Unit 20: Planning Accelerated Life Tests

Ramón V. León

Unit 20 Objectives

• Outline reasons and practical issues in planning Accelerated Life Tests (ALTs)
• Describe criteria for ALT planning
• Illustrate how to evaluate the properties of ALTs
• Describe methods of constructing and choosing among ALT plans
  – One-variable
  – (Two-variables)
• Present guidelines for developing practical ALT plans with good statistical properties
Possible Reasons for Conducting an Accelerated Test

Accelerated Tests (ATs) are used for different purposes. These include:

• ATs designed to identify failure modes and other weaknesses in product design
• ATs for improving reliability
• ATs to assess the durability of materials and components
• ATs to monitor and audit a production process to identify changes in design or process that might have a seriously negative effect on production reliability
Motivation/Example

Reliability Assessment of an Adhesive Bond

- **Need:** Estimate of the B10 of failure-time distribution at 50°C (expect $\geq$ 10 years).

- Constraints
  - 300 test units.
  - 6 months for testing.

- 50°C test expected to yield little relevant data.
Model and Assumptions

- Failure-time distribution is loglocation-scale
  \[
  \Pr(T \leq t) = F(t; \mu, \sigma) = \Phi \left[ \frac{\log(t) - \mu}{\sigma} \right]
  \]

- \(\mu = \mu(x) = \beta_0 + \beta_1 x\), where
  \[
  x = \frac{11605}{\text{temp} \, ^\circ C + 273.15}.
  \]

- \(\sigma\) does not depend on the experimental variables.

- Units tested simultaneously until censoring time \(t_c\).

- Observations statistically independent.
Assumed Planning Information for the Adhesive Bond Experiment

The objective is finding a test plan to estimate B10 with good precision.

- Weibull failure-time distribution with same shape parameter at each level of temperature $\sigma$ and location scale parameter $\mu(x) = \beta_0 + \beta_1 x$, where $x$ is °C in the Arrhenius scale.

- .1% failing in 6 months at 50°C.

- 90% failing in 6 months at 120°C.

**Result:** Defines failure probability in 6 months at all levels of temperature. If $\sigma$ is given also, defines all model parameters.
Justification

$$F(t_e) = \Phi \left( \frac{\log t_e - [\beta_0 + \beta_1 x]}{\sigma} \right)$$

$$= \Phi \left( \frac{\log t_e - \beta_0}{\sigma} - \frac{\beta_1}{\sigma} x \right)$$

Weibull shape parameter thought to be:

$$\beta^\circ = 1.667 \left( or \right. \quad \sigma^\circ = \frac{1}{\beta^\circ} = .6 \left.) \right)$$

Parameters A and C can be determined from two values of $$F(t_e)$$.

If $$\sigma$$ is also given can find planning values:

$$\beta_0^\circ = -16.733, \quad \beta_1^\circ = .7265$$

Time measured in days
Specify ALT model information (planning values)

Model specification
Distribution: Weibull
Number of accelerating variables: 1
Push to start choosing relationships

Optional inputs
Time units: days
Use condition level: 50
Save results in: last.Weibull.altpv

Relationship
Relationship 1: Arrhenius

Intercept is: -16.736
Relationship coefficients = 0.7265
weibull.beta = 1.667
sigma = 0.6
Use conditions: 50
DegreesC
Engineer’s Originally Proposed Test Plan for the Adhesive Bond

<table>
<thead>
<tr>
<th>Temp °C</th>
<th>Allocation Proportion $\pi_i$</th>
<th>Allocation Number $n_i$</th>
<th>Failure Probability $p_i$</th>
<th>Expected Number Failing $E(r_i)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td></td>
<td></td>
<td>0.001</td>
<td>50</td>
</tr>
<tr>
<td>110</td>
<td>$\frac{1}{3}$</td>
<td>100</td>
<td>0.60</td>
<td>60</td>
</tr>
<tr>
<td>130</td>
<td>$\frac{1}{3}$</td>
<td>100</td>
<td>1.00</td>
<td>100</td>
</tr>
<tr>
<td>150</td>
<td>$\frac{1}{3}$</td>
<td>100</td>
<td>1.00</td>
<td>100</td>
</tr>
</tbody>
</table>
Adhesive Bond

Engineers’ Originally Proposed Test Plan

\[ n = 300, \pi_i = 1/3 \text{ at each } 110\degree C, 130\degree C, 150\degree C \]
Critique of Engineers’ Original Proposed Plan

- Arrhenius model in doubt at high temperatures (above 120°C)
- Question ability to extrapolate to 50°C
- Data much above the B10 are of limited value.

Suggestion for improvement
- Test at lower more realistic temperatures (even if only small fraction will fail).
- Larger allocation to lower temperatures
Engineers’ Modified Traditional ALT Plan with a Maximum Test Temperature of 120°C

<table>
<thead>
<tr>
<th>Temp °C</th>
<th>Allocation Proportion $\pi_i$</th>
<th>Failure Probability $p_i$</th>
<th>Expected Number Failing $E(r_i)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>0</td>
<td>.04</td>
<td>4</td>
</tr>
<tr>
<td>80</td>
<td>1/3</td>
<td>.29</td>
<td>29</td>
</tr>
<tr>
<td>100</td>
<td>1/3</td>
<td>.90</td>
<td>90</td>
</tr>
<tr>
<td>120</td>
<td>1/3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For this plan and the Weibull-Arrhenius model, $\text{Ase}[\log(\hat{t}_{1}(50))] = .4167$
Temp number.units censor.times
1  80  100  183
2 100  100  183
3 120  100  183
Total number.units = 300
\[ \text{ase}(\log t_{1.1}) = \frac{\text{ase}(t_{1.1})}{t_{1.1}} = \frac{1246.5}{2991.8} = 0.4166 \]
Accelerated Test Plan

Levels = 80, 100, 120, n=100, 100, 100
Censor time = 183, parameters = -16.74, 0.7265, 0.6
Simulation of Engineers’ Modified Traditional ALT Plan

Levels = 80, 100, 120 Degrees C, n=100,100,100
Censor time=183,183,183, parameters= -16.74,0.7265,0.5999

Precision factors R for quantile estimates at 50 Degrees C
R( 0.1 quantile)= 2.288
R( 0.5 quantile)= 2.484
R(Ea)= 1.165

Results based on 300 simulations
Lines shown for 50 simulations
Accelerated life test simulation based on
last ALTplan last:Weibull.altpv
x:Arrhenius , Dist:Weibull
Failure time 0.1 quantile vs DegreesC

Days

DegreesC

Sat Nov 13 19:32:48 EST 2004
Methods of Evaluating Test Plan Properties

Assume inferences needed on a function $g(\theta)$ (one-to-one and all the first derivatives with respect to the elements of $\theta$ exist, and are continuous).

- Properties depend on test plan, model and (unknown) parameter values. Need **planning values**.

- Asymptotic variance of $g(\hat{\theta})$

  $$Avar[g(\hat{\theta})] = \left[ \frac{\partial g(\theta)}{\partial \theta} \right]^{'} \Sigma_{\theta} \left[ \frac{\partial g(\theta)}{\partial \theta} \right].$$

  Simple to compute (with software) and general results.

- Use Monte Carlo simulation. Specific results, provides picture of data, requires much computer time.
Statistically Optimum Plan for the Adhesive Bond

- **Objective:** Estimate B10 at 50°C with minimum variance
- **Constraint:** Maximum testing temperature of 120°C
- **Inputs:** Failure probabilities $p_U = .001$ and $p_H = .90$
Optimum test plan based on last Weibull.altpv

Test plan summary:

<table>
<thead>
<tr>
<th>DegreesC</th>
<th>n</th>
<th>ctime</th>
<th>zeta</th>
<th>p</th>
<th>efail</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>94.6522</td>
<td>212</td>
<td>183</td>
<td>-1.63</td>
<td>0.1782030</td>
</tr>
<tr>
<td>2</td>
<td>120.0000</td>
<td>88</td>
<td>183</td>
<td>0.83</td>
<td>0.9001869</td>
</tr>
</tbody>
</table>

Evaluation at use conditions 50 DegreesC

<table>
<thead>
<tr>
<th>Quantile</th>
<th>days</th>
<th>Ase</th>
<th>R.Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>2991.765</td>
<td>1134.752</td>
<td>2.10307</td>
</tr>
</tbody>
</table>

\[
ase(\log t_{1}) = \frac{ase(t_{1})}{t_{1}} = \frac{1134.572}{2991.765} = 0.3793
\]
Distribution: Weibull
Relationship: Arrhenius
Time units: days

For a censoring time of 183 days
the failure probability at 50 DegreesC is: 0.001

Intercept is: -16.73593
Relationship coefficients = 0.7265

weibull.beta = 1.666667
sigma = 0.6
Use conditions: 50 DegreesC

<table>
<thead>
<tr>
<th>Quantile</th>
<th>days</th>
<th>Quantile</th>
<th>days</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2991.765</td>
<td>1134.752</td>
<td>2.103070</td>
</tr>
<tr>
<td>2</td>
<td>9264.373</td>
<td>3963.577</td>
<td>2.312968</td>
</tr>
<tr>
<td>3</td>
<td>19039.264</td>
<td>8861.745</td>
<td>2.489936</td>
</tr>
</tbody>
</table>
Notation

Standardized Acceleration Level:

\[ \xi_i = \frac{x_i - x_U}{x_H - x_U} \] so that \( \xi_U = 0 \) and \( \xi_H = 1 \)

Allocation of Test Units:

\( \pi_i \) = proportion of units allocated to level \( x_i \) (standardized level \( \xi_i \))

Standardized Censoring Times:

\[ \zeta_i = \frac{\log(t_c) - \mu(x_i)}{\sigma} = \Phi^{-1}(p_i) \]

\( \Phi(\zeta_i) = p_i \)
Contour Plot Showing
$\log_{10}\{\text{Avar}[\log(\hat{t}_{1})] / \min \text{Avar}[\log(\hat{t}_{1})]\}$
as Function of $\xi_L, \pi_L$ to Find the Optimum ALT Plan
Adhesive Bond
Weibull Distribution Statistically Optimum Plan

Allocations: $\pi_{\text{Low}} = 0.71$ at $95^\circ C$, $\pi_{\text{High}} = 0.29$ at $120^\circ C$
Simulation of the Weibull Distribution Statistically Optimum Plan

Levels = 95,120 Degrees C, n=212,88
Censor time=183,183, parameters= -16.74,0.7265,0.5999

Precision factors R for quantile estimates at 50 Degrees C
R(0.1 quantile)= 2.103
R(0.5 quantile)= 2.309
R(Ea)= 1.155

Results based on 300 simulations
Lines shown for 50 simulations
Accelerated life test simulation based on
Optimum last: Weibull.altpv
\(x\); Arrhenius, Dist: Weibull
Failure time 0.1 quantile vs DegreesC
Weibull Distribution Statistically Optimum Plan

<table>
<thead>
<tr>
<th>Temp °C</th>
<th>Allocation</th>
<th>Failure Probability</th>
<th>Expected Number Failing</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>πᵢ</td>
<td>nᵢ</td>
<td>pᵢ</td>
</tr>
<tr>
<td>50</td>
<td>0.71</td>
<td>213</td>
<td>0.001</td>
</tr>
<tr>
<td>95</td>
<td>0.29</td>
<td>87</td>
<td>0.18</td>
</tr>
<tr>
<td>120</td>
<td>0.29</td>
<td>87</td>
<td>0.90</td>
</tr>
</tbody>
</table>

For this plan and the Weibull-Arrhenius model, \( \text{Ase}[\log(\hat{t}_1(50))] = 0.3794 \)
Adhesive Bond
Lognormal Distribution Statistically Optimum Plan

Allocations: $\pi_{\text{Low}} = .74 \text{ at } 78^\circ C$, $\pi_{\text{High}} = .26 \text{ at } 120^\circ C$
Lognormal Distribution Statistically Optimum Plan

<table>
<thead>
<tr>
<th>Temp °C</th>
<th>Allocation Proportion πᵢ</th>
<th>Allocation Number nᵢ</th>
<th>Failure Probability pᵢ</th>
<th>Expected Number Failing E(rᵢ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>.74</td>
<td>233</td>
<td>.001</td>
<td>30</td>
</tr>
<tr>
<td>78</td>
<td>.26</td>
<td>77</td>
<td>.13</td>
<td>30</td>
</tr>
<tr>
<td>120</td>
<td>.26</td>
<td>77</td>
<td>.90</td>
<td>69</td>
</tr>
</tbody>
</table>

For this plan and the Lognormal-Arrhenius model, $\text{Ase}[\hat{t}_{1}(50)] = .2002$
Critique of the Statistically Optimum Plan

• Still too much temperature extrapolation (to 50°C)
• Only two levels of temperature
• Optimum Weibull and lognormal plans quite different
  – 95°C and 120°C for Weibull versus
  – 78°C and 120°C for lognormal

In general, optimum plans are not robust to model departures
Want a Plan That

• Meets practical constraints and is intuitively appealing
• Is robust to deviations from assumed inputs
• Has reasonably good statistical properties
Criteria for Test Planning

Subject to constraints in time, sample size and ranges of experimental variables,

- Minimize \( \text{Var}[\log(\hat{t}_p)] \) under the assumed model.

- Maximize the determinant of the Fisher information matrix.

- Minimize \( \text{Var}[\log(\hat{t}_p)] \) under more general or higher-order model(s) (for robustness).

- Control the expected number of failures at each experimental condition (since a small expected number of failures at critical experimental conditions suggests potential for a failed experiment).
Types of Accelerated Life Test Plans

• **Optimum plans** – maximized statistical precision

• **Traditional plans** – Equal spacing and allocation; may be inefficient.

• **Optimized (best) compromise plans** – Require at least 3 levels of the accelerating variable (e.g., 20% constrained at middle) and optimized lower level and allocation.
General Guidelines for Planning ALTs
(Suggested from Optimum Plan Theory)

• Choose the highest level of the accelerating variable to be as high as possible
• Lowest level of the accelerating variable can be optimized
• Allocate more units to lower levels of the accelerating variable
• Test-plan properties and optimum plans depend on unknown inputs
Practical Guidelines for Compromise ALT Plans

• Use three or four levels of the accelerating variable
• Limit high level of the accelerating variable to maximum reasonable condition
• Reduce lowest level of the accelerating variable (to minimize extrapolation) – subject to seeing some action
• Allocate more units to lower levels of the accelerating variable
• Use statistically optimum plan as a starting point
• Evaluate plans in various meaningful ways
Adjusted Compromise Weibull ALT Plan for the Adhesive Bond (20% Constrained Allocation at the Middle and Constrained Expected Number of Failures at Lower Temperature to 5)

<table>
<thead>
<tr>
<th>Temp °C</th>
<th>Allocation Proportion $\pi_i$</th>
<th>Allocation Number $n_i$</th>
<th>Failure Probability $p_i$</th>
<th>Expected Number Failing $E(r_i)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td></td>
<td></td>
<td>.001</td>
<td></td>
</tr>
<tr>
<td>78</td>
<td>.52</td>
<td>156</td>
<td>.03</td>
<td>5</td>
</tr>
<tr>
<td>98</td>
<td>.20</td>
<td>60</td>
<td>.24</td>
<td>14</td>
</tr>
<tr>
<td>120</td>
<td>.28</td>
<td>84</td>
<td>.90</td>
<td>76</td>
</tr>
</tbody>
</table>

For this plan with the Weibull-Arrhenius model, $A_{SE}[\log(\hat{t}_{1}(50))] = .4375$. (15% larger than the optimum plan)

Could not get in SPLIDA. See next slide.
Generate an accelerated life test plan

Basic

Required inputs
- Plan values object: last.Weibull.alt
- Refresh list(s)

Plan type: Optimized comp
Use condition: 50
Highest level of accelerating var: 120

Required censoring information
- Censoring time: 183

Optional inputs
- Specify quantile of interest: .1
- Sample size: 300
- Proportion at middle level(s): .20

Save results in: last..ALTplan

OK Cancel Apply Help
Optimized compromise test plan based on last Weibull altpv

Test plan summary:

<table>
<thead>
<tr>
<th>DegreesC</th>
<th>n</th>
<th>ctime</th>
<th>zeta</th>
<th>p</th>
<th>efail</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>91.3862</td>
<td>160</td>
<td>183</td>
<td>-1.97</td>
<td>0.1300982</td>
</tr>
<tr>
<td>2</td>
<td>105.1528</td>
<td>60</td>
<td>183</td>
<td>-0.57</td>
<td>0.4326224</td>
</tr>
<tr>
<td>3</td>
<td>120.0000</td>
<td>80</td>
<td>183</td>
<td>0.83</td>
<td>0.9001869</td>
</tr>
</tbody>
</table>

Evaluation at use conditions 50 DegreesC

<table>
<thead>
<tr>
<th>Quantile</th>
<th>days</th>
<th>Ase R. Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>2991.765</td>
<td>1182.54</td>
</tr>
</tbody>
</table>

11/13/2004
Adhesive Bond

Adjusted Compromise Weibull ALT Plan

$\pi_{Low} = .52, \; \pi_{Mid} = .20, \; \pi_{High} = .28$
Simulation of the Adhesive Bond Compromise Weibull ALT Plan

Levels = 78, 98, 120 Degrees C, n=155, 60, 84
Censor time=183, 183, 183, parameters= -16.74, 0.7265, 0.5999

Precision factors R for quantile estimates at 50 Degrees C
R(0.1 quantile)= 2.381
R(0.5 quantile)= 2.645
R(Ea)= 1.177

Results based on 300 simulations
Lines shown for 50 simulations
Basic Issue 1: Choose Levels of Accelerating Variables

Need to Balance:

• Extrapolation in the acceleration variable (assumed temperature-time relationship)
• Extrapolation in time (assumed failure-time distribution)

Suggested Plan:

• Middle and high levels of the acceleration variable – expect to interpolate in time
• Low level of the acceleration variable – expect to extrapolate in time
Basic Issue 2: Allocation of Test Units

- Allocate more test units to low rather than high levels of accelerating variable
  - Tends to equalize the number of failures at experimental conditions
  - Testing more units near the use conditions is intuitively appealing
  - Suggested by statistically optimum plan
- Need to constrain a certain percentage of units to the middle level of the accelerating variable
Properties of Compromise ALT Plans Relative to Statistically Optimum Plans

• Increase asymptotic variance of estimator of log B10 at 50°C by 15% (if assumptions are correct)

However it also:

• Reduces low test temperature to 78°C (from 95°C)

• Uses three levels of accelerating variable, instead of two levels

• Is more robust to departures from assumptions and uncertain inputs
Generalizations and Comments

• Constraints on test positions (instead of test units): Consider replacement after 100p% failures at each level of accelerating variable
• Continue tests at each level of accelerating variable until at least 100p% units have failed
• Include some test at the use conditions
• Fine tune with computer evaluation and/or simulation of user-suggested plans
• Desire to estimate reliability (instead of a quantile) at use conditions
• Need to quantify robustness
ALT with Two or More Variables

• Moderate increases in two accelerating variables may be safer than using a large amount of a single accelerating variable
• There may be interest in assessing the effect of non-accelerating variables
• There may be interest in assessing joint effects of two more accelerating variables