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 Project Name: Balloon Bursting  
 Due Date: November 22, 1996

**EXECUTIVE SUMMARY:**

In this experiment we compared pre-blown balloons and out-of-bag balloons with respect to water capacity. “Pre-blown” refers to balloons which were inflated inside a glass jar prior to testing. “Out-of-bag” refers to balloons tested with no treatment. We define water capacity to be the amount of water a balloon holds until it bursts. The balloons were filled at a flow rate of 1.9 ounces per second. The table below summarizes our experimental findings.

<i>Test Group</i>	<i>Mean Capacity (ounces)</i>	<i>Standard Deviation</i>	<i>Mean Time (seconds)</i>	<i>Standard Deviation</i>
Out-Of-Bag	56.31	12.11	29.6022	6.366
Pre-Blown	67.84	14.33	35.6650	7.535

The pre-blown balloons have an additional holding capacity of 11.5 ounces on average. The pre-blown balloons held more water prior to bursting than the out-of-bag balloons.

**EXPERIMENTAL SETUP:**

Thirty-six balloons were filled with a constant flow rate of water (1.9 ounces per second) until bursting. The following procedure was used in this experiment:

1. Randomly select balloon from a test group
2. Operator fits balloon over faucet
3. Timer begins on operator’s queue
4. Operator holds balloon on faucet while filling continues
5. Balloon is filled until burst
6. Timer stops upon visual verification of burst
7. Time is recorded onto data sheet
8. Repeat steps 1 - 7 until all thirty-six balloons are burst

Consistent water flow was maintained by leaving the faucet on for the duration of the experiment. Five times during the experiment, constant flow rate was checked by measuring the time it took the faucet to fill a 28 oz cup of water. During the experiment , household water usage was stopped to control constant flow rate. The validation readings were as follows:

After Run Number:	Time to Fill Cup 28 OZ
# 0	14.38 seconds
# 8	14.85 seconds
# 16	14.81 seconds
# 24	14.88 seconds
# 36	14.68 seconds

For each balloon, the time from initial filling to burst was measured. One test group included eighteen balloons which were pre-blown prior to filling. One operator was used so that each balloon was filled to the same capacity. Each pre-blown balloon was inflated inside a jar until the edges met the glass. The second test group of eighteen balloons were untouched prior to filling (called out-of-bag = OOB balloons).

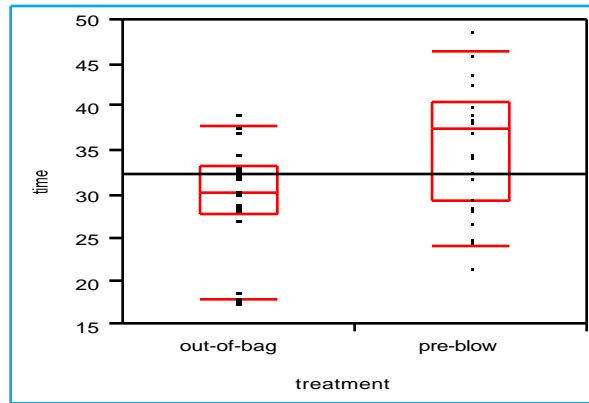
In order to avoid confounding of color and treatments, six different colors of balloons were used in each test group. Each set of test groups included Three blue, green, pink, red, orange, and yellow balloons.

**RESULTS:**

Descriptive Statistic Analysis:

From Figure #1 below we see the median time to bursting for the pre-blown test group appears higher. The raw data for each test group can be found in Appendix A. The average time to burst for the OOB balloons was 29.6 seconds while the mean for pre-blown balloons was 35.7 seconds.

Figure #1: Time By Treatment



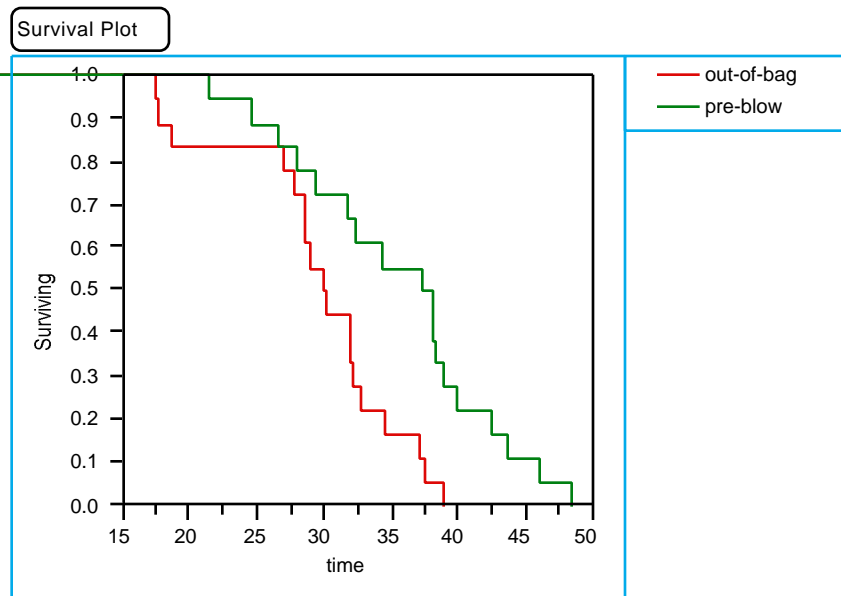
Level	Number	Means and Standard Deviations		
		Mean	Std Dev	Std Err Mean
out-of-bag	18	29.6022	6.36689	1.5007
pre-blow	18	35.6650	7.53501	1.7760

Distribution Verification Analysis:

Probability plots of the data were used to analyze model fits (see Appendix B). The data does not deviate from the assumptions of either the normal (Pvalue oob = 0.0884, Pvalue pre-blown = 0.8974) or the weibull distribution. We used both methods of analysis to determine if the treatment is significant.

Kaplan-Meier Graphical Analysis:

Below is the survival plot for both test groups, and a comparison of the survival rates. Half of the out-of-bag balloons burst prior to 30 seconds. None of the out-of-bag balloons exceeded 38 seconds of filling. Nine (50%) of the pre-blown balloons survived. Three of the eighteen (16.6 %) out-of-bag balloons failed before 20 seconds. The survival plot (OOB) has a significant shift after the three early failures.



The Exponential, Weibull, and Lognormal distributions were considered in fitting the data (see Appendix C). The Exponential distribution was a poor fit. Both the Weibull and Lognormal distributions seemed to characterize the

data appropriately. The Weibull distribution fit both groups, but the three early failures were not properly fitted for the out of bag test group. The Lognormal plot did not fit the data as well as the Weibull plot.

Treatment Effect Analysis:

The data does not deviate from assumptions of either the Weibull or Normal distribution. Three testing methods were used to analyze the data:

- Nonparametric Methods Analysis (no distribution assumed)
  - Wilcoxon test
  - Log-Rank test
- Normality Assumption Analysis
  - Homogeneity of Variance test
  - 2-Sample T test
  - Model equation and residual analysis
- Weibull Assumption Analysis
  - Log-likelihood ratio tests
  - Model equation and residual analysis

Nonparametric Methods Analysis:

Both the Log-Rank Test and the Wilcoxon Test show that the two medians for the test groups are statistically different (at  $\alpha = 0.05$ ). The median pre-blown balloon burst time is longer than the balloons out of the bag.

Test	Chi-Square	DF	Prob>ChiSq
Log-Rank Test	8.4879	1	0.0036
Wilcoxon Test	5.3762	1	0.0204

Normality Assumption Analysis:

The table below indicates the equal variances assumption (at  $\alpha = 0.05$ ) is satisfied for the out of bag balloons and the pre-blown balloons. Accordingly, the 2 Sample-T test can be used to compare average burst times for the two groups.

Test ( <i>Homogeneity of Variance</i> )	F Ratio	DF Num	DF Den	Prob>F
O'Brien[.5]	0.6340	1	34	0.4314
Brown-Forsythe	0.8786	1	34	0.3552
Levene	1.3029	1	34	0.2617
Bartlett	0.4664	1	34	0.4947

We conclude that there is a significant difference between the average burst times according to the 2-sample T-test.

2-Sample T-Test (OOB VS. Pre):	Difference	t-Test	DF	Prob> t
Estimate	-6.0628	-2.607	34	0.0134
Std Error	2.3251			

Using the parameter estimates below, the following model can be used to predict the average burst times (where 1 = pre-blown & 0 = out-of-bag):

Model: **Burst Time = 29.60 + 6.06 \* (Treatment)**

*Parameter Estimates*

Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	29.602222	1.644128	18.00	<.0001
Pre=1&OOB=0	6.0627778	2.325148	2.61	0.0134

The residual plot for this model is shown in Appendix D. This plot shows the effect of pre-blowing the balloons on burst time. All other sources of variation are randomly distributed. As noticed earlier, the three out-of-bag outliers stand out.

Weibull Assumption Analysis:

Below is the table summarizing the weibull log-likelihood ratio testing applied to our data sets. See Appendix E for contour plots verifying the accuracy of our initial estimates used for weibull regression.

The P-Value = 0.002946 implies that model  $M_1$  is more appropriate than  $M_0$ . The first model ( $M_0$ ) assumes that the two groups of data come from one weibull distribution and implies no effect due to pre-blowing. The second model ( $M_1$ ) assumes the location parameter ( $\mu$ , of the gumbel data) is different for each group, but the shape parameter ( $\sigma$ , of the gumbel data) is the same. Since there is a significant difference (at  $\alpha = 0.05$ ) between these two models, we conclude that pre-blowing the balloons significantly affects the burst time. Since the second P-Value is very close to 1.0, we conclude that the assumptions of  $M_2$  do not prove  $M_1$  assumptions to be inappropriate. See Appendix F for ML estimates, 95% confidence intervals for these estimates, and log-likelihood estimates ( $l_0$ ).

Model	Assumption	# of Params	$l_0$	$2(l_1 - l_{(i-1)})$	df	$\text{Chi}_{(0.05)}^2$ df = 1	P-Value
$M_0$	$\mu_{oob} = \mu_{pre}$ $\sigma_{oob} = \sigma_{pre}$ ( $\mu_{oob} = \mu_{pre}$ )	2	1.21586				
$M_1$	$\mu_{oob} \neq \mu_{pre}$ $\sigma_{oob} = \sigma_{pre}$ ( $\mu_{oob} \neq \mu_{pre}$ )	3	5.63606	8.84040	1	3.84	0.002946
$M_2$	$\mu_{oob} \neq \mu_{pre}$ $\sigma_{oob} \neq \sigma_{pre}$ ( $\mu_{oob} \neq \mu_{pre}$ )	4	$(2.91514 + 2.73549) = 5.65063$	0.02914	1	3.84	0.864456

Appendix D shows the Weibull residual plot. Both the Weibull residual plot and the transformed normal quantile plot hint that there is a lack of fit problem with the Weibull model:  $\log(\text{Burst Time}) = \alpha + \beta * \text{Treatment}$ .

Appendix F shows the results of the non-linear fit and variance-covariance matrix. The variance-covariance matrix was obtained using the standard error estimates and the correlation matrix. Using the Delta Method the median quantile and standard errors were estimated for both test groups (see Summary and Conclusions).

**SUMMARY AND CONCLUSION:**

By pre-blowing the balloons we increased the amount of water the balloons could hold. On average, the pre-blown balloons held 67.8 ounces of water prior to bursting while the out-of-bag balloons held only 56.3 ounces. The treatment method improves the water capacity of the balloons by 11.5 oz (approximately the size of a can of soda pop).

Using three methods of analysis (Weibull regression, two-sample T tests, and nonparametric testing) we have proven that the two test groups are statistically different. The table below summarizes the estimated medians of the fitted weibull and normal models.

Model	Treatment	Median Time	Standard Error	95% C.I. on Median
Weibull	Out Of Bag	26.34	7.28	[ 12.07 , 40.62 ]
Weibull	Pre-Blown	36.30	9.99	[ 16.72 , 55.88 ]
Normal	Out Of Bag	29.60	1.50	[ 26.26 , 32.94 ]
Normal	Pre-Blown	35.66	1.77	[ 32.32 , 39.01 ]

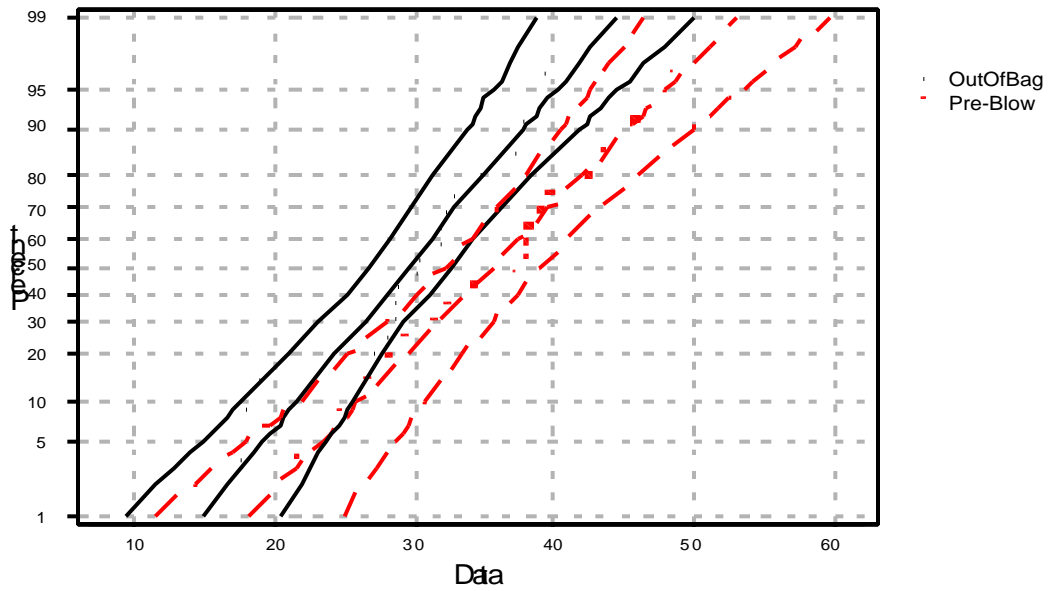
The 95% C.I.'s for the normal model are much tighter than those for the weibull. The model based on normality is the appropriate one to use when predicting burst time.

**Appendix A: Raw Data**

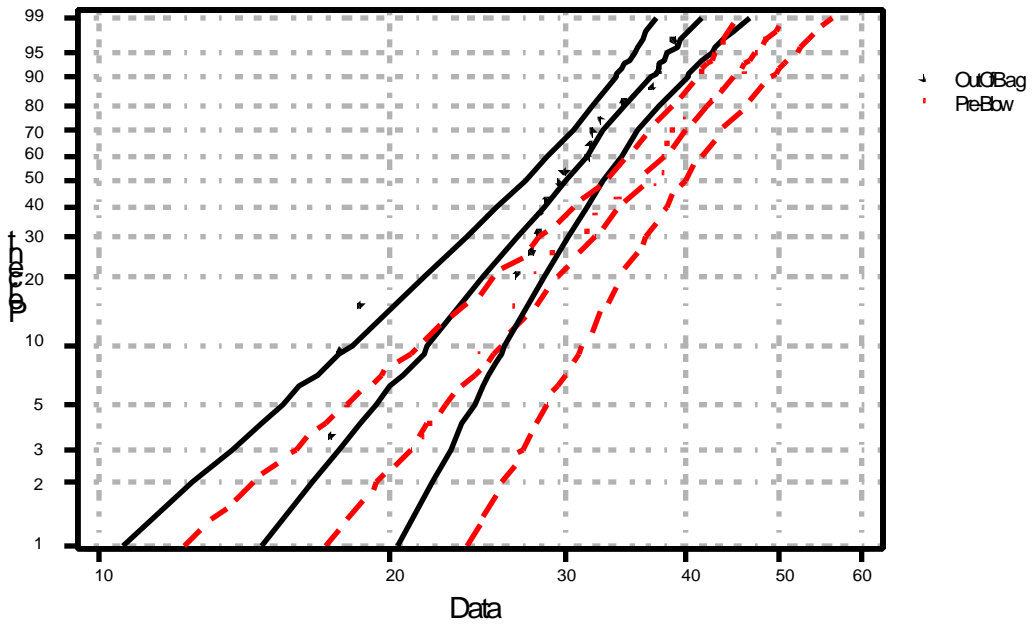
Color	Run Order #	Out Of Bag Balloons (seconds)	Water Capacity (ounces)	Run Order #	Pre-Blown Balloons (seconds)	Water Capacity (ounces)
Blue	1	28.97	55.11	2	34.37	65.38
Red	3	32.94	62.66	4	29.5	56.11
Pink	5	17.47	33.23	6	38.22	72.70
Blue	7	26.91	51.19	8	37.28	70.91
Orange	9	31.85	60.58	10	48.62	92.48
Green	11	29.96	56.99	12	39.97	76.03
Red	13	30.29	57.62	14	39.21	74.58
Pink	15	17.81	33.88	16	38.53	73.29
Yellow	17	28.59	54.38	18	43.81	83.33
Orange	19	34.62	65.85	20	28.25	53.74
Blue	21	18.63	35.44	22	31.75	60.39
Green	23	28.43	54.08	24	38.25	72.76
Yellow	25	32.25	61.35	26	42.59	81.01
Orange	27	37.72	71.75	28	26.72	50.83
Pink	29	37.18	70.72	30	24.69	46.96
Red	31	28.03	53.32	32	21.6	41.09
Green	33	39.19	74.55	34	46.13	87.75
Yellow	35	32	60.87	36	32.48	61.78
Mean		29.60	56.31		35.67	67.84
Std Dev		6.37	12.11		7.54	14.33

**Appendix B: Probability Plots with 95% Group Confidence Intervals:**

Normal Probability Plot for OutOfBag - Pre-Blow

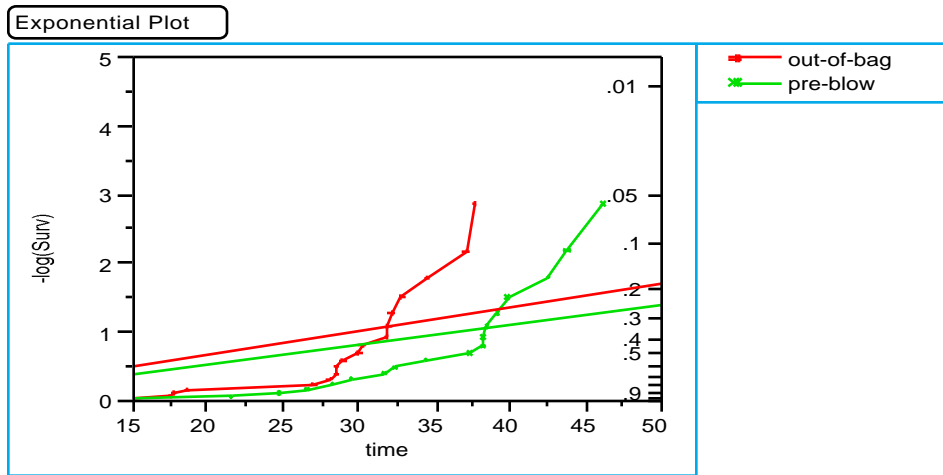


Weibull Probability Plots

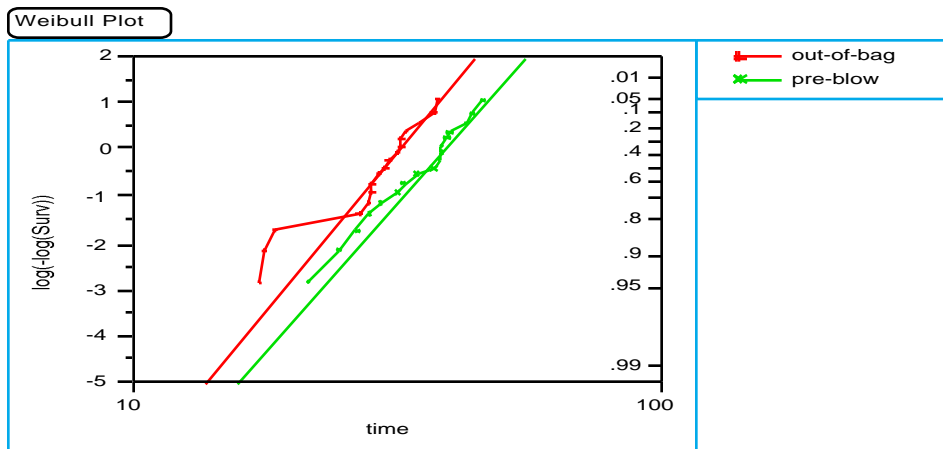


## Appendix C: Kaplan-Meier Fitted Distributions:

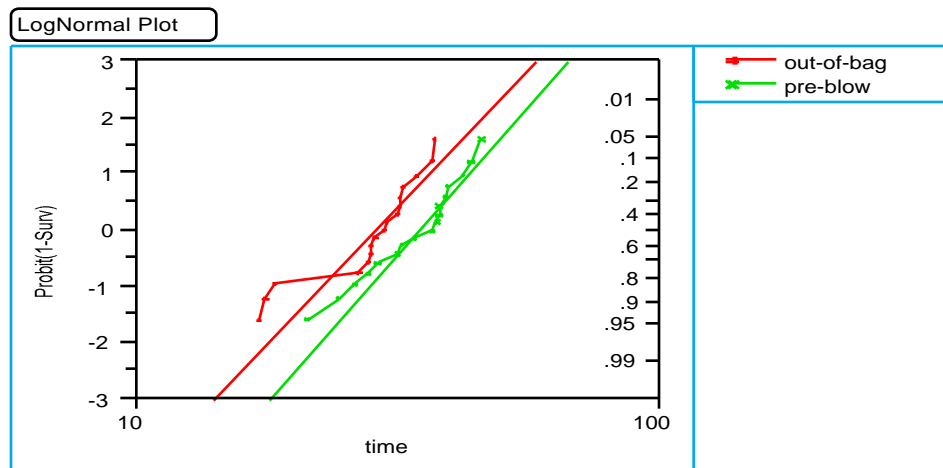
### Exponential



### Weibull

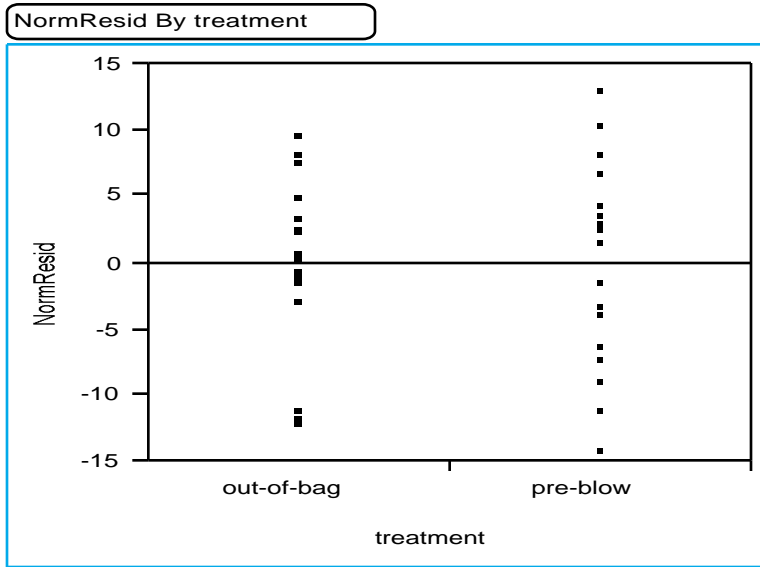


### Lognormal

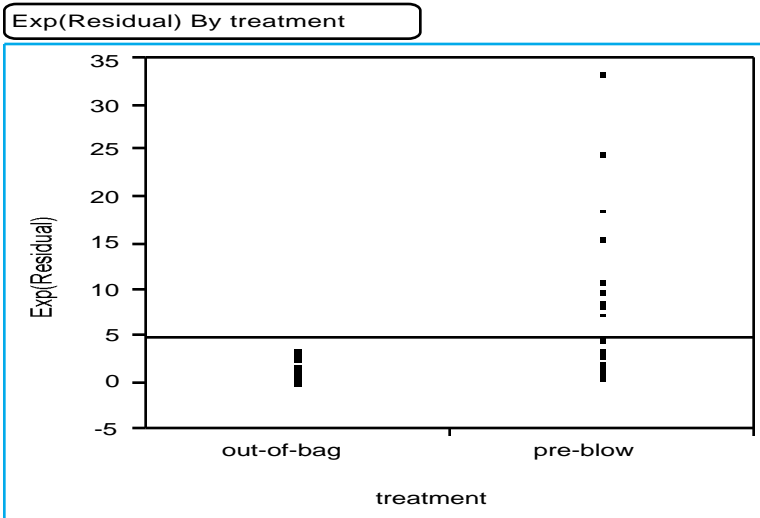
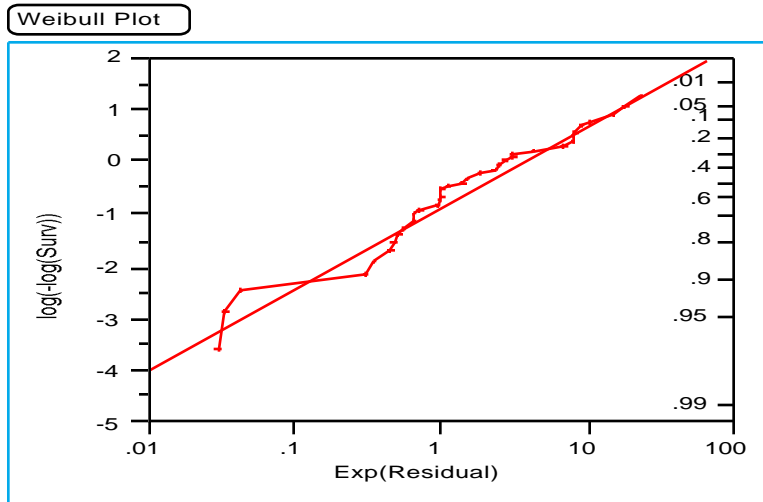


**Appendix D: Residual Plots:**

Assumes Normality of Data:

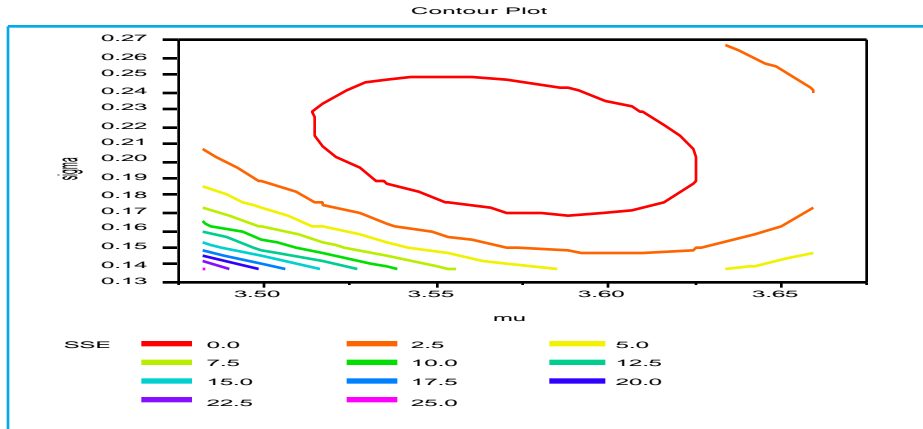


Assumes Weibull Distribution of Data:

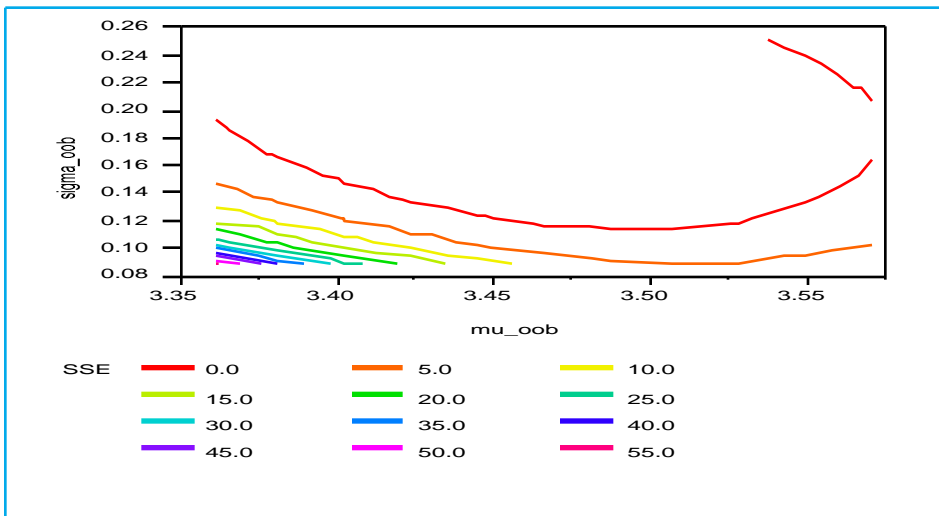


## Appendix E: Contour Plots:

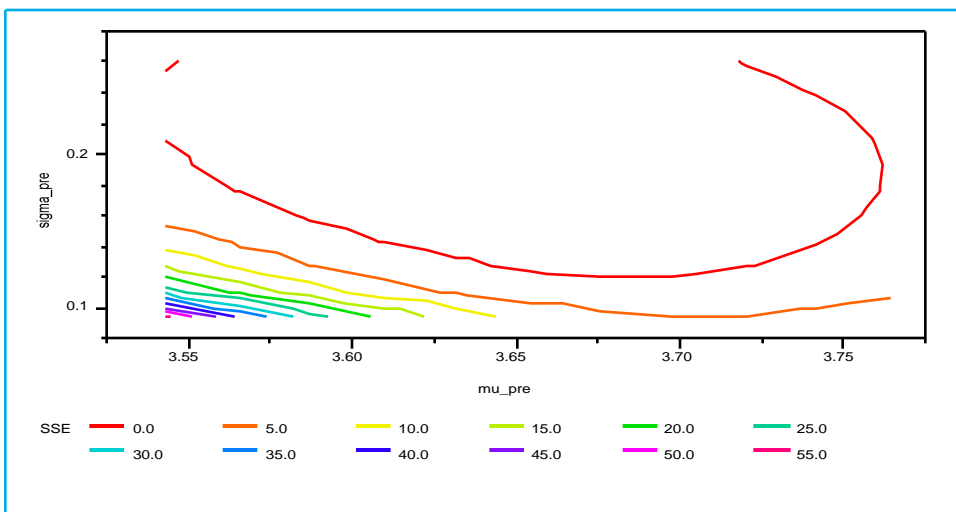
### Contour Plot of $M_0$ Parameters:



### Contour Plot of Out of Bag Parameters:



### Contour Plot of Pre-Blown Parameters:



**Appendix F: Log-Likelihood, Parametric Survival Fit Output, & Non-Linear Fit:**

2 Parameter Model: Solution

SSE	DFE	MSE	RMSE
-1.215862424	34	-0.035761	?

Parameter	Estimate	ApproxStdErr	Lower CL	Upper CL
mu	3.571078028	0.03556077	3.49778592	3.6411135
sigma	0.2022692569	0.02598175	0.1596728	0.2648646

Out of Bag Parameters Model: Solution

SSE	DFE	MSE	RMSE
-2.915141509	16	-0.182196	?

Parameter	Estimate	ApproxStdErr	Lower CL	Upper CL
mu_oob	3.4662263668	0.04187652	3.37622194	3.55138713
sigma_oob	0.1695093156	0.03240365	0.12012747	0.25576288

Pre-Blown Parameters Model: Solution

SSE	DFE	MSE	RMSE
-2.73549838	16	-0.170969	?

Parameter	Estimate	ApproxStdErr	Lower CL	Upper CL
mu_pre	3.6539253147	0.04409277	3.55939423	3.74287073
sigma_pre	0.1774194964	0.03306809	0.12666591	0.26480802

Parametric Survival Fit: Weibull Distribution: Whole Model Test

Model	-LogLikelihood	Chi-Square	DF	Prob>ChiSq
Difference	4.42019734	8.8404	1	0.0029
Full	-5.6360598			
Reduced	-1.2158624			

Initial Parameter Estimates

Term	Estimate	Std Error
( o ) = Intercept	3.46467421	0.0418257
( 1 ) = Pre=1,OOB=0	0.19089789	0.0578782
( ) = Delta	0.17358283	0.0231667

Non-Linear Fit: Solution

SSE	DFE	MSE	RMSE
-5.636059763	33	-0.17079	?

Parameter	Estimate	ApproxStdErr	Lower CL	Upper CL
Beta0	3.4646743395	0.0418258	3.38131435	3.55172944
Beto1	0.1908979085	0.05787838	0.07186369	0.30856846
Eta	5.7609235447	0.76886518	4.35861629	7.37162529

Correlation of Estimates

	Beta0	Beto1	Eta
Beta0	1.0000	-0.6864	0.2077
Beto1	-0.6864	1.0000	0.0244
Eta	0.2077	0.0244	1.0000

Varaince - Covariance Matrix

	Beta0	Beto1	Eta
Beta0	0.001749	-0.001662	0.006679
Beta1	-0.001662	0.003350	0.001086
Eta	0.006679	0.001086	0.591154