{\omega}_y^2 = \sum_{h=-q}^{q}\left(1 - \frac{|h|}{q}\right)\hat{\gamma}(h)$$

where $\hat{\gamma}(h) = \frac{1}{n}\sum_{i=1}^{n-|h|}\hat{y}_i\hat{y}_{i+h}$, Under H_1, $\hat{\gamma}(h) = \mathrm{O}_p(n^d)$ and consequently $\hat{\omega}_y^2 = \mathrm{O}_p(n^d q)$. Thus, it can verified that, under H_1

$$\frac{1}{\sqrt{n}}\sum_{i=1}^{[nr]}\hat{y}_i = \mathrm{O}_p(n^d)$$

and

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$$\frac{1}{\hat{\omega}_y}\frac{1}{\sqrt{n}}\sum_{i=1}^{[nr]}\hat{y}_i = \mathrm{O}_p\left(\frac{n^d}{\sqrt{n^d q}}\right) = \mathrm{O}_p\left(\sqrt{\frac{n^d}{q}}\right).$$

Given that $q = \mathrm{o}_p(n^d)$, we get a consistent test, that is, $\widehat{Q}_n \to \infty$ as $n \to \infty$. This proves Theorem 3. □

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