Unit 18: Accelerated Test Models


Review: Log Location-Scale Model Representation

\[
\log T = \mu + \sigma Z, \quad P(Z \leq t) = \Phi(t)
\]

\[
\Leftrightarrow
\]

\[
P(T \leq t) = P(\log T \leq \log t)
\]

\[
= P(\mu + \sigma Z \leq \log t)
\]

\[
= P\left(Z \leq \frac{\log t - \mu}{\sigma}\right)
\]

\[
= \Phi\left(\frac{\log t - \mu}{\sigma}\right)
\]
Review: Log Location-Scale Families and the SAFT Model

Baseline Distribution at $x_0$:

$$P(T(x_0) \leq t) = \Phi\left(\frac{\log t - \mu(x_0)}{\sigma}\right)$$

SAFT Model:

$$T(x) = \frac{T(x_0)}{AF(x)}$$
Unit 18 Objectives

- Describe motivation and application of accelerated reliability testing
- Explain the connection between degradation, physical failure, and acceleration of reliability tests
- Examine the basis for temperature and humidity acceleration
- Examine the basis for voltage and pressure stress acceleration
- Show how to compute time-acceleration factors
- Review other accelerated test models and assumptions

Accelerated Test Increasingly Important

- Today’s manufacturers need to develop newer, higher technology products in record time while improving productivity, reliability, and quality
- Important Issues:
  - Rapid product development
  - Rapidly changing technologies
  - More complicated products with more components
  - Higher customer expectations for better reliability
Need for Accelerated Tests

• Need timely information on high reliability products
  – Modern products designed to last for years or decades
  – Accelerated Tests (ATs) used for timely assessment of reliability of product components and materials
  – Test at high levels of use rate, temperature, voltage, pressure, humidity, etc.
  – Estimate life at use conditions
• Note: Estimation/prediction from ATs involves extrapolation

Application of Accelerated Tests

• Applications of Accelerated Tests include:
  – Evaluating the effect of stress on life
  – Assessing component reliability
  – Demonstrating component reliability
  – Detecting failure modes
  – Comparing two or more competing products
  – Establishing safe warranty times
Methods of Acceleration

- Three fundamentally different methods of accelerating a reliability test:
  - Increase the use-rate of the product (e.g., test a toaster 400 times/day). Higher use rate reduces test time.
  - Use elevated temperature or humidity to increase rate of failure-causing chemical/physical process.
  - Increase stress (e.g., voltage or pressure) to make degrading units fail more quickly.
- Use a physical/chemical (preference) or empirical model relating degradation or lifetime at use conditions.

Change in Resistance Over Time of Carbon-Film Resistors (Shiorni and Yanagisawa, 1979)
Accelerated Degradation Tests (ADTs)

- **Response**: Amount of degradation at points in time
- **Model components**
  - Model for degradation over time
  - A definition of failure as a function of degradation variable
  - Relationship(s) between degradation model parameters (e.g., chemical process reaction rates) and acceleration variables (e.g., temperature or humidity).

Breakdown Times in Minutes of a Mylar-Polyurethane Insulating Structure (from Kalkanis and Rosso, 1989)
Accelerated Life Tests (ALTs)

- Response
  - Failure time (or interval) for units that fail
  - Censoring time for units that do not fail
- Model Components:
  - Constant-stress time-to-failure distribution
  - Relationship(s) between one (or more) of the constant-stress model parameters and the accelerating variables

Use-Rate Acceleration

- **Basic Idea:** Increase use-rate to accelerate failure-causing wear or degradation
- **Examples:**
  - Running automobile engines or appliances continuously
  - Rapid cycling of relays and switches
  - Cycles to failure in fatigue testing
- **Simple assumption:**
  - Useful if life adequately modeled by cycles of operation. Reasonable if cycling simulates actual use and if test units return to steady state after each cycle.
- **More complicated situation:**
  - Wear rate or degradation rate depends on cycling frequency or product deteriorates in stand-by as well as during actual use
Elevated Temperature Acceleration of Chemical Reaction Rates

- The **Arrhenius model Reaction Rate**, \( R(\text{temp}) \), is:

\[
R(\text{temp}) = \gamma_0 \exp \left( \frac{-E_a}{k_B (\text{temp} \cdot ^\circ C + 273.15)} \right) = \gamma_0 \exp \left( \frac{-E_a \times 11605}{\text{temp} \cdot K} \right)
\]

where \( \text{temp} \cdot K = \text{temp} \cdot ^\circ C + 273.15 \) is temperature in degrees Kelvin and \( k_B = 1/11605 \) is Boltzmann’s constant in units of electron volts per K. The reaction activation energy, \( E_a \), and \( \gamma_0 \) are characteristics of the product or material being tested.

- The reaction rate **Acceleration Factor** is:

\[
AF(\text{temp}, \text{temp}_U, E_a) = \frac{R(\text{temp})}{R(\text{temp}_U)} = \exp \left[ \frac{E_a \left( \frac{11605}{\text{temp}_U \cdot K} - \frac{11605}{\text{temp} \cdot K} \right)}{\text{temp} \cdot K} \right]
\]

- When \( \text{temp} > \text{temp}_U \), \( AF(\text{temp}, \text{temp}_U, E_a) > 1 \).

Acceleration Factors for the SAFT Arrhenius Model

- Table 18.2 gives the **Temperature Differential Factors** (TDF):

\[
\text{TDF} = \left( \frac{11605}{\text{temp}_{\text{Low}} \cdot K} - \frac{11605}{\text{temp}_{\text{High}} \cdot K} \right).
\]

- Figure 18.3 gives

\[
AF(\text{temp}_{\text{High}}, \text{temp}_{\text{Low}}, E_a) = \exp (E_a \times \text{TDF})
\]

- We use \( AF(\text{temp}) = AF(\text{temp}, \text{temp}_U, E_a) \) when \( \text{temp}_U \) and \( E_a \) are understood to be, respectively, product use temperature and reaction-specific activation energy.
Nonlinear Degradation Reaction-Rate Acceleration

- Consider the simple chemical degradation path model

\[ D(t; \text{temp}) = D_\infty \times (1 - \exp \left[ -R_U \times AF(\text{temp}) \times t \right] ) \]

where \( R_U \) is the rate reaction at use temperature (temp\(_U\)) and for temp > temp\(_U\), \( AF(\text{temp}) > 1 \).

- For \( D_\infty > 0 \), failure occurs when \( D(t; \text{temp}) > D_T \). Equating \( D(T; \text{temp}) \) to \( D_T \) and solving for failure time, \( T(\text{temp}) \), gives

\[ T(\text{temp}) = \frac{T(\text{temp}_U)}{AF(\text{temp})} = \left[ -\frac{1}{R_U} \log \left( 1 - \frac{D_T}{D_\infty} \right) \right] \]

where \( T(\text{temp}_U) \) is failure time at use conditions.

- This is an SAFT model.
SAFT Model from Nonlinear Degradation Path

\[ D(t; \text{temp}) = D_\infty \times \left\{ 1 - \exp \left[ -R_U \times A_F(\text{temp}) \times t \right] \right\} \]

\[ D_\infty < 0 \]

The Arrhenius-Lognormal Regression Model

The Arrhenius-lognormal regression model is

\[ \Pr[I^* \leq I; \text{temp}] = \Phi_{\text{nor}} \left[ \frac{\log(t) - \mu}{\sigma} \right] \]

where

- \( \mu = \beta_0 + \beta_1 x \),
- \( x = 11605/(\text{temp K}) = 11605/(\text{temp } ^\circ\text{C} + 273.15) \)
- and \( \beta_1 = E_a \) is the activation energy
- \( \sigma \) is constant
- This implies that

\[ t_p(\text{temp}_U) = t_p(\text{temp}) \times A_F(\text{temp}) \]
Example Arrhenius-Lognormal Life Model

\[
\log[t_{\text{temp}}] = \beta_0 + \beta_1 x + \Phi_{\text{log}}(\mu) \sigma
\]

\[
\log T = \beta_0 + \beta_1 x + \sigma Z, \quad Z \sim N(0,1)
\]
Linear Degradation Reaction-Rate Acceleration

Note:
\[ \exp(-x) = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \ldots \]
\[ \Rightarrow x \approx 1 - \exp(-x) \text{ if } x \text{ is small} \]

If \( R_U \times A F(t_{\text{temp}}) \times t \) is small so that \( D(t) \) is small relative to \( D_\infty \), then
\[
D(t; \text{temp}) = D_\infty \times \{1 - \exp\left[-R_U \times A F(t_{\text{temp}}) \times t\right]\} \\
\approx D_\infty \times R_U \times A F(t_{\text{temp}}) \times t = R_U^+ \times A F(t_{\text{temp}}) \times t
\]
is approximately linear in \( t \).

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Linear Degradation Reaction-Rate Acceleration

- Also some degradation processes are linear in time:
  \[ D(t; \text{temp}) = R_U \times A F(t_{\text{temp}}) \times t. \]
- Failure occurs when \( D(T; \text{temp}) > D_f \). Equating \( D(T; \text{temp}) \)
to \( D_f \) and solving for failure time, \( T(\text{temp}) \),
  \[ T(\text{temp}) = \frac{T(t_{\text{temp}})}{A F(\text{temp})} \]
where \( T(t_{\text{temp}}) = D_f / R_U \) is failure time at use conditions.
- This is an SAEFT model and, for example, \( T(t_{\text{temp}}) \sim \text{WEIB}(\mu, \sigma) \) implies \( T(\text{temp}) \sim \text{WEIB}[\mu - \log(A F(\text{temp})), \sigma] \).
Non-SAFT Degradation Reaction-Rate Acceleration

Consider the more complicated chemical degradation path

\[ D(t; \text{temp}) = D_{1\infty} \times \{1 - \exp[-R_{1U} \times AF_1(\text{temp}) \times t]\} \\
+ D_{2\infty} \times \{1 - \exp[-R_{2U} \times AF_2(\text{temp}) \times t]\} \]

\( R_{1U}, R_{2U} \) are the rates of the reactions contributing to failure.

This is not an SAFT model. Temperature affects the two degradation processes differently, inducing a nonlinearity into the acceleration function relating times at two different temperatures.

Voltage Acceleration and Voltage Stress Inverse Power Relationship

- Depending on the failure mode, voltage can be raised to:
  - Increase the strength of electric fields. This can accelerate some failure-causing reactions.
  - Increase the stress level (e.g., voltage stress relative to declining voltage strength).

- An empirical model for life at volt relative to use conditions volt_U is

\[ T(\text{volt}) = \frac{T(\text{volt}_U)}{AF(\text{volt})} = \left(\frac{\text{volt}}{\text{volt}_U}\right)^{\beta_1} T(\text{volt}_U) \]

where \( AF(\text{volt}) = AF(\text{volt}, \text{volt}_U, \beta_1) \),

\[ AF(\text{volt}) = AF(\text{volt}, \text{volt}_U, \beta_1) = \frac{T(\text{volt}_U)}{T(\text{volt})} = \left(\frac{\text{volt}}{\text{volt}_U}\right)^{-\beta_1} \]

and \( \beta_1 \) is a material characteristic. \( T(\text{volt}), T(\text{volt}_U) \) are the failure times at increased voltage and use conditions.
Inverse Power Relationship-Weibull Model

The inverse power relationship-Weibull model is

\[
\Pr[T \leq t; \text{volt}] = \Phi_{\text{SEV}} \left[ \frac{\log(t) - \mu}{\sigma} \right]
\]

where

- \( \mu = \beta_0 + \beta_1 x \), and

- \( x = \log(\text{Voltage Stress}) \).

- \( \sigma \) assumed to be constant.

Example Weibull Inverse Power Relationship Between Life and Voltage Stress

\[
\log[t_\mu(\text{volt})] = \beta_0 + \beta_1 x + \Phi_{\text{SEV}}^{-1}(p)\sigma
\]

\[
\log T = \beta_0 + \beta_1 x + \sigma Z, \quad Z \sim \text{Gumbel}(0,1)
\]
Other Commonly Used Life-Stress Relationships

Other commonly used SAFT models have the simple form:

\[ T(x) = T(x_U) \frac{\mathcal{A}(x)}{\mathcal{A}(x_U)} \]

where \( \mathcal{A}(x) = \mathcal{A}(x, x_U, \beta_1) = \exp[\beta_1(x - x_U)] \). \( \beta_1 \) is a material characteristic.

Examples include:

- Cycling rate: \( x = \log(\text{frequency}) \).
- Current density: \( x = \log(\text{current}) \).
- Size: \( x = \log(\text{thickness}) \).
- Humidity 1: \( x = \log(\text{RH}) \), RH is % relative humidity/100.
- Humidity 2: \( x = \log[\text{RH}/(1 - \text{RH})] \).

Some of these models are empirical. For a location-scale time-to-failure distribution \( \mu = \beta_0 + \beta_1 x \).

Eyring Temperature Relationship

- Arrhenius relationship obtained from empirical observation.
- Eyring developed physical theory describing the effect that temperature has on a reaction rate:

\[ \mathcal{R}(\text{temp}) = \gamma_0 \times A(\text{temp}) \times \exp\left(\frac{-E_0}{k_B \times \text{temp K}}\right) \]

- \( A(\text{temp}) \) is a function of temperature depending on the specifics of the reaction dynamics; \( \gamma_0 \) and \( E_0 \) are constants.

- Applications in the literature have used \( A(\text{temp}) = (\text{temp K})^m \) with a fixed value of \( m \) ranging between \( m = 0 \) (Boccaccetti et al. 1989), \( m = .5 \) (Klinger 1991a), to \( m = 1 \) (Nelson 1990a and Mann Schafer and Singpurwalla 1974). Difficult to identify \( m \) from limited data.

- Eyring showed how to include other accelerating variables.
The Eyring Regression Model (e.g., for Weibull or Lognormal Distribution)

The Eyring temperature-acceleration regression model is

$$Pr(T \leq t; \text{temp}) = \Phi \left( \frac{\log(t) - \mu}{\sigma} \right)$$

where

- $\mu = -m \log(\text{temp}^\circ C + 273.15) + \beta_0 + \beta_1 x$.
- $x = 11605/((\text{temp}^\circ C + 273.15)$.
- $\beta_1 = E_a$ is the activation energy.
- $m$ is usually given; $\sigma$ is constant, but usually unknown.
- With $m > 0$, Arrhenius provides a useful first order approximation to the Eyring model, with conservative extrapolation to lower temperatures.

Humidity Acceleration Models

- Useful for accelerating failure mechanisms involving corrosion and certain other kinds of chemical degradation.
- Often used in conjunction with elevated temperature.
- Most humidity models have been developed empirically.
- Empirical and limited theoretical results for corrosion on thin films (Gillen and Mead 1980, Peck 1986, and Klinger 1991b) suggest the use of RH instead of $P_v$ (vapor pressure) as the independent (or experimental) variable in humidity relationships when temperature is also to be varied.
- RH is the preferred variable because the change in life, as a function of RH, does not depend on temperature. That is,

$$\frac{\partial^2 \text{Life}}{\partial \text{RH} \partial \text{temp}} = 0$$

or no statistical interaction.
Humidity Regression Relationships

- Consider the Weibull/lognormal lifetime regression model
  \[ \Pr(T \leq t; \text{humidity}) = \Phi_{\text{weibull}} \left( \frac{\log(t) - \mu}{\sigma} \right) \]
  where \( \mu = \beta_0 + \beta_1 x_1 \) and \( \sigma \) is constant. Letting \( 0 < \text{RH} < 1 \) denote relative humidity, possible humidity relationships are:
  \[ x_1 = \text{RH} \] [Intel, empirical].
  \[ x_1 = \log(\text{RH}) \] [Peck, empirical].
  \[ x_1 = \log(\text{RH}/(1 - \text{RH})) \] [Klinger, corrosion on thin films].

- For temperature and humidity acceleration, possible relationships include
  \[ \mu = \beta_0 + \beta_1 x_1 + \beta_2 x_2 \]
  or \( \mu = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2 \)
  or \( \mu = \beta_0 + \beta_2 x_2 + \beta_3 x_1 x_2 \)
  where \( x_2 = \frac{11605}{\text{temp} \cdot \text{K} + 273.15} \).

Temperature/Humidity Acceleration Factors with RH and Temp K and No Interaction

- Peck’s relationship
  \[ \mathcal{A}(\text{temp}, \text{RH}) = \frac{\mathcal{R}(\text{temp}, \text{RH})}{\mathcal{R}(\text{temp}_U, \text{RH}_U)} \]
  \[ = \left( \frac{\text{RH}_U}{\text{RH}} \right)^{\beta_1} \exp \left[ E_a \left( \frac{11605}{\text{temp}_U \cdot \text{K}} - \frac{11605}{\text{temp} \cdot \text{K}} \right) \right]. \]

- Klinger’s relationship
  \[ \mathcal{A}(\text{temp}, \text{RH}) = \frac{\mathcal{R}(\text{temp}, \text{RH})}{\mathcal{R}(\text{temp}_U, \text{RH}_U)} \]
  \[ = \left( \frac{\text{RH}_U}{1 - \text{RH}_U} \left( \frac{1 - \text{RH}}{\text{RH}} \right) \right)^{\beta_1} \times \]
  \[ \exp \left[ E_a \left( \frac{11605}{\text{temp}_U \cdot \text{K}} - \frac{11605}{\text{temp} \cdot \text{K}} \right) \right]. \]
Thermal Cycling

- Fatigue is an important failure mechanism for many products and materials.
- Mechanical expansion and contraction from thermal cycling can lead to fatigue cracking and failure.
- Applications include:
  - Power-on/power-off cycling of electronic equipment and effect on component encapsulement and solder joints.
  - Take-off power-thrust in jet engines and its effect on crack initiation and growth in fan disks.
  - Power-up/power-down of nuclear power plants and effect on the growth of cracks in heat generator tubes.
  - Thermal inkjet printhead delamination could be caused by temperature cycling during normal use.

Coffin-Manson Relationship

- The Coffin-Manson relationship says that the typical number of cycles to failure is
  \[ N = \frac{\delta}{(\Delta\text{temp})^{\beta_1}} \]
  where \( \Delta\text{temp} \) is the temperature range and \( \delta \) and \( \beta_1 \) are properties of the material and test setup. This power-rule relationship explains the effect that temperature range has on thermal-fatigue life. For some metals, \( \beta_1 \approx 2 \).
- Letting \( T \) be the random number of cycles to failure (e.g., \( T = N \times \epsilon \) where \( \epsilon \) is a random variable), the acceleration factor when \( \Delta\text{temp} \), relative to the number of cycles when \( \Delta\text{temp}_0 \), is
  \[ \frac{T(\Delta\text{temp})}{T(\Delta\text{temp}_0)} = \left( \frac{\Delta\text{temp}}{\Delta\text{temp}_0} \right)^{\beta_1} \]
- There may be a \( \Delta\text{temp} \) threshold below which little or no fatigue damage is done during thermal cycling.
Generalized Coffin-Manson Relationship

- Empirical evidence has shown that the effect of temperature cycling can depend importantly on \( t_{\text{max}} \), the maximum temperature in the cycling (e.g., if \( t_{\text{max}} \) is more than .2 or .3 times a metal’s melting point).

- The effect of temperature cycling can also depend on the cycling rate (e.g., due to heat buildup).

- An empirical extension of the Coffin-Manson relationship is

\[
N = \frac{\delta}{(\Delta \text{temp})^{\beta_1}} \times \frac{1}{(\text{freq})^{\beta_2}} \times \exp\left(\frac{E_a \times 11605}{t_{\text{max}} \text{K}}\right)
\]

where \( \text{freq} \) is the cycling frequency, and \( E_a \) is an activation energy.

- Caution must be used when using such a model outside the range of available data and past experience.

Other Topics in Chapter 18

- Other accelerated degradation models and relationships to accelerated time models

- Discussion of stress-cycling models

- Other models for two or more experimental factors.