Statistics 567 Midterm Exam

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Name: _____

This exam is open book and open notes. You need a calculator to take this test.

1. Suppose that a Weibull plot has slope 3 and intercept -2. What are the graphical estimates of the parameters μ and σ of the Gumbel distribution of log failure times? What are the graphical estimates for the parameters α and η of the Weibull distribution of failure times?

2. Suppose that a lognormal plot has slope 3 and intercept -2. What are the graphical estimates of the lognormal parameters μ and σ ?

3. A number of units known to have an exponential distribution was subjected to a life test involving right censoring. The units were observed for a total of 1000 hours during which ten failures were observed. Estimate the units' failure rate and mean time to failure.

4. Ten units were subjected to a life test. Seven units were observed to fail at times $t_i, i = 1,2,...,7$. One units was removed from the test before failure at time *c*. The last two units are only known to have failed somewhere between times *a* and *b*. Assuming the units have cumulative distribution function *F* and density *f* write down the likelihood function for these data.

5. A life test of forty units ran until ten failures were observed. Weibull and lognormal distributions were fitted by the method of maximum likelihood to the observed failure and censoring times. The maximum likelihood estimates are:

lognormal:	$\hat{\mu}$ = 6 and $\hat{\sigma}$ = 1
Weibull:	$\hat{\alpha}$ = 20 and $\hat{\eta}$ =5

On the basis of this information which distribution do you think fits the data better?

6. Suppose that you have $n = 100$ observations and that the empirical survival function calculated from this data has the value $\hat{S}(50) = .86$ at fifty hours. What is the standard error of this estimate, i.e., of $\hat{S}(50)$? What is the standard error of the empirical cumulative hazard function at fifty hours?

7. Suppose you have some failure data which we believe to have a Weibull distribution. Suppose that we want to check if we can actually simplify the model further and assume an exponential distribution. Let $\hat{\mu}$ and $\hat{\sigma}$ be the MLEs for the Gumbel parameters of the distribution of log-failure times. Let $\hat{\mu}_1$ be the MLE of μ when $\sigma = 1$. Suppose that the maximized log-likelihoods have the following values:

$$l(\hat{\mu}, \hat{\sigma}) = -17$$

$$l(\hat{\mu}_1, 1) = -24.$$

Do you think that the exponential model is adequate? Explain (Hint: Use the following partial table of χ^2 upper tail values.)

Deg. of Freedom	1	2	3	4	5	6
$\chi^2_{.05}(n)$	3.84	5.99	7.81	9.49	11.1	12.6

8. Maximum likelihood estimates for a Gumbel distribution applied to the log failure times observed in a life test are $\hat{\mu} = 5$ and $\hat{\sigma} = .06$ with variance-covariance matrix

$$10^{-4}\begin{pmatrix} .9 & -.2 \\ -.2 & .6 \end{pmatrix}$$

   a. Find the maximum likelihood estimate for the probability of surviving 60 hours.

   b. Find an approximate value for the standard error of this estimate.

   c. Find the maximum likelihood estimate of b_{10}, the time at which ten percent of the population is expected to fail.

   d. Find an approximate value for the standard error of this estimate.