

**Project Title : Mechanical Properties of
Blended Single Wall Carbon Nanotube
(SWCNT) Composites Admixed in Epoxy**

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Summary

In this present study we analyze the behavior of blended composite specimens comprising of 0.0%, 0.1% , 0.5%, and 1%, single walled carbon nanotubes (SWCNT) admixed in epoxy resin (epichlorohydrin and bisphenol F). Nanoindentation using nanoindenter XP is done on the surface of these samples and from the study of SWCNT%, applied load, depth of indentation and hardness, the nature of the young's modulus of the samples has been analyzed. The data used here are the average of 10 indentation performed on each sample. The source data is the data generated by the ongoing research at the department of civil and environment engineering in The University of Tennessee [2,3].

The purpose of this project is to examine the relationship of modulus(y) in epoxy/carbon-nanotube composites with variables of SWCNT% (x1), Depth (x2), Load (x3) and Hardness (x4) and apply the best fit model to represent the response variable with the several predictors as explained.

The bivariate fit of the respective predictor variables with the response variable modulus(y) indicates that the Depth (x2) and Load (x3) data are not linear whereas a natural log transformation of these data [i.e. $\ln x_2(\text{Depth})$ and $\ln x_3(\text{Load})$] gives a linear relationship. While checking multicollinearity between the various predictor variables, we observed that the correlation coefficient between $\ln x_2(\text{Depth})$ and $\ln x_3(\text{Load})$ is very high (0.9996) indicating a serious multicollinearity problem. We also checked several related issues and realized that to avoid such multicollinearity, we need to delete one of the predictor variable between $\ln x_2(\text{Depth})$ and $\ln x_3(\text{Load})$ from our final model. Through stepwise regression for a full second degree model, we got initial idea about our model selection. However, for our final model selection we give emphasis on Type I and Type III error tests, PRESS statistics, R^2 Value etc. besides several other criteria. We tests several test models and conclude that the model (Model 2) comprising SWCNT%(x1), $\ln x_2$ (Depth), Hardness(x4) and the interaction terms between SWCNT%(x1) & Hardness(x4) as well as between $\ln x_2(\text{Depth})$ & Hardness(x4) best suits the purpose. Through several model characteristics viz. R^2 value, PRESS statistics etc., we observed that $\ln x_2$ is better predictor than $\ln x_3(\text{load})$ and thus deleted $\ln x_3(\text{load})$ from our model. To reduce any multicollinearity existing in our model we considered centering the data for the interaction terms.

From the residual plot for the selected model, we see that the residuals are having few outliers. Based on the studentized residuals data, we identify that there are nine observations that shows values greater than two standard deviations. We examined the data once again and it reveals through scanning electron microscopy (SEM) that these outliers are the erratic responses of modulus arising out due to the non homogeneous mixing of SWCNT in epoxy while preparing samples. Considering our present purpose is to establish the best fit model of response variable depending on the several predictors, we realize that deleting these outliers from our model will not hamper much the characteristic of the material. So our final model is devoid of any outliers.

We run the model deleting these outliers and the model further improves the R-Square to 0.936981 which means that 93.70% of the variability in modulus(y) is accounted for by the SWCNT%, \ln (Depth), Hardness and the interaction of SWCNT% & Hardness and of \ln (Depth) & Hardness. The P-Value of F-test is $<.0001$ which is highly significant. The VIF values also well below 10 indicating that there is no multicollinearity problem. The

Type I error and type III error tests demonstrate that each independent variable are significant at $\alpha = 0.05$ level.

The PRESS statistics is minimum with 0.3967 which is pretty low in comparison to the previous value of 0.7259 suggesting it is a better model.

The residual by predicted plot shows no obvious shape and the dispersion of the residuals are approximately constant with respect to the predicted values. So, the constant variance assumption is satisfied.

The normal quantile plot of residuals also shows clear linear shape indicating residuals are normally distributed. In the Goodness-of-Fit Test, the Shapiro-Wilk W test, the attained significance level is .3267 which is highly non-significant. So we can not reject the null hypothesis H_0 and the residual normality assumption is accepted.

From the residuals vs Xi plot, all of them exhibit random scatter around zero and there is no symmetric pattern implying linearity assumption is also satisfied.

However, we noticed few influential observations in our model and we see that the lower and higher depth samples are mostly showing the influential behavior. This implies that during the initial stage of indentation and at the last stage of indentation some sort of unknown activity was going on that makes the data influential. One reason for these data being influential at the initial stage might be that the indenter approaches the surface of the sample, and detection of surface is made manually for indentation test which is not a very precise process, giving rise to the influential observation. However, we are not very sure about the true reason for such influential responses of the data at this initial & final stage of indentation which demands further research work is needed. So we did not delete this influential data from our model at this stage.

The model has a Durbin-Watson statistic of 1.3312473 and a corresponding autocorrelation of 0.3175 indicating autocorrelation might be a problem. Moreover, we know that the indentation testing is time dependent where a continuous load is applied on the sample and corresponding modulus at different depth is measured. So time dependent machine drift with resultant autocorrelation can not be ruled out. However, due to some practical constraints and paucity of data it is not possible at this stage to explore such time dependent autocorrelation.

So, the two major constraints for our model are the presence of few influential observations and an unknown source of autocorrelation. Beside these the model shows a very good overall fit with 93.70% of the variability in modulus(y) is accounted for by the SWCNT%, ln (Depth), Hardness and the interaction of SWCNT% & Hardness and ln (Depth) & Hardness.

Introduction

Since the discovery of carbon nanotubes in 1991 by Iijima [1] with subsequent observation regarding its characteristic mechanical and electrical properties, an extensive research in the field of blended carbon-epoxy nanocomposites has started. Nanotubes are long, slender fullerenes where the walls of the tubes are hexagonal carbon (graphite structure) and often capped at each end. Two principal types of nanotubes are of prime importance for the researchers viz. Single walled carbon nanotube (SWCNT) and multiwalled carbon nanotubes (MWCNT). A SWCNT is a graphene sheet rolled-over into a cylinder with typical diameter on the order of 1.4 nm, similar to that of a C₆₀ buckyball.

In this present study the characteristic flexural property (young's modulus) of nanotube/epoxy blended nanocomposites prepared using a commercially available low viscosity, liquid epoxy resin (epichlorohydrin and Bisphenol-F) and unpurified single walled carbon nanotubes (SWCNT) will be evaluated.

Blended composites specimens comprising of 0.1% , 0.5%, and 1%, single walled carbon nanotubes (SWCNT) admixed in epoxy resin (epichlorohydrin and bisphenol F) were prepared along with a control samples (without any nanotube i.e. 0% SWCNT). The samples were then subjected to nanoindentation using nanoindenter XP (with a pyramid shape Berkovich indenter) on the surface of the samples and from the study of load, displacement (i.e. depth of indentation) and hardness, the nature of the young's modulus of the samples will be analyzed. The data to be collected are the average of 10 indentation performed on each sample. The source data is the data generated by the ongoing research at the department of civil and environment engineering in The University of Tennessee [2,3].

Thus the response variables for this study are Young's modulus (measured in GPa) of the different composites samples. Altogether 4 different types of samples are analyzed which are :

- a) 0.1% SWCNT admixed in epoxy
- b) 0.5% SWCNT admixed in epoxy
- c) 1.0% SWCNT admixed in epoxy
- d) Control epoxy (0.0% SWCNT)

The different treatment factors that might affect the response variables, to be analyzed are:

- a) Different weight fraction of SWCNT composite (i.e. % SWCNT admixed in epoxy).
- b) Applied load [in milli Newton (mN)] during indentation.
- c) The corresponding depth [in nanometer (nm)] of loading.
- d) The hardness [in gigapascal (GPa)] of the sample at respective depth

Finally, our objective for this study will be to establish the relationship between the various predictor variables on the response variable. An enthusiastic approach might be to predict the young's modulus using the optimum level of treatment variables as well as by controlling the parameters to predict the response of outcome.

References :

- [1] S. Iijima, Nature 354 (1991) 56-57.
- [2] D. Penumadu, A. Dutta, G.M. Pharr, B.S. Files, J. Mater. Res., 18 (2003) 1849-1853.
- [3] A.K.Dutta, D.Penumadu, B.S.Files, J.Mater. Res., (Article in press).

Data

From the nature of indentation and experience it is seen that during the initial stage of indentation (when the surface detection is an important part for the indentation technique), the load vs depth data is not reliable. Thus we only considered the data beyond a depth of 500 nm that are considered well representative of the sample characteristic. As said before, each sample is subjected for 10 indentations (using the nanoindenter XP) and the measured depth-wise data based on the average of those 10 indentations is shown in Annexure I.

Altogether 169 samples covering 0% SWCNT, 0.1% SWCNT, 0.5% SWCNT and 1.0% SWCNT are considered for the analysis.

Data Analysis

The bivariate fit of the respective predictor variables with the response variable modulus(y) is shown below. It is seen that the bivariate fit of modulus(y) with Depth (x2) [fig 1a] and Load (x3) [fig 2a] showing curvature whereas their natural log transformation shows more linear relationship [Fig 1b & 2b]. Thus for our model, we will consider this natural log transformation (defined as ln) for variable Depth (x2) and load (x3). The bivariate fit for hardness (x4) with modulus(y) [Fig 3] shows linear relationship suggesting no transformation is needed for the data.

Bivariate Fit of Modulus(y) By Depth(x2)

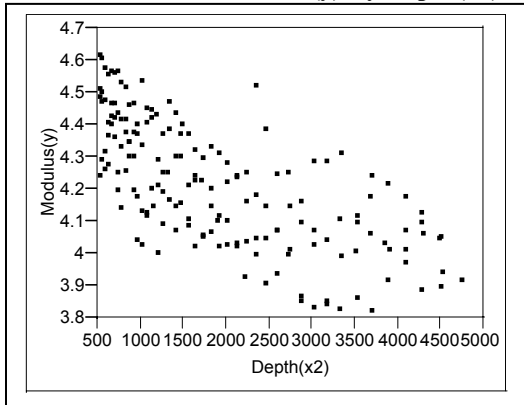


Fig 1a

Bivariate Fit of Modulus(y) By ln x2 (Depth)

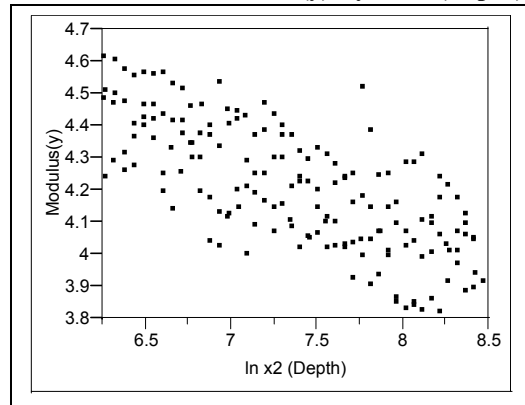


Fig 1b

Bivariate Fit of Modulus(y) By Load(x3)

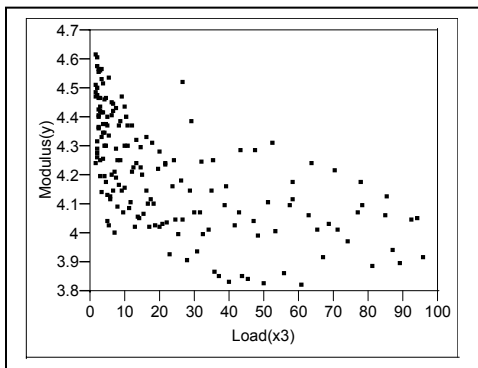


Fig 2a

Bivariate Fit of Modulus(y) By ln x3(Load)

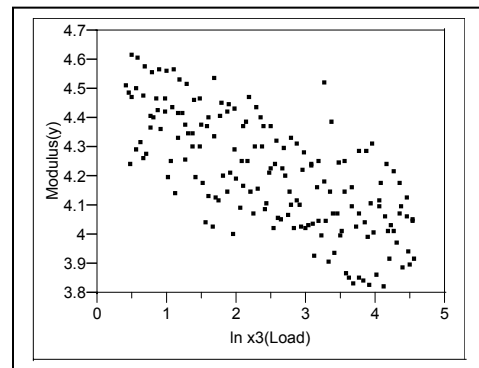


Fig 2b

Bivariate Fit of Modulus(y) By Hardness(x4)

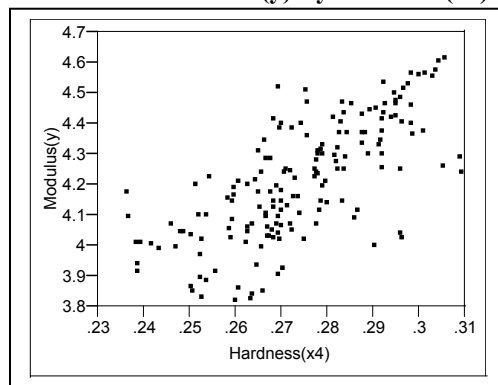


Fig 3

Correlation between response variable and each of predictor variable is shown below:

	R-Square	F-Ratio	Prob> t
SWCNT%(x1)	.296615	70.4233	< .0001
ln x2(Depth)	.530423	188.6389	< .0001
Ln x3 (Load)	.513268	176.1046	< .0001
Hardness (x4)	.500512	167.3425	< .0001

Table 1

It is seen from the correlation Table that R^2 value for all the predictor variables are high showing good correlation with the response variable.

Checking Multicollinearity

Multicollinearity between the predictor variables causes serious numerical and statistical difficulties in fitting regression model and in such cases we need to delete the extra predictors. From the correlation table [Table 2], we see that correlation coefficient between ln x2(Depth) and ln x3(Load) is very high (0.9996) indicating a serious multicollinearity problem. The scatter plot matrix [Fig 4] also shows the strong positive correlation between ln x2(Depth) and ln x3(Load).

Multivariate Correlations

	SWCNT%(x1)	ln x2 (Depth)	ln x3(Load)	Hardness(x4)
SWCNT%(x1)	1.0000	-0.0105	0.0120	0.4476
ln x2 (Depth)	-0.0105	1.0000	0.9996	-0.7876
ln x3(Load)	0.0120	0.9996	1.0000	-0.7722
Hardness(x4)	0.4476	-0.7876	-0.7722	1.0000

Table 2

Scatterplot Matrix

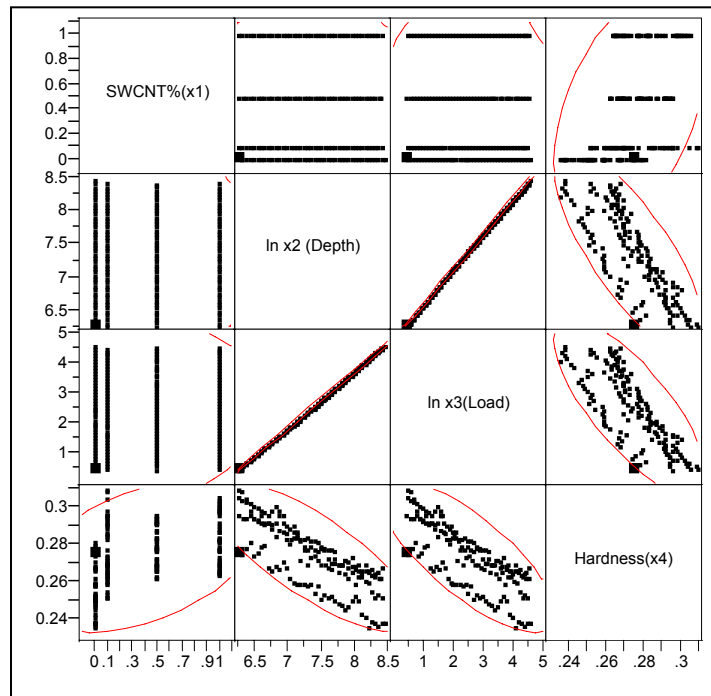


Fig 4

We also checked the Variance inflation factor (VIF) to measure the multicollinearity by using all terms in a multiple regression and the values are shown in Table 3. We see that the VIF values for ln x2(Depth) and ln x3(Load) are much greater than 10 indicating the serious multicollinearity problem between them.

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob> t	Lower 95%	Upper 95%	VIF
Intercept	29.965423	3.367115	8.90	<.0001	23.316937	36.613908	.
SWCNT%(x1)	0.3253171	0.019301	16.85	<.0001	0.287206	0.3634282	2.541603
ln x2 (Depth)	-3.741453	0.527343	-7.09	<.0001	-4.782709	-2.700197	4972.8205
ln x3 (Load)	1.7552889	0.272675	6.44	<.0001	1.2168822	2.2936956	4761.4385
Hardness(x4)	-9.849091	0.85873	-11.47	<.0001	-11.54468	-8.153498	9.0169709

Table 3

Model Identification:

For initial identification of the model, we run the stepwise regression for the full second degree model of the form :

$$y = \beta_0 + \beta_1x_1 + \beta_2x_2 + \beta_3x_3 + \beta_4x_4 + \beta_{12}x_1x_2 + \beta_{13}x_1x_3 + \beta_{14}x_1x_4 + \beta_{23}x_2x_3 + \beta_{24}x_2x_4 + \beta_{34}x_3x_4 + \beta_{11}x_1^2 + \beta_{22}x_2^2 + \beta_{33}x_3^2 + \beta_{44}x_4^2$$

Here we use centering of the interaction terms to help remove the multicollinearity from the interaction terms. The result of the Stepwise regression is shown in Table 4.

Stepwise Fit

Response:
Modulus(y)

Stepwise Regression Control

Prob to Enter 0.250
 Prob to Leave 0.100
 Direction:
 Rules:

0.250
0.100

Current Estimates

Lock	Entered	Parameter	Estimate	nDF	SS	"F Ratio"	"Prob>F"
		Intercept	136.459228	1	0	0.000	1.0000
X	X	SWCNT%(x1)	-0.0135325	3	0.091475	27.943	0.0000
	X	ln x2 (Depth)	-20.493236	4	0.462922	106.056	0.0000
	X	ln x3(Load)	10.3973692	4	0.491975	112.712	0.0000
	X	Hardness(x4)	-25.912325	4	0.843895	193.337	0.0000
		(SWCNT%(x1)-0.39763)*(ln x2 (Depth)-7.36824)	0	1	0.001076	0.986	0.3222
	X	(SWCNT%(x1)-0.39763)*(ln x3(Load)-2.50112)	-0.0606261	1	0.036409	33.366	0.0000
		(SWCNT%(x1)-0.39763)*(Hardness(x4)-0.27406)	0	1	0.000569	0.520	0.4721
	X	(ln x2 (Depth)-7.36824)*(ln x3(Load)-2.50112)	3.28121485	1	0.076875	70.448	0.0000
	X	(ln x2 (Depth)-7.36824)*(Hardness(x4)-0.27406)	87.576971	1	0.013874	12.714	0.0005
	X	(ln x3(Load)-2.50112)*(Hardness(x4)-0.27406)	-50.047294	1	0.017794	16.307	0.0001
	X	(SWCNT%(x1)-0.39763)*(SWCNT%(x1)-0.39763)	0.15614504	1	0.018154	16.636	0.0001
	X	(ln x2 (Depth)-7.36824)*(ln x2 (Depth)-7.36824)	-6.7731031	1	0.085585	78.430	0.0000
		(ln x3(Load)-2.50112)*(ln x3(Load)-2.50112)	0	1	0.000227	0.207	0.6495
	X	(Hardness(x4)-0.27406)*(Hardness(x4)-0.27406)	77.191802	1	0.00313	2.868	0.0923

Step History

Step	Parameter	Action	"Sig Prob"	Seq SS	RSquare	Cp	p
1	(SWCNT%(x1)-0.39763)*(ln x3(Load)-2.50112)	Entered	0.0000	5.156172	0.8197	867.76	4
2	(ln x2 (Depth)-7.36824)*(Hardness(x4)-0.27406)	Entered	0.0000	0.523282	0.9029	399.17	7
3	(ln x2 (Depth)-7.36824)*(ln x2 (Depth)-7.36824)	Entered	0.0000	0.210066	0.9363	210.65	8
4	(ln x2 (Depth)-7.36824)*(ln x3(Load)-2.50112)	Entered	0.0000	0.195406	0.9673	35.431	9
5	(SWCNT%(x1)-0.39763)*(SWCNT%(x1)-0.39763)	Entered	0.0004	0.015637	0.9698	23.249	10
6	(ln x3(Load)-2.50112)*(Hardness(x4)-0.27406)	Entered	0.0003	0.015472	0.9723	11.217	11
7	(Hardness(x4)-0.27406)*(Hardness(x4)-0.27406)	Entered	0.0923	0.00313	0.9728	10.379	12

Table 4

From the step history it is seen that the second order terms for $\ln x_3(\text{load})$ and $\text{Hardness}(x_4)$ are not significant. The interaction terms for $\text{SWCNT}\%(x_1)$ with $\ln x_2(\text{Depth})$ and $\text{Hardness}(x_4)$ are also non-significant. All other variables are showing significant.

Based on the above discussions we see that there is serious multicollinearity problem existing and we should delete one of the variables between $\ln x_2(\text{Depth})$ and $\ln x_3(\text{Load})$ from our model.

Now we will check the best fit models in view of finding the most significant variable between $\ln x_2(\text{Depth})$ and $\ln x_3(\text{Load})$ for our final model. The stepwise regression method here only provide us the initial guideline about the model selection but for our final model selection we give emphasis on Type I and Type III error tests, PRESS statistics, R^2 Value etc. besides several other criteria.

Model 1 [Deleting $\ln x_2(\text{Depth})$ from the model]

Based on the above discussion, the stepwise regression and other considerations we choose our first model that contains variable $\text{SWCNT}\%(x_1)$, $\ln x_3(\text{Load})$, $\text{Hardness}(x_4)$ and there interaction terms. As the stepwise history indicates that the higher order terms are non-significant we did not considered the second order terms into our model. To check this we also run several models considering second order terms as well and in all those cases the higher order terms either does not passes the F-test (Prob >F non-significant) or VIF values greater than 10. We also did not considered the interaction term between $\text{SWCNT}\%(x_1)$ & $\ln x_3(\text{Load})$ as it shows non-significant responses.

The output of this model is shown below in Table 5 and fig 5 shows the residual vs predicted plot.

Summary of Fit

RSquare	0.890004
RSquare Adj	0.88663
Root Mean Square Error	0.065153
Mean of Response	4.217856
Observations (or Sum Wgts)	169

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	5	5.5985565	1.11971	263.7738
Error	163	0.6919298	0.00424	Prob > F
C. Total	168	6.2904864		<.0001

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob> t	VIF
Intercept	6.942564	0.25633	27.08	<.0001	.
SWCNT%(x1)	0.450573	0.022252	20.25	<.0001	3.0559218
$\ln x_3(\text{Load})$	-0.207312	0.010088	-20.55	<.0001	5.895892
$\text{Hardness}(x_4)$	-8.733697	0.876631	-9.96	<.0001	8.5007537
$(\text{SWCNT}\%(x_1)-0.39763)*(\text{Hardness}(x_4)-0.27406)$	-5.280867	1.101817	-4.79	<.0001	1.8762514
$(\ln x_3(\text{Load})-2.50112)*(\text{Hardness}(x_4)-0.27406)$	-1.524858	0.312996	-4.87	<.0001	1.3898951

Effect Tests

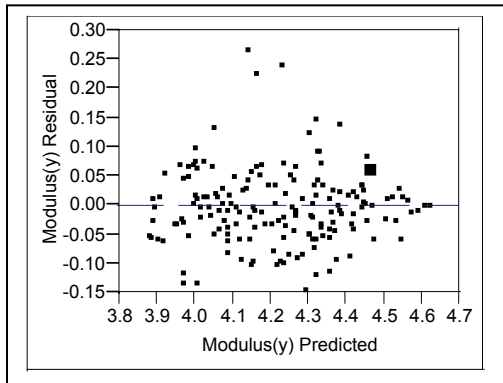
Source	Nparm	DF	Sum of Squares	F Ratio	Prob > F
SWCNT%(x1)	1	1	1.7404987	410.0145	<.0001
$\ln x_3(\text{Load})$	1	1	1.7926634	422.3031	<.0001
$\text{Hardness}(x_4)$	1	1	0.4213440	99.2573	<.0001
SWCNT%(x1)* $\text{Hardness}(x_4)$	1	1	0.0975138	22.9716	<.0001
$\ln x_3(\text{Load})*\text{Hardness}(x_4)$	1	1	0.1007526	23.7346	<.0001

Sequential (Type 1) Tests

Source	Nparm	DF	Seq SS	F Ratio	Prob > F
SWCNT%(x1)	1	1	1.8658520	439.5444	<.0001
$\ln x_3(\text{Load})$	1	1	3.2883957	774.6573	<.0001
$\text{Hardness}(x_4)$	1	1	0.3131438	73.7682	<.0001
SWCNT%(x1)* $\text{Hardness}(x_4)$	1	1	0.0304124	7.1643	0.0082
$\ln x_3(\text{Load})*\text{Hardness}(x_4)$	1	1	0.1007526	23.7346	<.0001

Table 5

Residual by Predicted Plot



Press
0.7412399131

Fig 5

Model 2 [Deleting ln x3(Load) from the model]

In this model we took the variables SWCNT%(x1), ln x2 (Depth), Hardness(x4) and there interaction terms. However, the interaction term between SWCNT%(x1) and ln x2 being non-significant does not considered for the model.

The output of this model is shown below:

Summary of Fit

RSquare	0.892312
RSquare Adj	0.889008
Root Mean Square Error	0.064466
Mean of Response	4.217856
Observations (or Sum Wgts)	169

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	5	5.6130734	1.12261	270.1250
Error	163	0.6774130	0.00416	Prob > F
C. Total	168	6.2904864		<.0001

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob> t	VIF
Intercept	9.4253371	0.365478	25.79	<.0001	.
SWCNT%(x1)	0.4403096	0.021612	20.37	<.0001	2.9445387
ln x2 (Depth)	-0.397562	0.019049	-20.87	<.0001	5.9959602
Hardness(x4)	-8.974643	0.87447	-10.26	<.0001	8.640172
(SWCNT%(x1)-0.39763)*(Hardness(x4)-0.27406)	-5.016709	1.087813	-4.61	<.0001	1.868052
(ln x2 (Depth)-7.36824)*(Hardness(x4)-0.27406)	-2.516581	0.574546	-4.38	<.0001	1.3783352

Effect Tests

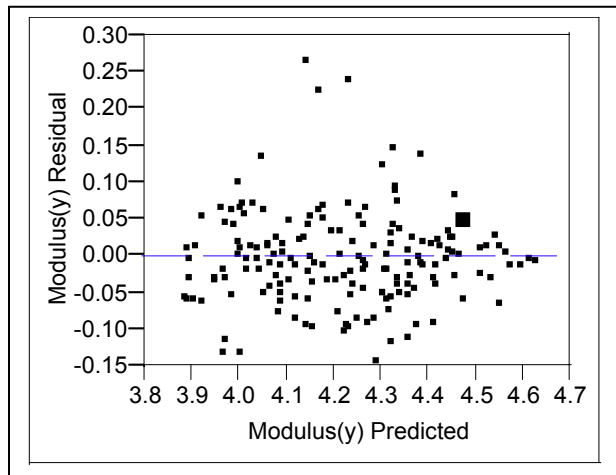
Source	Nparm	DF	Sum of Squares	F Ratio	Prob > F
SWCNT%(x1)	1	1	1.7249826	415.0676	<.0001
ln x2 (Depth)	1	1	1.8101713	435.5658	<.0001
Hardness(x4)	1	1	0.4377336	105.3280	<.0001
SWCNT%(x1)*Hardness(x4)	1	1	0.0883884	21.2681	<.0001
ln x2 (Depth)*Hardness(x4)	1	1	0.0797331	19.1855	<.0001

Sequential (Type 1) Tests

Source	Nparm	DF	Seq SS	F Ratio	Prob > F
SWCNT%(x1)	1	1	1.8658520	448.9638	<.0001
ln x2 (Depth)	1	1	3.2847849	790.3893	<.0001
Hardness(x4)	1	1	0.3509291	84.4410	<.0001
SWCNT%(x1)*Hardness(x4)	1	1	0.0317742	7.6456	0.0063
ln x2 (Depth)*Hardness(x4)	1	1	0.0797331	19.1855	<.0001

Table 6

Residual by Predicted Plot



Press
0.7258538383

Fig 6

Model Selection

From the above two models it is seen that both the model passes the F – test:

$H_0 : \beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5 = 0$ vs. $H_1 : \text{at least one of the term is not equal to zero}$ with

$\text{Prob} > F$ is less than .0001 for both the models. So we can reject H_0 , and accept the hypothesis that there is at least one parameter estimates is not equal to zero.

From the type I error of sequential tests and type III error of effect tests we also see that each independent variable are significant at $\alpha = .05$ level. Therefore, each variable provides additional information about the dependent variable Modulus(y).

The centering of the data provides the VIF values less than 10 for all the terms considered and so there is no multicollinierity.

The residual vs predicted plot also shows no specific pattern. So both the models have a good overall fit.

However, comparing the above two models we see that the model 2 is giving better R^2 value than the model 1. We also see that the PRESS statistic for the model 2 (0.7259) is smaller than model 1 (0.7412). **All these indicating that ln x2 (Depth) is better predictor than ln x3(load) and thus we will consider model 2 for our final model consideration.**

From the residual normal plot for the selected model (Model 2), we see that the residuals are having few outliers. From the studentized residuals data (Shown in Annexure I) we check the outliers and see that there are nine outliers.

Distributions Residual Modulus(y)

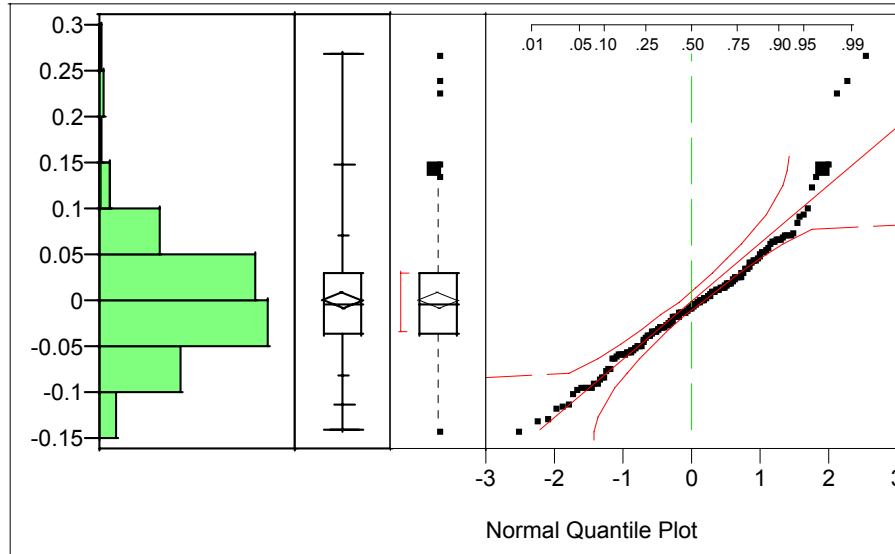
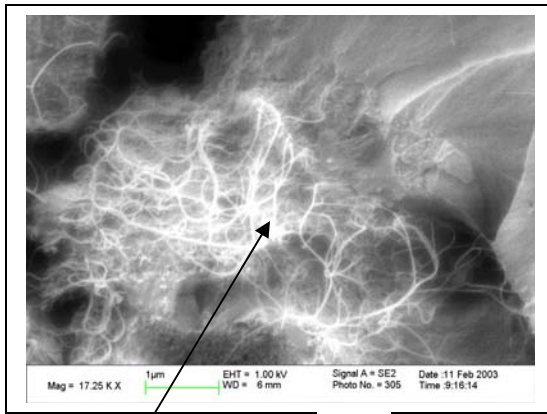


Fig 7

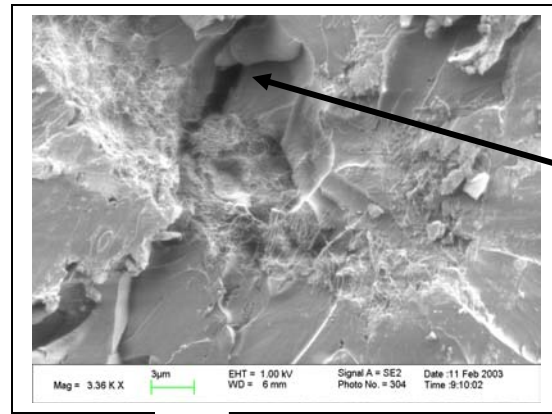
We go back and check the data to explore why they are outliers and observe the following characteristics about these outliers :

There are altogether nine outliers: three of them are from the data set of 0% SWCNT (observation numbers 8, 33 and 38), 5 from the data set of 0.1% SWCNT (observation numbers 48,49,54,61 & 63) and 1 from the data set of 1.0 % SWCNT (observation number 156).

We noticed that these outliers is the results of the abnormal behavior of the modulus(y) at these particular depth of indentation. In 0% SWCNT (except one case) they are the results of sudden decrease of modulus and in other cases they are sudden increase of modulus. We know from the material behavior that the modulus is responsive of material characteristic at that particular depth. So their abnormality reflects that it is not homogeneous in nature. We check the four group of samples (i.e, 0% SWCNT, 0.1%SWCNT 0.5% SWCNT & 1.0% SWCNT samples) through scanning electron microscopy (SEM) to find their morphological characteristics and we observed that the mixing in the samples are not homogeneous throughout. In some cases we noticed presence of micro-voids and in other cases we observed the local concentration of carbon nanotubes. A typical SEM image on 0.1% SWCNT is shown in Fig. 8, where the presence of microvoids and localized concentration of nanotubes are clearly shown. If the indenter during its traverse experienced a microvoids the modulus will be low in that case and if it experienced the zone of localized concentration of nanotubes it will reflect an increase of modulus. Thus a sample with more non-homogeneity will produce more number of erratic responses. We observed these non-homogeneity is more adverse in cases of 0% SWCNT and 0.1% SWCNT giving rise to more number of outliers in these two groups of samples.



A



B

Micro Void

Localized concentration of Carbon nanotubes

Fig. 8 : SEM Images of 0.1% SWCNT composite sample A) High Magnification B) Low Magnification

So based on the above study we realized that the outliers are the results of erratic responses of the modulus(y) and as our present purpose is to establish the best fit model of response variable depending on the several predictors, we can exclude these outliers from our model.

We run the model after excluding the outliers the results obtained is shown in Table 7.

Summary of Fit					
RSquare		0.936981			
RSquare Adj		0.934935			
Root Mean Square Error		0.048821			
Mean of Response		4.211928			
Observations (or Sum Wgts)		160			
Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Ratio	Prob > F
Model	5	5.4574291	1.09149	457.9389	
Error	154	0.3670551	0.00238		
C. Total	159	5.8244842			<.0001
Parameter Estimates					
Term	Estimate	Std Error	t Ratio	Prob> t	VIF
Intercept	9.2876111	0.285135	32.57	<.0001	.
SWCNT%(x1)	0.4400427	0.016802	26.19	<.0001	2.9482746
In x2 (Depth)	-0.388258	0.014678	-26.45	<.0001	5.8886941
Hardness(x4)	-8.75031	0.687318	-12.73	<.0001	8.7085191
(SWCNT%(x1)-0.41063)*(Hardness(x4)-0.27415)	-4.035346	0.847876	-4.76	<.0001	1.862303
(In x2 (Depth)-7.37688)*(Hardness(x4)-0.27415)	-2.3046	0.449449	-5.13	<.0001	1.3919268
Effect Tests					
Source	Nparm	DF	Sum of Squares	F Ratio	Prob > F
SWCNT%(x1)	1	1	1.6348595	685.9143	<.0001
In x2 (Depth)	1	1	1.6676369	699.6662	<.0001
Hardness(x4)	1	1	0.3863151	162.0806	<.0001
SWCNT%(x1)*Hardness(x4)	1	1	0.0539892	22.6515	<.0001
In x2 (Depth)*Hardness(x4)	1	1	0.0626674	26.2924	<.0001
Sequential (Type 1) Tests					
Source	Nparm	DF	Seq SS	F Ratio	Prob > F
SWCNT%(x1)	1	1	1.9489247	817.6821	<.0001
In x2 (Depth)	1	1	3.0808193	1292.575	<.0001
Hardness(x4)	1	1	0.3502222	146.9377	<.0001
SWCNT%(x1)*Hardness(x4)	1	1	0.0147954	6.2075	0.0138
In x2 (Depth)*Hardness(x4)	1	1	0.0626674	26.2924	<.0001

Table 7

This model further improves the R-Square to 0.936981 which means that 93.70% of the variability in modulus(y) is accounted for by the SWCNT%, ln (Depth), Hardness and the interaction of SWCNT% & Hardness and of ln (Depth) & Hardness. The P-Value of F-test is <.0001 which is highly significant. The VIF values also well below 10 indicating that there is no multicollinierity problem. The Type I error and type III error tests demonstrate that each independent variable are significant at $\alpha = 0.05$ level.

The PRESS statistics is 0.3967 which is pretty low in comparison to the previous value of 0.7259 suggesting it is a better model.

The residual by predicted plot (Fig 10) shows no obvious shape and the dispersion of the residuals are approximately constant with respect to the predicted values. So, the constant variance assumption is satisfied.

**Response Modulus(y)
Whole Model
Actual by Predicted Plot**

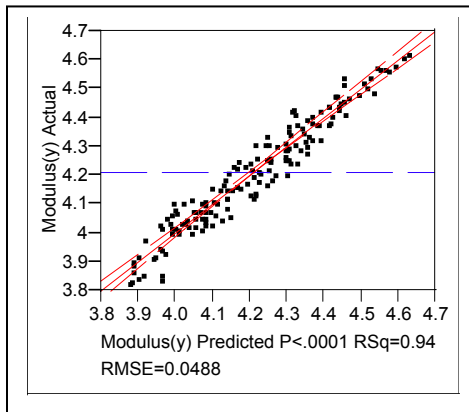


Fig 9

Residuals by Predicted Plot

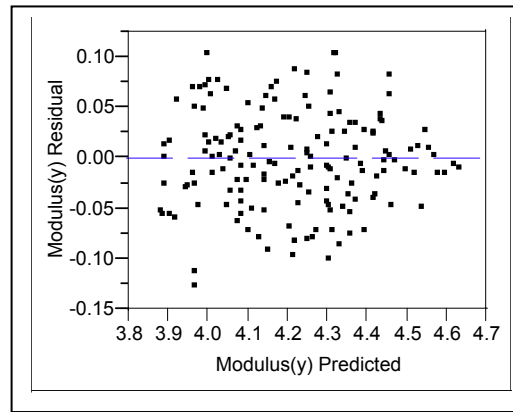
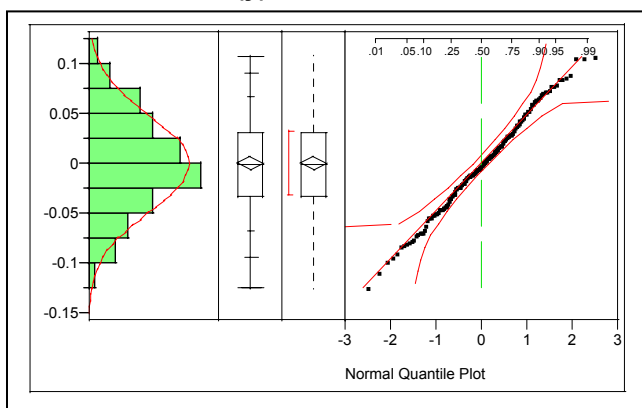


Fig 10

The normal quantile plot (Fig 11) of residuals has clear linear shape indicating residuals are normally distributed. In the Goodness-of-Fit Test, the Shapiro-Wilk W test also proves that for the hypothesis H_0 : the residuals are normal vs H_1 : the residuals are not normal, the attained significance level is .3267 which is highly non-significant. So we can not reject the null hypothesis H_0 and residual normality assumption is accepted.

**Distributions
Residual Modulus(y)**



**Goodness-of-Fit Test
Shapiro-Wilk W Test**

W	Prob<W
0.978931	0.3267

Table 8

Fig 11

Normal(-8e-16,0.04805)

**Bivariate Fit of Residual Modulus(y)
By ln x2 (Depth)**

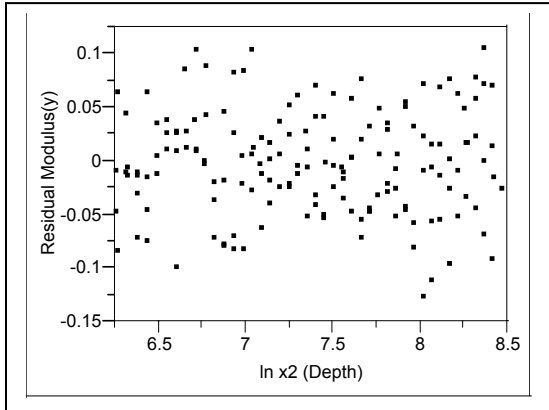


Fig 12

**Bivariate Fit of Residual Modulus(y)
By Hardness(x4)**

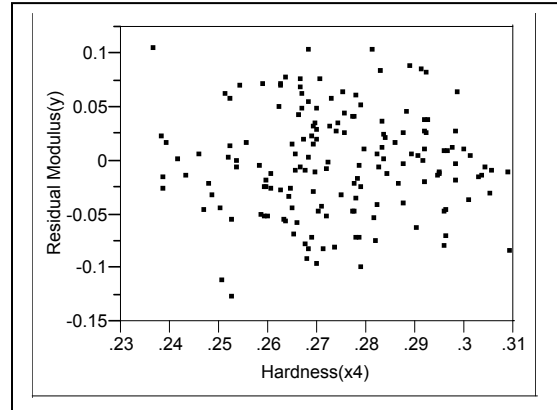


Fig 13

**Bivariate Fit of Residual Modulus(y)
By SWCNT%(x1)**

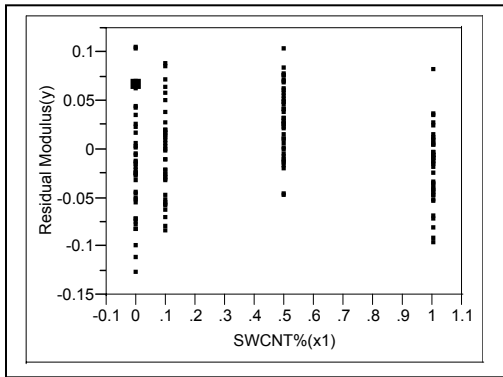


Fig 14

**Bivariate Fit of Residual Modulus(y) By
[(SWCNT%(x1)-0.41063]*[Hardness(x4) -0.27415]**

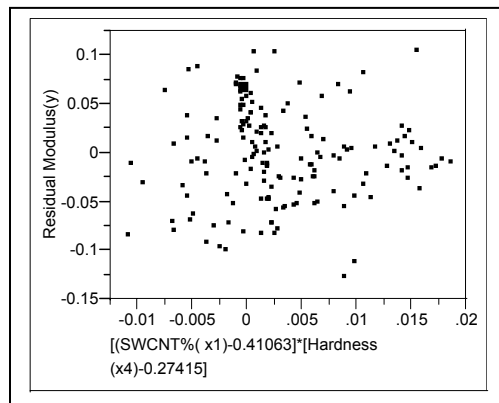


Fig 15

Bivariate Fit of Residual Modulus(y) By [ln x2(Depth)-7.37688]*[Hardness(x4) -0.27415]

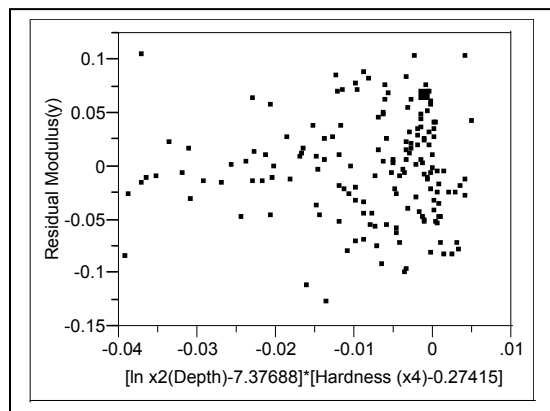


Fig 16

From the residuals vs Xi plot, all of them exhibit random scatter around zero and there is no symmetric pattern implying linearity assumption is also satisfied.

However, the residual vs predicted plot as well as the leverage plots also indicates possibility of influential data. We check the influential observation by comparing the hat column (Table 9). Where $h_{ii} > 2 \frac{(k+1)}{n} = 2 \frac{(5+1)}{160} = 0.075$ is influential. We see here few of the values are greater than 0.075, indicating there is influential data.

**SWCNT%(x1)
Leverage Plot**

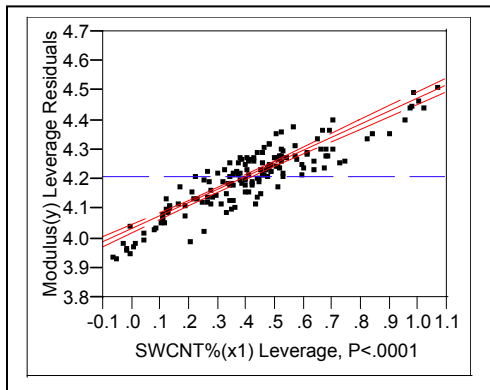


Fig 17

**In x2 (Depth)
Leverage Plot**

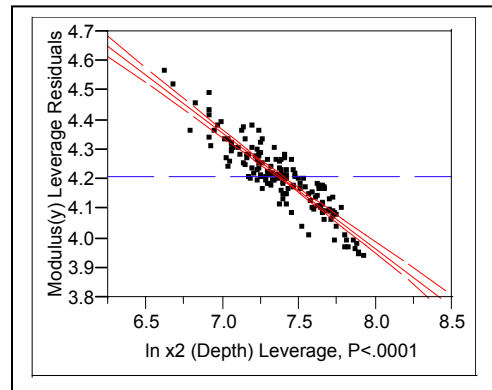


Fig 18

**Hardness(x4)
Leverage Plot**

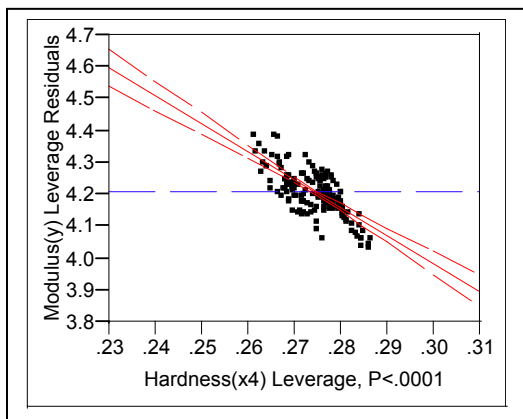


Fig 19

**SWCNT%(x1)*Hardness(x4)
Leverage Plot**

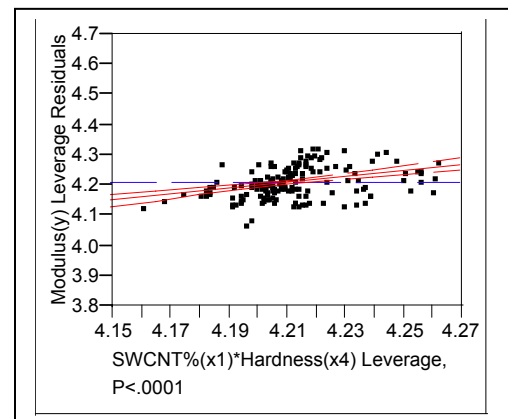


Fig 20

**In x2 (Depth)*Hardness(x4)
Leverage Plot**

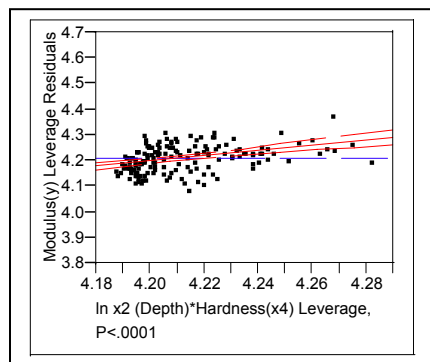


Fig 20

An interesting thing reveals when we go through the influential data. It is seen that for a given sample (i.e, for a particular SWCNT% dataset) the lower and higher depth samples are mostly showing the influential behavior. This implies that during the initial stage of indentation and at the last stage of indentation some sort of unknown activity was going on that makes the data influential. One reason for these data being influential might be that at the initial stage the indenter approaches the surface of the sample and detection of surface being made manually which was not so precise for indentation test, resulting the influential observations. However, we are not sure about the true reason for such influential responses of the data at this initial & final stage of indentation which demands further research work is needed to identify the specific reason for this behavior. So we will not delete this influential data from our model at this stage.

Durbin-Watson

Durbin-Watson	Number of Obs.	AutoCorrelation
1.3312473	160	0.3175

The model has a Durbin-Watson statistic of 1.3312473 and a corresponding autocorrelation of 0.3175 indicating autocorrelation might be a problem. Moreover, we know that the indentation testing is time dependent where a constant load is applied on the sample and corresponding modulus at different depth is measured. So time dependent machine drift with resultant autocorrelation can not be ruled out. However, due to some practical constraint and paucity of data it is not possible at this stage to explore such time dependent autocorrelation.

Thus our final model is devoid of any outliers with the following equation :

$$\begin{aligned} \text{Modulus}(y) = & 9.4253 + [0.4403 * (\text{SWCNT}\%(x1))] + [-0.3976 * (\ln x2 (\text{Depth}))] \\ & + [-8.9746 * (\text{Hardness}(x4))] \\ & + [(\text{SWCNT}\%(x1) - 0.3976) * (\text{Hardness}(x4) - 0.2741) * (-5.0167)] \\ & + [(\ln x2 (\text{Depth}) - 7.3682) * (\text{Hardness}(x4) - 0.2741) * (-2.5166)] \end{aligned}$$

Through this project we examined the relationship of modulus(y) in epoxy/carbon-nanotube composites with variables of percentage of single walled carbon nanotubes (SWCNT) present in the sample, the depth of indentation and the hardness of the material.

Our preferred model estimates regressions for modulus (y) with $R^2 = 0.9370$ i.e, 93.70% of the variability in modulus(y) is accounted for by the SWCNT%, ln (Depth), Hardness and the interaction of SWCNT% & Hardness and of ln (Depth) & Hardness.

Table 9

Sample No.	SWCNT % \times 1	h Modulus(y)
1	0	0.07267348
2	0	0.06419544
3	0	0.05105917
4	0	0.04203167
5	0	0.04848597
6	0	0.03992862
7	0	0.03131309
8	0	0.04568918
9	0	0.04754487
10	0	0.03483234
11	0	0.03426613
12	0	0.02461471
13	0	0.02652143
14	0	0.0366759
15	0	0.03856275
16	0	0.03737873
17	0	0.03324936
18	0	0.0305816
19	0	0.03003563
20	0	0.02488929
21	0	0.03331418
22	0	0.02237063
23	0	0.03390269
24	0	0.02634056
25	0	0.02742348
26	0	0.02429764
27	0	0.02741169
28	0	0.03031898
29	0	0.03134424
30	0	0.03586238
31	0	0.03281027
32	0	0.02618359
33	0	0.02944007
34	0	0.04928818
35	0	0.05807987
36	0	0.0776076
37	0	0.08749839
38	0	0.10460934
39	0	0.09762481
40	0	0.10425421

Sample No.	SWCNT % \times 1	h Modulus(y)
41	0.1	0.15551016
42	0.1	0.14035164
43	0.1	0.10600409
44	0.1	0.06691542
45	0.1	0.04709276
46	0.1	0.03317254
47	0.1	0.03356782
48	0.1	0.02696606
49	0.1	0.05178351
50	0.1	0.05691484
51	0.1	0.02770318
52	0.1	0.02367839
53	0.1	0.04721153
54	0.1	0.03640829
55	0.1	0.02201945
56	0.1	0.02001509
57	0.1	0.02417501
58	0.1	0.02150772
59	0.1	0.01932574
60	0.1	0.02068075
61	0.1	0.01952828
62	0.1	0.01992435
63	0.1	0.03015442
64	0.1	0.02058957
65	0.1	0.03569248
66	0.1	0.02336944
67	0.1	0.02083676
68	0.1	0.03486269
69	0.1	0.02018616
70	0.1	0.03392055
71	0.1	0.03753415
72	0.1	0.03319739
73	0.1	0.03492714
74	0.1	0.03142725
75	0.1	0.03541685
76	0.1	0.03857645
77	0.1	0.04346042

Table 9 Continued.....

Sample No.	SWCNT %\times1	h Modulus(y)
78	0.5	0.05150916
79	0.5	0.04326331
80	0.5	0.03637889
81	0.5	0.03117433
82	0.5	0.02576042
83	0.5	0.02172071
84	0.5	0.01856049
85	0.5	0.0162569
86	0.5	0.01474847
87	0.5	0.01369893
88	0.5	0.01409428
89	0.5	0.01132257
90	0.5	0.01114877
91	0.5	0.01114022
92	0.5	0.01173893
93	0.5	0.01088753
94	0.5	0.01177215
95	0.5	0.01166746
96	0.5	0.01077187
97	0.5	0.01086318
98	0.5	0.01284415
99	0.5	0.01204286
100	0.5	0.01258207
101	0.5	0.01721475
102	0.5	0.01873818
103	0.5	0.01170579
104	0.5	0.0115253
105	0.5	0.01352837
106	0.5	0.0122422
107	0.5	0.01292174
108	0.5	0.01790897
109	0.5	0.01408281
110	0.5	0.01658649
111	0.5	0.01854913
112	0.5	0.02188089
113	0.5	0.01880633
114	0.5	0.0214239
115	0.5	0.02448639
116	0.5	0.0276973
117	0.5	0.02655107
118	0.5	0.02928334
119	0.5	0.03178577

Sample No.	SWCNT %\times1	h Modulus(y)
120	1	0.09699723
121	1	0.08520595
122	1	0.07751275
123	1	0.07211523
124	1	0.06283941
125	1	0.05581059
126	1	0.04902223
127	1	0.04675977
128	1	0.04337656
129	1	0.05359778
130	1	0.07727213
131	1	0.06113915
132	1	0.03468533
133	1	0.03107404
134	1	0.02930158
135	1	0.02750907
136	1	0.02842231
137	1	0.0263475
138	1	0.02556187
139	1	0.02605884
140	1	0.02544465
141	1	0.02562419
142	1	0.02563278
143	1	0.0259467
144	1	0.02669755
145	1	0.02653004
146	1	0.02629285
147	1	0.02670758
148	1	0.05439028
149	1	0.03660804
150	1	0.03922081
151	1	0.0306949
152	1	0.05813594
153	1	0.06549839
154	1	0.07903153
155	1	0.03984539
156	1	0.06831735
157	1	0.07889368
158	1	0.07138339
159	1	0.06608105
160	1	0.05042676

Annexure I

Sample No.	SWCNT %	Depth (nm)	Load (mN)	Hardness (Gpa)	Modulus (Gpa)	Studentized Residual
	x1	x2	x3	x4	y	
1	0	523	1.49	0.275219	4.51974	0.76212414
2	0	552.9	1.65	0.275588	4.47651	0.44916987
3	0	587.1	1.83	0.278373	4.32405	-1.3966113
4	0	621.8	2.03	0.281705	4.28188	-1.4417428
5	0	657.2	2.24	0.274128	4.40745	0.33955699
6	0	694.4	2.47	0.275617	4.36864	0.18628903
7	0	733.5	2.73	0.278924	4.20419	-1.7941657
8	0	776.4	3.02	0.279845	4.14985	-2.2034076
9	0	822.4	3.35	0.268044	4.4227	1.48137335
10	0	867.7	3.69	0.266055	4.35302	0.53003791
11	0	914.6	4.07	0.268816	4.20287	-1.2851935
12	0	965.4	4.49	0.267565	4.18384	-1.3703251
13	0	1019	4.96	0.27106	4.1376	-1.4442367
14	0	1077	5.5	0.268278	4.13365	-1.4331849
15	0	1136	6.08	0.26236	4.20678	-0.536097
16	0	1196	6.69	0.260336	4.21553	-0.2978687
17	0	1259	7.37	0.259379	4.19746	-0.3865773
18	0	1326	8.15	0.259523	4.17259	-0.4677452
19	0	1401	9.06	0.259159	4.15469	-0.4731822
20	0	1474	9.97	0.258048	4.16444	-0.1520315
21	0	1556	11.1	0.259309	4.09029	-0.8754562
22	0	1635	12.1	0.254001	4.23268	1.05670679
23	0	1720	13.4	0.258442	4.06208	-0.8481997
24	0	1814	14.8	0.25133	4.20793	0.92754531
25	0	1904	16.3	0.253492	4.107	-0.1492186
26	0	2015	18.2	0.251843	4.10937	-0.0019944
27	0	2114	19.9	0.252653	4.02926	-0.9026665
28	0	2227	21.9	0.250288	4.04163	-0.7201432
29	0	2343	24.2	0.248481	4.05019	-0.5431362
30	0	2457	26.5	0.247704	4.05141	-0.3743755
31	0	2600	29.6	0.24584	4.07654	0.07562631
32	0	2727	32.5	0.24685	4.00086	-0.7383736
33	0	2867	35.8	0.250277	3.87357	-2.0208271
34	0	3020	39.6	0.252542	3.838	-1.9974395
35	0	3165	43.4	0.250363	3.85634	-1.7568763
36	0	3346	48.3	0.243049	3.9989	-0.2411553
37	0	3510	53.1	0.241504	4.01365	0.01517318
38	0	3683	58.2	0.236165	4.18405	2.2339245
39	0	3900	65.2	0.239134	4.01678	0.24868088
40	0	4084	71.2	0.23821	4.01803	0.36049384
41	0	4278	78.1	0.236464	4.10233	1.70055342
42	0	4523	87	0.23845	3.94642	-0.2531272
43	0	4743	95.6	0.238407	3.92134	-0.4319688

Annexure I continued....

Sample No.	SWCNT %	Depth (nm)	Load (mN)	Hardness (Gpa)	Modulus (Gpa)	Studentized Residual
	x1	x2	x3	x4	y	
44	0.1	523.4	1.58	0.309307	4.24634	-1.8292172
45	0.1	553.1	1.74	0.308696	4.29745	-0.5674975
46	0.1	588.1	1.94	0.305065	4.26703	-0.8639314
47	0.1	622	2.14	0.298645	4.37164	0.72674579
48	0.1	656.2	2.35	0.294752	4.47221	2.38097452
49	0.1	696.2	2.61	0.293704	4.42796	2.00534765
50	0.1	734.7	2.87	0.295905	4.25685	-0.1280222
51	0.1	774.7	3.16	0.29129	4.33724	1.10384726
52	0.1	819.7	3.5	0.291794	4.26283	0.37053797
53	0.1	869	3.9	0.288713	4.30637	1.16978628
54	0.1	917.8	4.3	0.28511	4.47302	3.80401191
55	0.1	965.8	4.72	0.295938	4.04938	-1.4554221
56	0.1	1018	5.2	0.296233	4.03216	-1.3230609
57	0.1	1075	5.75	0.286478	4.12227	-0.4969248
58	0.1	1139	6.39	0.283322	4.15267	0.03784471
59	0.1	1198	7.02	0.290256	4.00935	-1.1508405
60	0.1	1260	7.7	0.285904	4.09826	0.13465274
61	0.1	1329	8.51	0.272026	4.39348	3.57694867
62	0.1	1404	9.44	0.277338	4.07602	-0.4550445
63	0.1	1477	10.4	0.269668	4.40981	4.20821053
64	0.1	1551	11.4	0.27378	4.11231	0.33263045
65	0.1	1636	12.6	0.274692	4.02544	-0.5891695
66	0.1	1721	13.9	0.27228	4.05525	-0.0971974
67	0.1	1816	15.4	0.269955	4.07409	0.24414622
68	0.1	1910	17	0.26963	4.02627	-0.2373185
69	0.1	2010	18.7	0.268006	4.03478	-0.0018523
70	0.1	2115	20.6	0.267182	4.03775	0.24453845
71	0.1	2218	22.6	0.27006	3.93051	-0.7818362
72	0.1	2348	25.2	0.26537	4.00173	0.06296397
73	0.1	2468	27.7	0.269331	3.91418	-0.485987
74	0.1	2590	30.5	0.264484	3.94105	-0.4338695
75	0.1	2733	33.8	0.262148	4.01804	0.76462267
76	0.1	2867	37.1	0.265951	3.85775	-0.944365
77	0.1	3028	41.3	0.258829	4.03106	1.0847143
78	0.1	3179	45.3	0.263393	3.8456	-0.8933571
79	0.1	3330	49.6	0.263312	3.83038	-0.874555
80	0.1	3526	55.5	0.260509	3.86537	-0.4065379
81	0.1	3694	60.8	0.259745	3.829	-0.830002
82	0.1	3878	66.7	0.255523	3.92116	0.25239194
83	0.1	4087	73.9	0.252089	3.97715	0.89293802
84	0.1	4283	80.9	0.253469	3.89071	-0.0010815
85	0.1	4496	88.9	0.25204	3.90133	0.20438255

Annexure I continued....

Sample No.	SWCNT %	Depth (nm)	Load (mN)	Hardness (Gpa)	Modulus (Gpa)	Studentized Residual
	x1	x2	x3	x4	y	
86	0.5	522.4	1.57	0.295944	4.4909	-0.9602339
87	0.5	554.7	1.74	0.294664	4.50653	-0.416825
88	0.5	587.4	1.93	0.294739	4.48485	-0.3429023
89	0.5	622	2.14	0.29609	4.41482	-0.8878241
90	0.5	659.7	2.38	0.294679	4.4324	-0.352863
91	0.5	696.9	2.63	0.292599	4.47457	0.4536509
92	0.5	736.5	2.91	0.292038	4.44243	0.26736715
93	0.5	776	3.2	0.291864	4.42292	0.30273423
94	0.5	821.1	3.55	0.291986	4.38084	0.05057521
95	0.5	866.9	3.92	0.29146	4.35146	-0.0981583
96	0.5	913.5	4.32	0.291709	4.30641	-0.4091051
97	0.5	967.7	4.81	0.28805	4.37669	0.61382962
98	0.5	1020	5.3	0.287375	4.34038	0.31412643
99	0.5	1077	5.87	0.282901	4.41141	1.19577058
100	0.5	1132	6.44	0.281016	4.4271	1.51274567
101	0.5	1194	7.12	0.283809	4.29993	0.25726042
102	0.5	1260	7.87	0.283623	4.25885	-0.0594792
103	0.5	1324	8.64	0.28228	4.25502	0.01743911
104	0.5	1403	9.66	0.278951	4.3068	0.7296922
105	0.5	1473	10.6	0.277877	4.30625	0.87616246
106	0.5	1553	11.7	0.279503	4.2194	0.0972037
107	0.5	1637	13	0.277627	4.24683	0.57956263
108	0.5	1717	14.2	0.277076	4.23294	0.5859057
109	0.5	1819	15.9	0.278435	4.15203	-0.1066674
110	0.5	1909	17.4	0.278022	4.12449	-0.2910558
111	0.5	2011	19.3	0.272821	4.22858	0.84622038
112	0.5	2124	21.4	0.270569	4.24735	1.11621122
113	0.5	2226	23.4	0.272524	4.16642	0.45453254
114	0.5	2347	25.9	0.269999	4.18801	0.70113618
115	0.5	2461	28.4	0.269774	4.15288	0.40321901
116	0.5	2589	31.4	0.271814	4.07623	-0.1451487
117	0.5	2738	35	0.268144	4.15476	0.80031557
118	0.5	2870	38.4	0.269021	4.10457	0.44946508
119	0.5	3025	42.6	0.268858	4.07846	0.33044625
120	0.5	3172	46.7	0.269223	4.04799	0.2040344
121	0.5	3320	51.2	0.266398	4.1148	1.02275452
122	0.5	3522	57.5	0.26634	4.10251	1.16985497
123	0.5	3681	62.7	0.266688	4.06816	0.95811383
124	0.5	3850	68.4	0.266819	4.03589	0.74522996
125	0.5	4088	76.9	0.263576	4.07918	1.17730327
126	0.5	4302	84.8	0.262496	4.06702	1.08069526
127	0.5	4487	92.2	0.262386	4.05093	1.05220031

Annexure I continued....

Sample No.	SWCNT %	Depth (nm)	Load (mN)	Hardness (Gpa)	Modulus (Gpa)	Studentized Residual
	x1	x2	x3	x4	y	
128	1.0	522.2	1.61	0.305637	4.62338	-0.0671296
129	1.0	553.8	1.79	0.30427	4.61091	-0.022051
130	1.0	587.1	1.98	0.303396	4.58331	-0.1543074
131	1.0	621.7	2.19	0.302736	4.56203	-0.1646097
132	1.0	657.2	2.42	0.301099	4.57097	0.14106555
133	1.0	694.5	2.68	0.299672	4.56646	0.25283982
134	1.0	734.9	2.96	0.298035	4.57287	0.50690864
135	1.0	776.4	3.27	0.297419	4.5393	0.27312039
136	1.0	820.1	3.62	0.296345	4.5218	0.21516702
137	1.0	866	3.99	0.298175	4.46843	0.06482679
138	1.0	914.2	4.41	0.300885	4.38429	-0.4272358
139	1.0	966.5	4.89	0.298073	4.40567	-0.1514406
140	1.0	1020	5.41	0.292227	4.54184	1.38188428
141	1.0	1076	5.97	0.290379	4.45877	0.12317331
142	1.0	1131	6.56	0.289072	4.45323	0.1531632
143	1.0	1192	7.22	0.287484	4.43886	0.00223122
144	1.0	1261	8.03	0.287455	4.3786	-0.5670987
145	1.0	1331	8.9	0.283205	4.47789	0.58692441
146	1.0	1402	9.84	0.283347	4.44254	0.40260516
147	1.0	1473	10.8	0.284025	4.37757	-0.1660505
148	1.0	1553	11.9	0.282572	4.37831	-0.0771842
149	1.0	1632	13.1	0.282154	4.32759	-0.6247172
150	1.0	1719	14.5	0.281349	4.30408	-0.8054431
151	1.0	1817	16.1	0.278967	4.33743	-0.3840925
152	1.0	1908	17.7	0.277852	4.31981	-0.5646581
153	1.0	2014	19.6	0.27751	4.28993	-0.7538177
154	1.0	2117	21.6	0.277825	4.24318	-1.1089896
155	1.0	2229	23.9	0.27715	4.25515	-0.7263864
156	1.0	2345	26.3	0.269289	4.52516	2.27171945
157	1.0	2460	28.8	0.269368	4.3924	0.46112592
158	1.0	2597	32	0.271824	4.25483	-0.8709672
159	1.0	2725	35.2	0.270946	4.25611	-0.737844
160	1.0	2870	38.9	0.273459	4.16741	-1.2814377
161	1.0	3029	43.2	0.26744	4.29352	-0.2527888
162	1.0	3172	47.3	0.266352	4.29305	-0.2205282
163	1.0	3336	52.2	0.26477	4.31607	0.10347956
164	1.0	3519	58	0.269848	4.12	-1.5588984
165	1.0	3688	63.6	0.265381	4.24809	-0.2466996
166	1.0	3881	70.1	0.264173	4.2204	-0.6638884
167	1.0	4087	77.6	0.264764	4.18256	-0.822706
168	1.0	4286	85.1	0.265264	4.13467	-1.1863601
169	1.0	4504	93.8	0.267734	4.05979	-1.4977896