

Unit 18: Accelerated Test Models

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Notes largely based on
“Statistical Methods for Reliability Data”
by W.Q. Meeker and L. A. Escobar,
Wiley, 1998 and on their class notes.

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Review: Log Location-Scale Model Representation

$$\log T = \mu + \sigma Z, \quad P(Z \leq t) = \Phi(t)$$

\Leftrightarrow

$$\begin{aligned} P(T \leq t) &= P(\log T \leq \log t) \\ &= P(\mu + \sigma Z \leq \log t) \\ &= P\left(Z \leq \frac{\log t - \mu}{\sigma}\right) \\ &= \Phi\left(\frac{\log t - \mu}{\sigma}\right) \end{aligned}$$

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Review: Log Location-Scale Families and the SAFT Model

Baseline Distribution at x_0 :

$$P(T(x_0) \leq t) = \Phi\left(\frac{\log t - \mu(x_0)}{\sigma}\right)$$

SAFT Model:

$$T(x) = \frac{T(x_0)}{AF(x)}$$

Review:
Log
Location-
Scale
Families
and the
SAFT
Model

$$\begin{aligned} P(T(x) \leq t) &= P\left(\frac{T(x_0)}{AF(x)} \leq t\right) \\ &= P(T(x_0) \leq tAF(x)) \\ &= \Phi\left(\frac{\log tAF(x) - \mu(x_0)}{\sigma}\right) \\ &= \Phi\left(\frac{\log t - (\mu(x_0) - \log AF(x))}{\sigma}\right) \\ &= \Phi\left(\frac{\log t - \mu(x)}{\sigma}\right) \\ &\Leftrightarrow \\ \mu(x) &= \mu(x_0) - \log AF(x) \end{aligned}$$

Unit 18 Objectives

- Describe motivation and application of accelerated reliability testing
- Explain the connection between degradation, physical failure, and acceleration of reliability tests
- Examine the basis for temperature and humidity acceleration
- Examine the basis for voltage and pressure stress acceleration
- Show how to compute time-acceleration factors
- Review other accelerated test models and assumptions

Accelerated Test Increasingly Important

- Today's manufacturers need to develop newer, higher technology products in record time while improving productivity, reliability, and quality
- Important Issues:
 - Rapid product development
 - Rapidly changing technologies
 - More complicated products with more components
 - Higher customer expectations for better reliability

Need for Accelerated Tests

- Need timely information on high reliability products
 - Modern products designed to last for years or decades
 - Accelerated Tests (ATs) used for timely assessment of reliability of product components and materials
 - Test at high levels of use rate, temperature, voltage, pressure, humidity, etc.
 - Estimate life at **use conditions**
- Note: Estimation/prediction from ATs involves **extrapolation**

Application of Accelerated Tests

- Applications of Accelerated Tests include:
 - Evaluating the effect of stress on life
 - Assessing component reliability
 - Demonstrating component reliability
 - Detecting failure modes
 - Comparing two or more competing products
 - Establishing safe warranty times

Methods of Acceleration

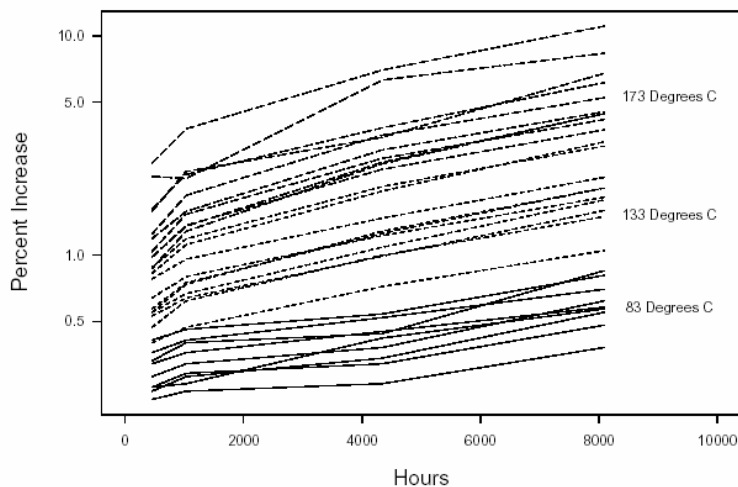
- Three fundamentally different methods of accelerating a reliability test:
 - Increase the use-rate of the product (e.g., test a toaster 400 times/day). Higher use rate reduces test time
 - Use elevated temperature or humidity to increase rate of failure-causing chemical/physical process
 - Increase stress (e.g., voltage or pressure) to make degrading units fail more quickly.
- Use a **physical/chemical** (preference) or **empirical** model relating degradation or lifetime at **use conditions**.

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Change in Resistance Over Time of Carbon-Film Resistors
(Shiorni and Yanaqisawa, 1979)



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Accelerated Degradation Tests (ADTs)

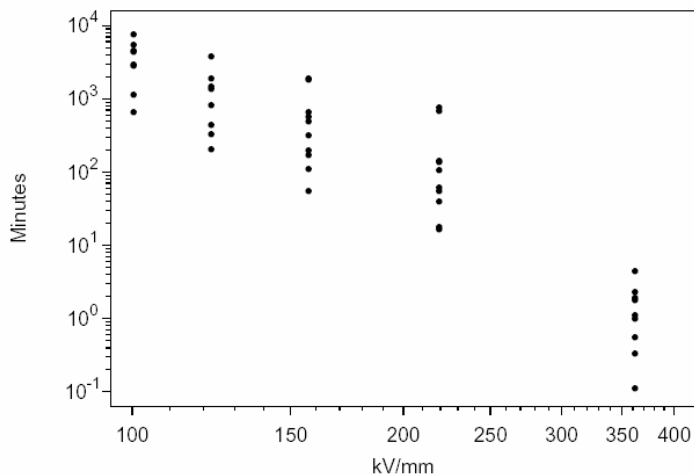
- **Response:** Amount of degradation at points in time
- Model components
 - Model for degradation over time
 - A definition of failure as a function of degradation variable
 - Relationship(s) between degradation model parameters (e.g., chemical process reaction rates) and acceleration variables (e.g., temperature or humidity).

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Breakdown Times in Minutes of a Mylar-Polyurethane Insulating Structure (from Kalkanis and Rosso, 1989)



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Accelerated Life Tests (ALTs)

- Response
 - Failure time (or interval) for units that fail
 - Censoring time for units that do not fail
- Model Components:
 - Constant-stress time-to-failure distribution
 - Relationship(s) between one (or more) of the constant-stress model parameters and the accelerating variables

Use-Rate Acceleration

- **Basic Idea:** Increase use-rate to accelerate failure-causing wear or degradation
- **Examples:**
 - Running automobile engines or appliances continuously
 - Rapid cycling of relays and switches
 - Cycles to failure in fatigue testing
- **Simple assumption:**
 - Useful if life adequately modeled by cycles of operation.
Reasonable if cycling simulates actual use and if test units return to steady state after each cycle.
- **More complicated situation:**
 - Wear rate or degradation rate depends on cycling frequency or product deteriorates in stand-by as well as during actual use

Elevated Temperature Acceleration of Chemical Reaction Rates

- The **Arrhenius** model **Reaction Rate**, $\mathcal{R}(\text{temp})$, is

$$\mathcal{R}(\text{temp}) = \gamma_0 \exp\left(\frac{-E_a}{k_B(\text{temp}^\circ\text{C} + 273.15)}\right) = \gamma_0 \exp\left(\frac{-E_a \times 11605}{\text{temp K}}\right)$$

where $\text{temp K} = \text{temp}^\circ\text{C} + 273.15$ is temperature in degrees Kelvin and $k_B = 1/11605$ is Boltzmann's constant in units of electron volts per K. The reaction activation energy, E_a , and γ_0 are characteristics of the product or material being tested.

- The reaction rate **Acceleration Factor** is

$$\begin{aligned} \mathcal{AF}(\text{temp}, \text{temp}_U, E_a) &= \frac{\mathcal{R}(\text{temp})}{\mathcal{R}(\text{temp}_U)} \\ &= \exp\left[E_a \left(\frac{11605}{\text{temp}_U \text{ K}} - \frac{11605}{\text{temp K}}\right)\right] \end{aligned}$$

- When $\text{temp} > \text{temp}_U$, $\mathcal{AF}(\text{temp}, \text{temp}_U, E_a) > 1$.

Acceleration Factors for the SAFT Arrhenius Model

- Table 18.2 gives the **Temperature Differential Factors** (TDF)

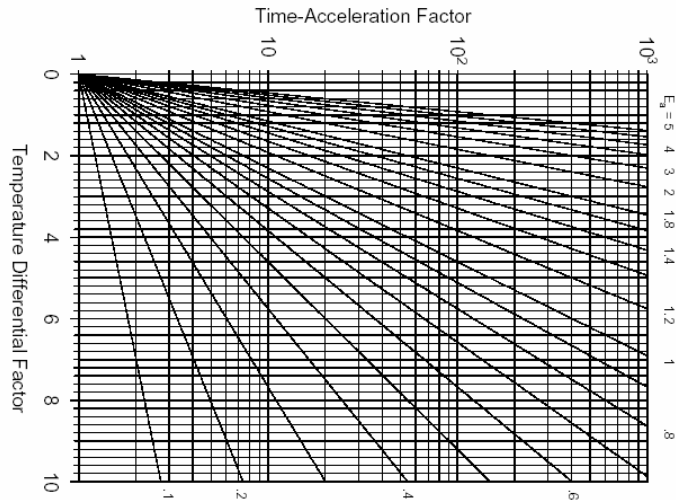
$$\text{TDF} = \left(\frac{11605}{\text{temp}_{\text{Low}} \text{ K}} - \frac{11605}{\text{temp}_{\text{High}} \text{ K}}\right).$$

- Figure 18.3 gives

$$\mathcal{AF}(\text{temp}_{\text{High}}, \text{temp}_{\text{Low}}, E_a) = \exp(E_a \times \text{TDF})$$

- We use $\mathcal{AF}(\text{temp}) = \mathcal{AF}(\text{temp}, \text{temp}_U, E_a)$ when temp_U and E_a are understood to be, respectively, product use temperature and reaction-specific activation energy.

Time-Acceleration Factor as a Function of Temperature Factor (Figure 18.3)



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Nonlinear Degradation Reaction-Rate Acceleration

- Consider the simple chemical degradation path model

$$D(t; \text{temp}) = D_{\infty} \times \{1 - \exp[-\mathcal{R}_U \times \mathcal{AF}(\text{temp}) \times t]\}$$

where \mathcal{R}_U is the rate reaction at use temperature (temp_U) and for $\text{temp} > \text{temp}_U$, $\mathcal{AF}(\text{temp}) > 1$.

- For $D_{\infty} > 0$, failure occurs when $D(T; \text{temp}) > D_f$. Equating $D(T; \text{temp})$ to D_f and solving for failure time, $T(\text{temp})$, gives

$$T(\text{temp}) = \frac{T(\text{temp}_U)}{\mathcal{AF}(\text{temp})} = \frac{\left[-\frac{1}{\mathcal{R}_U} \log\left(1 - \frac{D_f}{D_{\infty}}\right)\right]}{\mathcal{AF}(\text{temp})}$$

where $T(\text{temp}_U)$ is failure time at use conditions.

- This is an SAFT model.

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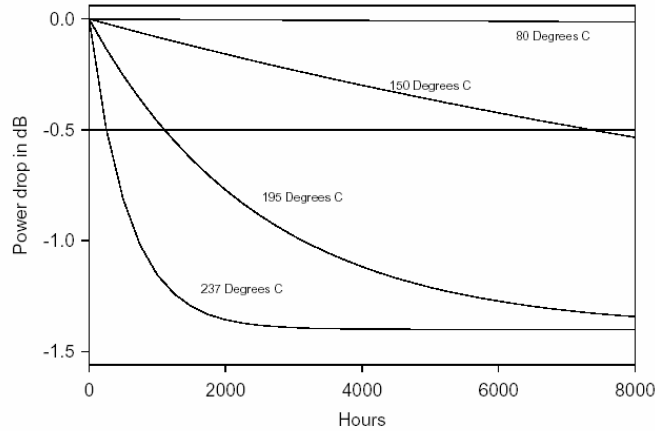
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SAFT Model from Nonlinear Degradation Path

$$D(t; \text{temp}) = D_{\infty} \times \{1 - \exp[-\mathcal{R}_U \times \mathcal{AF}(\text{temp}) \times t]\}$$

$$D_{\infty} < 0$$



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The Arrhenius-Lognormal Regression Model

The Arrhenius-lognormal regression model is

$$\Pr[T \leq t; \text{temp}] = \Phi_{\text{nor}} \left[\frac{\log(t) - \mu}{\sigma} \right]$$

where

- $\mu = \beta_0 + \beta_1 x$,
- $x = 11605 / (\text{temp K}) = 11605 / (\text{temp } ^\circ\text{C} + 273.15)$
- and $\beta_1 = E_a$ is the activation energy
- σ is constant
- This implies that

$$t_p(\text{temp}_U) = t_p(\text{temp}) \times \mathcal{AF}(\text{temp})$$

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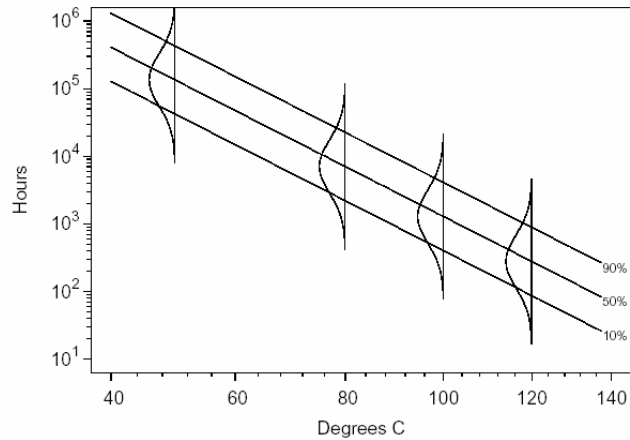
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Example Arrhenius-Lognormal Life Model

$$\log[t_p(\text{temp})] = \beta_0 + \beta_1 x + \Phi_{\text{nor}}^{-1}(p)\sigma$$

$$\log T = \beta_0 + \beta_1 x + \sigma Z, \quad Z \sim N(0,1)$$

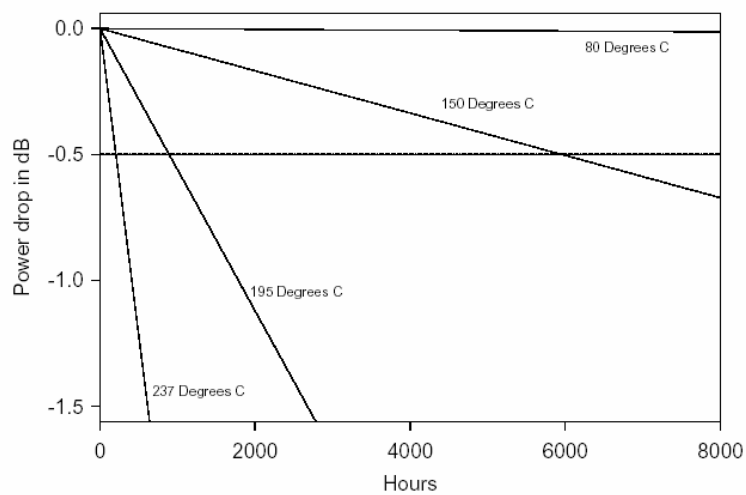


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SAFT Model from Linear Degradation Paths



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Linear Degradation Reaction-Rate Acceleration

Note:

$$\exp(-x) = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots$$

\Rightarrow

$$x \doteq 1 - \exp(-x) \text{ if } x \text{ is small}$$

If $\mathcal{R}_U \times \mathcal{A}\mathcal{F}(\text{temp}) \times t$ is small so that $\mathcal{D}(t)$ is small relative to \mathcal{D}_∞ , then

$$\begin{aligned} \mathcal{D}(t; \text{temp}) &= \mathcal{D}_\infty \times \{1 - \exp[-\mathcal{R}_U \times \mathcal{A}\mathcal{F}(\text{temp}) \times t]\} \\ &\approx \mathcal{D}_\infty \times \mathcal{R}_U \times \mathcal{A}\mathcal{F}(\text{temp}) \times t = \mathcal{R}_U^+ \times \mathcal{A}\mathcal{F}(\text{temp}) \times t \end{aligned}$$

is approximately linear in t .

Linear Degradation Reaction-Rate Acceleration

- Also some degradation processes are linear in time:

$$\mathcal{D}(t; \text{temp}) = \mathcal{R}_U \times \mathcal{A}\mathcal{F}(\text{temp}) \times t.$$

- Failure occurs when $\mathcal{D}(T; \text{temp}) > \mathcal{D}_f$. Equating $\mathcal{D}(T; \text{temp})$ to \mathcal{D}_f and solving for failure time, $T(\text{temp})$,

$$T(\text{temp}) = \frac{T(\text{temp}_U)}{\mathcal{A}\mathcal{F}(\text{temp})}$$

where $T(\text{temp}_U) = \mathcal{D}_f / \mathcal{R}_U$ is failure time at use conditions.

- This is an SAFT model and, for example, $T(\text{temp}_U) \sim \text{WEIB}(\mu, \sigma)$ implies $T(\text{temp}) \sim \text{WEIB}[\mu - \log(\mathcal{A}\mathcal{F}(\text{temp})), \sigma]$.

Non-SAFT Degradation Reaction-Rate Acceleration

Non-SAFT Degradation Reaction-Rate Acceleration

Consider the more complicated chemical degradation path

$$\mathcal{D}(t; \text{temp}) = \mathcal{D}_{1\infty} \times \{1 - \exp[-\mathcal{R}_{1U} \times \mathcal{AF}_1(\text{temp}) \times t]\} \\ + \mathcal{D}_{2\infty} \times \{1 - \exp[-\mathcal{R}_{2U} \times \mathcal{AF}_2(\text{temp}) \times t]\}$$

$\mathcal{R}_{1U}, \mathcal{R}_{2U}$ are the rates of the reactions contributing to failure.

This is **not** an SAFT model. Temperature affects the two degradation processes differently, inducing a nonlinearity into the acceleration function relating times at two different temperatures.

Voltage Acceleration and Voltage Stress Inverse Power Relationship

- Depending on the failure mode, voltage can be raised to:
 - ▶ Increase the strength of electric fields. This can accelerate some failure-causing reactions.
 - ▶ Increase the stress level (e.g., voltage stress relative to declining **voltage strength**).
- An **empirical** model for life at volt relative to use conditions volt_U is

$$T(\text{volt}) = \frac{T(\text{volt}_U)}{\mathcal{AF}(\text{volt})} = \left(\frac{\text{volt}}{\text{volt}_U}\right)^{\beta_1} T(\text{volt}_U)$$

where $\mathcal{AF}(\text{volt}) = \mathcal{AF}(\text{volt}, \text{volt}_U, \beta_1)$,

$$\mathcal{AF}(\text{volt}) = \mathcal{AF}(\text{volt}, \text{volt}_U, \beta_1) = \frac{T(\text{volt}_U)}{T(\text{volt})} = \left(\frac{\text{volt}}{\text{volt}_U}\right)^{-\beta_1}$$

and β_1 is a material characteristic. $T(\text{volt}), T(\text{volt}_U)$ are the failure times at increased voltage and use conditions.

Inverse Power Relationship-Weibull Model

The inverse power relationship-Weibull model is

$$\Pr[T \leq t; \text{volt}] = \Phi_{\text{sev}} \left[\frac{\log(t) - \mu}{\sigma} \right]$$

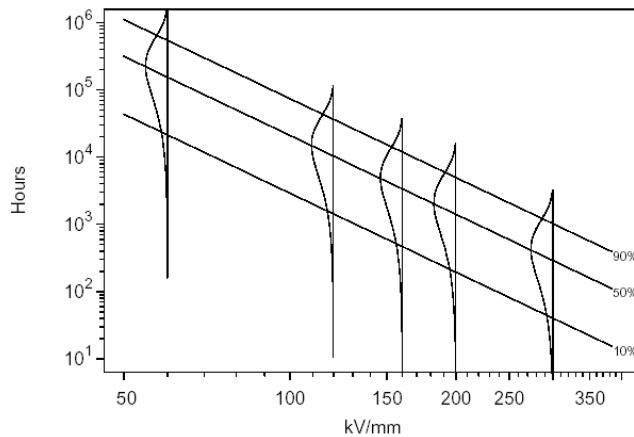
where

- $\mu = \beta_0 + \beta_1 x$, and
- $x = \log(\text{Voltage Stress})$.
- σ assumed to be constant.

Example Weibull Inverse Power Relationship Between Life and Voltage Stress

$$\log[t_p(\text{volt})] = \beta_0 + \beta_1 x + \Phi_{\text{sev}}^{-1}(p)\sigma$$

$$\log T = \beta_0 + \beta_1 x + \sigma Z, \quad Z \sim \text{Gumbel}(0,1)$$



Other Commonly Used Life-Stress Relationships

Other commonly used SAFT models have the simple form:

$$T(x) = \frac{T(x_U)}{\mathcal{AF}(x)}$$

where $\mathcal{AF}(x) = \mathcal{AF}(x, x_U, \beta_1) = \exp[\beta_1(x - x_U)]$. β_1 is a material characteristic.

Examples include:

- Cycling rate: $x = \log(\text{frequency})$.
- Current density: $x = \log(\text{current})$.
- Size: $x = \log(\text{thickness})$.
- Humidity 1: $x = \log(\text{RH})$, RH is % relative humidity/100.
- Humidity 2: $x = \log[\text{RH}/(1 - \text{RH})]$.

Some of these models are empirical. For a location-scale time-to-failure distribution $\mu = \beta_0 + \beta_1 x$.

Eyring Temperature Relationship

- Arrhenius relationship obtained from **empirical** observation.
- Eyring developed **physical theory** describing the effect that temperature has on a reaction rate:

$$\mathcal{R}(\text{temp}) = \gamma_0 \times A(\text{temp}) \times \exp\left(\frac{-E_a}{k_B \times \text{temp K}}\right)$$

- $A(\text{temp})$ is a function of temperature depending on the specifics of the reaction dynamics; γ_0 and E_a are constants.
- Applications in the literature have used $A(\text{temp}) = (\text{temp K})^m$ with a fixed value of m ranging between $m = 0$ (Boccaletti et al. 1989), $m = .5$ (Klinger 1991a), to $m = 1$ (Nelson 1990a and Mann Schafer and Singpurwalla 1974). Difficult to identify m from limited data.
- Eyring showed how to include other accelerating variables.

The Eyring Regression Model (e.g., for Weibull or Lognormal Distribution)

The Eyring temperature-acceleration regression model is

$$\Pr(T \leq t; \text{temp}) = \Phi \left[\frac{\log(t) - \mu}{\sigma} \right]$$

where

- $\mu = -m \log(\text{temp}^\circ\text{C} + 273.15) + \beta_0 + \beta_1 x$.
- $x = 11605/(\text{temp}^\circ\text{C} + 273.15)$.
- $\beta_1 = E_a$ is the activation energy.
- m is usually given; σ is constant, but usually unknown.
- With $m > 0$, Arrhenius provides a useful first order approximation to the Eyring model, with conservative extrapolation to lower temperatures.

Humidity Acceleration Models

- Useful for accelerating failure mechanisms involving **corrosion** and certain other kinds of **chemical degradation**.
- Often used in conjunction with elevated temperature.
- Most humidity models have been developed empirically.
- Empirical and limited theoretical results for corrosion on thin films (Gillen and Mead 1980, Peck 1986, and Klinger 1991b) suggest the use of RH instead of P_v (vapor pressure) as the independent (or experimental) variable in humidity relationships when temperature is also to be varied.
- RH is the preferred variable because the change in life, as a function of RH, does not depend on temperature. That is,

$$\frac{\partial^2 \text{Life}}{\partial \text{RH} \partial \text{temp}} = 0$$

or no **statistical interaction**.

Humidity Regression Relationships

- Consider the Weibull/lognormal lifetime regression model

$$\Pr(T \leq t; \text{humidity}) = \Phi_{\text{sev}} \left[\frac{\log(t) - \mu}{\sigma} \right]$$

where $\mu = \beta_0 + \beta_1 x_1$ and σ is constant. Letting $0 < \text{RH} < 1$ denote **relative humidity**, possible humidity relationships are:

- ▶ $x_1 = \text{RH}$ [Intel, empirical].
 - ▶ $x_1 = \log(\text{RH})$ [Peck, empirical].
 - ▶ $x_1 = \log[\text{RH}/(1 - \text{RH})]$ [Klinger, corrosion on thin films].
- For temperature and humidity acceleration, possible relationships include

$$\begin{aligned} \mu &= \beta_0 + \beta_1 x_1 + \beta_2 x_2 \\ \text{or } \mu &= \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2 \\ \text{or } \mu &= \beta_0 + \beta_2 x_2 + \beta_3 x_1 x_2 \end{aligned}$$

where $x_2 = 11605/(\text{temp}^\circ\text{C} + 273.15)$.

Temperature/Humidity Acceleration Factors with RH and Temp K and No Interaction

- Peck's relationship

$$\begin{aligned} \mathcal{AF}(\text{temp}, \text{RH}) &= \frac{\mathcal{R}(\text{temp}, \text{RH})}{\mathcal{R}(\text{temp}_U, \text{RH}_U)} \\ &= \left(\frac{\text{RH}_U}{\text{RH}} \right)^{\beta_1} \exp \left[E_a \left(\frac{11605}{\text{temp}_U \text{ K}} - \frac{11605}{\text{temp K}} \right) \right]. \end{aligned}$$

- Klinger's relationship

$$\begin{aligned} \mathcal{AF}(\text{temp}, \text{RH}) &= \frac{\mathcal{R}(\text{temp}, \text{RH})}{\mathcal{R}(\text{temp}_U, \text{RH}_U)} \\ &= \left[\left(\frac{\text{RH}_U}{1 - \text{RH}_U} \right) \left(\frac{1 - \text{RH}}{\text{RH}} \right) \right]^{\beta_1} \times \\ &\quad \exp \left[E_a \left(\frac{11605}{\text{temp}_U \text{ K}} - \frac{11605}{\text{temp K}} \right) \right]. \end{aligned}$$

Thermal Cycling

- Fatigue is an important failure mechanism for many products and materials.
- Mechanical expansion and contraction from thermal cycling can lead to fatigue cracking and failure.
- Applications include:
 - ▶ Power-on/power-off cycling of electronic equipment and effect on component encapsulation and solder joints.
 - ▶ Take-off power-thrust in jet engines and its effect on crack initiation and growth in fan disks.
 - ▶ Power-up/power-down of nuclear power plants and effect on the growth of cracks in heat generator tubes.
 - ▶ Thermal inkjet printhead delamination could be caused by temperature cycling during normal use.

Coffin-Manson Relationship

- The **Coffin-Manson** relationship says that the **typical** number of cycles to failure is

$$N = \frac{\delta}{(\Delta\text{temp})^{\beta_1}}$$

where Δtemp is the temperature range and δ and β_1 are properties of the material and test setup. This power-rule relationship explains the effect that temperature range has on thermal-fatigue life. For some metals, $\beta_1 \approx 2$.

- Letting T be the random number of cycles to failure (e.g., $T = N \times \epsilon$ where ϵ is a random variable), the acceleration factor when Δtemp , relative to the number of cycles when Δtemp_U , is

$$\mathcal{AF}(\Delta\text{temp}) = \frac{T(\Delta\text{temp}_U)}{T(\Delta\text{temp})} = \left(\frac{\Delta\text{temp}}{\Delta\text{temp}_U} \right)^{\beta_1}$$

- There may be a Δtemp threshold below which little or no fatigue damage is done during thermal cycling.

Generalized Coffin-Manson Relationship

- Empirical evidence has shown that effect of temperature cycling can depend importantly on $\text{temp}_{\text{max}} \text{K}$, the maximum temperature in the cycling (e.g., if $\text{temp}_{\text{max}} \text{K}$ is more than .2 or .3 times a metal's melting point).
- The effect of temperature cycling can also depend on the cycling rate (e.g., due to heat buildup).
- An **empirical** extension of the Coffin-Manson relationship is

$$N = \frac{\delta}{(\Delta \text{temp})^{\beta_1}} \times \frac{1}{(\text{freq})^{\beta_2}} \times \exp\left(\frac{E_a \times 11605}{\text{temp}_{\text{max}} \text{K}}\right)$$

where freq is the cycling frequency, and E_a is an activation energy.

- Caution must be used when using such a model outside the range of available data and past experience.

Other Topics in Chapter 18

- Other accelerated degradation models and relationships to accelerated time models
- Discussion of stress-cycling models
- Other models for two or more experimental factors.